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A Location Problem of Fast Charging Stations Considering Congestion

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ARTICLE INFO	ABSTRACT
	This paper considers locating congested fast charging stations (FCSs) and deploying chargers in a stochastic environment, while the related studies have predominantly focused on problems in deterministic environments. Reducing the inconvenience caused by congestion at FCSs is an important challenge for FCS service provider. This is the underlying motivation for this study to consider a problem for FCS network design with the congestion restriction in a stochastic environment. We proposed a maximal coverage problem subject to budget constraints and a congestion restriction in order to maximize the demand coverage. With the derivation of the congestion restriction in the considered stochastic environment, the problem is formulated into an integer programming model. A real-life case study is conducted and managerial implications are drawn from its results.

1. Introduction

Electric Vehicles (EVs) have received much attention as eco-friendly vehicles to replace combustion engine vehicles. Combustion engine vehicles have caused several environmental problems. They require fossil energy that is being depleted. Such vehicles emit greenhouse gases and cause severe air pollution. However, since EVs, which are divided into hybrid EVs, plugin hybrid EVs, and battery-powered EVs, use little fossil energy or none at all, governments in many countries are promoting EV use by offering various incentives. This is due to environmental problems being averted or improved as a result of replacing combustion engine vehicles with EVs. However, people seem reluctant to adopt EVs since charging EVs require longer charging times compared with refuelling combustion engine vehicles.

Fast charging technologies, referred to as the level 3 charging standard, such as the Combined Coupler Standard (CCS) and the CHArge de Move standard (CHAdeMO), have reduced charging time substantially compared with previous charging standards. The previous level 1 and level 2 charging standards require 11-36 hours and 2-3 hours for full charging, respectively, but fast charging technologies require 0.2-1 hours to charge 80% of EV battery capacity (Yilmaz and Krein, 2013). Therefore, deploying fast charging stations (FCSs) on a large scale is being considered as an effective solution to mitigate the inconvenience caused by long charging times and to increase the EV adoption rate. For example, the Korean government has deployed many FCSs for public charging service and plans to deploy more. In Korea, 337 public FCSs are have been deployed as of 2015 and 300 additional FCSs are planned to be introduced by 2017 (Ministry of Environment of South Korea, 2015). Therefore, the design problem regarding the service network of FCSs is a timely and urgent issue. In particular, we focus on the problem in a stochastic environment.

Related studies are reviewed concerning network design problems for charging or refuelling service facilities and problems, which can be divided into inter-city and intra-city problems according to whether the charging facilities support long-distance or short-distance trips (Ghamami et al., 2016). In this context, the word "city" represents a small

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region for general trips to be completed via single-stop refuelling.

Inter-city problems focus on multi-stop refuelling for long-distance origin-destination (O-D) trips to be completed on a large network with many nodes representing cities. Inter-city problems have been extensively studied in deterministic environments as follows. Kuby and Lim (2005) and Kim and Kuby (2012) have studied on inter-city problems to maximize the total coverage for flow-based demands, each of which is associated with a path and the number of vehicles on it, with a fixed number of facilities. Furthermore, Wang and Lin (2009, 2013) and Li and Huang (2014) examined inter-city problems to minimize the number of stations while satisfying all flow-based demands. On the other hand, studies on inter-city problems in stochastic environments are very limited. Hosseini and MirHassani (2015) considered the stochastic nature in the actual number of service requests. However, the stochastic nature in the actual timing of service requests and the service time at facilities were neglected. The authors dealt with a problem to maximize the expected coverage in the situation where there are two types of stations (permanent and portable) and the actual number of service requests for each flow-based demand varies with scenarios each having its occurring probability.

On the other hand, intra-city problems consider a network representing a small region such as a metropolitan city or a few small cities. In the network, nodes represent administrative districts or specific locations within the region. Since a small region is considered, the problems assume single-stop refuelling for refuelling demands originating from nodes during the analysis time period or short-distance O-D trips. Similar to inter-city problems, most intra-city problems have been studied in a deterministic environment as follows. Dashora et al. (2010) and Zhu et al. (2016) considered capacitated fixed-charge location problems in order to minimize the total construction cost of charging stations and the travel costs for node-based demand. Here, node-based demand indicates demand originating from nodes. In addition, Frade et al. (2011) and Cavadas et al. (2015) dealt with capacitated maximal coverage problems in order to maximize the coverage weighted with the associated traveling cost for node-based demands. Moreover, Zockaie et al. (2016) considered a problem of locating fuel stations in order to minimize time associated with refuelling detours while satisfying all flow-based demands via single-stop refuelling. In addition, an intra-city problem in a stochastic environment was also studied. Ghamami et al. (2016) considered a capacitated fixed-charge location problem where node-based demand varies according to the scenario. However, similarily to Hosseini and MirHassani (2015), they considered the stochastic nature in the actual number of service requests, but neglected the stochastic nature in the actual timing of service requests and the service time at facilities.

Although various problems for locating refuelling or charging facilities have been considered, as shown in the above literature review, most have primarily been examined in a deterministic environment. Limited research has dealt with problems in a stochastic environment. In the inter-city problem of Hosseini and MirHassani (2015) and the intra-city problem of Ghamami et al. (2016), only the actual number of service requests was considered as a stochastic element. They neglected the stochastic nature in the actual timing of service requests and the service time at facilities However, facilities serving customers become congested more severely as more customers arrive, customers arrive at more similar timings and the service time become longer. Therefore, we fill this research gap by considering the actual timing of service requests and the service time at facilities as well as the actual number of service requests as stochastic elements in the proposed intra-city problem. In order to reflect the real-life service time elapsed at FCSs, the service time following a general distribution is considered.

We deal with an intra-city problem for locating congested FCSs and deploying chargers in a stochastic environment in which service time at FCSs follows a real-life general distribution. EV users using FCSs that takes about 30 minutes to fully charge generally charge their EVs in the middle of O-D trips as in gasoline refuelling (Yang, 2016). To reflect such a behavior, flow-based demand, which is associated with the O-D pair and the average rate of the O-D trip possibly requiring to charge (i.e., the average rate of charging requests) over the analysed time period, is assumed. It is assumed that charging requests follow a homogeneous Poisson process (that is, inter-arrival time of charging requests follows an exponential distribution). The objective is to maximize the coverage defined as the sum of the average rates of charging requests for served demands while satisfying the congestion restriction at FCSs. The proposed problem is formulated as an integer programming model and with the model, we conduct a real-life case study during peak times on weekdays when EV users are the most sensitive to congestion.

This paper is organized as follows. The next section presents the problem description and explains the main modelling elements. The problem is formulated into an integer programming model in Section 3. Section 4 presents a case study based in Jeju Island, South Korea using the model. Based on the case study, managerial implications are drawn. Finally, conclusions are provided in section 5.

2. Problem description

We model charging demands during the analysis time period. Since congestion at FCSs is considered in this paper, peak times during the day is appropriate for the analysis time period. We focus on peak times on weekdays, since EV mobility on weekends is deeply related to EV users' activities in their spare time. It is known that the evening rush hour on weekdays generally correspond to peak times (Idaho National Laboratory, 2015). Moreover, since O-D trips during

the evening rush hour correspond to commuting trips from workplaces to residences, we focus on commuting trips during the evening rush hour.

We make the following assumptions during the evening rush hour. Flow-based demands, each of which is associated with the O-D pair and the average rate of charging requests (i.e. the average rate of the O-D trip possibly requiring to charge), are assumed to be known. Each demand can be served while satisfying the following two restrictions: (1) each demand can be served in open FCSs on paths within a pre-specified deviation (which is called the maximum detour distance in this paper) from the shortest path (the detour distance restriction); (2) if, and only if, there are any open FCSs satisfying the detour distance restriction for some demand, the demand should be served at one of such FCSs (the service acceptance restriction). The maximum detour distance is assumed known and identical for all demands. Moreover, it is assumed that such charging requests follow a homogeneous Poisson process (that is, inter-arrival time of charging requests follows an exponential distribution), one or more chargers can be deployed in a FCS, the time (i.e., charging duration) served by deployed chargers at a FCS follows a real-life general distribution, and there is enough parking space at FCSs. Also, we assumed that there is enough space for EVs to wait in a FCS, and EV users do not give up the charge while wait for charging. Therefore, in this paper, a FCS is modelled as the M/G/k queuing system in which M, G, and k represent a Poisson arrival process (that is, the inter-arrival time distribution is an exponential distribution), a general service time distribution, and the number of chargers to be determined, respectively.

Under the above assumptions, we proposed a problem to determine the following three decision issues: (1) locating FCSs at given candidate nodes (FCS location decision), (2) deploying chargers at FCSs (charger deployment decision), and (3) determining charging demands to be served and allocating them to open FCSs (demand allocation decision). There are costs incurred by the two former decisions as follows: land acquisition costs for the FCS location decision, which is dependent on the FCS construction area, and charger install costs, which is dependent on the total number of chargers installed at the constructed FCSs. The sum of these costs must not exceed the given budget. Congestion should be managed such that the congestion restriction, $\langle \alpha, \beta \rangle$-CR, for given target parameters α and β is satisfied at operating FCSs. The restriction implies that the probability of the waiting time exceeding the target value α is not greater than the target probability β. However, it is difficult to derive the restriction exactly since an FCS is modelled as a M/G/k queuing system. For a M/G/k queuing system, no exact analytical method has been found to obtain stationary probabilities for waiting times in a queue. Therefore, we use the approximate formula proposed by Kimura (1994), and derive the congestion restriction which looks like a capacity constraint. The derivation will be explained in more detail later. While satisfying the restrictions explained above, the related decisions are determined to maximize the demand coverage, which is defined as the sum of the average rates of charging requests (i.e. the average rate of the O-D trip possibly requiring to charge) for served demand.

3. Formulation

In this section, the described problem to locate congested FCSs and deploy chargers, called PLCFDC, is formulated as an integer programming model. To formulate the problem [PLCFDC], the following notations are used:

Sets and indices

Q, q Demand set and the index (i.e., $q \in Q$)

I FCS candidate node set

Q_i Subset of Q, each demand of which can be serviced at FCS candidate node $i \in I$ within the maximum detour distance (i.e., $Q_i = \{q \in Q | d_{qi} \leq \bar{d}\}$ for $i \in I$ (refer to the definitions of d_{qi} and $\bar{d}$ to be explained soon))

I_q Subset of I, each FCS candidate node of which can service demand $q \in Q$ within the maximum detour distance (i.e., $I_q = \{i \in Q | d_{qi} \leq \bar{d}\}$ for $q \in Q$ (refer to the definitions of d_{qi} and $\bar{d}$ to be explained soon))

N, i, j Demand node set defined by $N = \bigcup_{q \in Q} \{O_q, D_q\}$ and the indices for I or N (i.e., $i, j \in I \cup N$)

K_i, k Set of the number of chargers to be possibly deployed for FCS candidate node $i \in I$ and the index (i.e., $k \in K_i$)

Parameters

Q_q, D_q Origin and destination node for demand $q \in Q$

d_{ij}, d_{qi} Distance between two nodes $i \in I \cup N$ and $j \in I \cup N$ on a road network, and detour distance, defined as $d_{O_q i} + d_{i D_q} - d_{O_q D_q}$, for demand $q \in Q$ to be serviced at FCS candidate node $i \in I$ [unit: km]

$\bar{d}$ Maximum detour distance required for demand [unit: km]

S, W Random variables representing service time, or charging duration, and waiting time at an FCS when a queuing analysis is conducted [unit: min]

α, β	The targeted value of the waiting time at an FCS and the targeted probability for the congestion restriction $\langle \alpha, \beta \rangle$-CR (i.e. Prob[$W > \alpha$] $\leq \beta$) [unit: min and %]
λ_q, μ	Average rate of charging requests for demand q and average service rate at an FCS (i.e. $\mu = 1/E(S)$) [unit: number/min]
$\lambda_{max,k}$	Maximum rate of charging requests to satisfy $\langle \alpha, \beta \rangle$-CR when k chargers are deployed in an FCS [unit: number/min]
f_i, g	Land acquisition cost for FCS $i \in I$ and charger deployment unit cost [unit: $]
H	Maximum allowable budget for constructing the FCS service network [unit: $]
B	A big number

Decision variables

$\bar{X}_q$	1, if demand $q \in Q$ is serviced at any FCS in I; 0 otherwise
X_{qi}	1, if demand $q \in Q$ is charged at FCS $i \in I$; 0 otherwise
Y_i	1, if FCS $i \in I$ is opened; 0 otherwise
Z_{ik}	1, if $k \in K_i$ chargers are deployed at FCS $i \in I$; 0 otherwise

The [PLCFDC] is now formulated as the following integer programming model:

[PLCFDC] Maximize $\quad \sum_{q \in Q} \lambda_q \bar{X}_q$ (1)

subject to

$$\sum_{i \in I_q} X_{qi} = \bar{X}_q \qquad \forall q \in Q \qquad (2)$$

$$Y_i \geq X_{qi} \qquad \forall q \in Q, i \in I_q \qquad (3)$$

$$\bar{X}_q \geq Y_i \qquad \forall i \in I, q \in Q_i \qquad (4)$$

$$Y_i = \sum_{k \in K_i} Z_{ik} \qquad \forall i \in I \qquad (5)$$

$$\sum_{q \in Q_i} \lambda_q X_{qi} \leq \sum_{k \in K_i} \lambda_{max,k} Z_{ik} \qquad \forall i \in I \qquad (6)$$

$$\sum_{i \in I} f_i Y_i + \sum_{i \in I} \sum_{k \in K_i} kg Z_{ik} \leq h \qquad (7)$$

$$\bar{X}_q \in \{0,1\} \qquad \forall q \in Q \qquad (8)$$

$$X_{qi} \in \{0,1\} \qquad \forall q \in Q, i \in I \qquad (9)$$

$$Y_i \in \{0,1\} \qquad \forall i \in I \qquad (10)$$

$$Z_{ik} \in \{0,1\} \qquad \forall i \in I, k \in K_i \qquad (11)$$

The objective function (1) is to maximize the coverage, which is defined as the sum of the average rates of charging requests for served demands. Constraints (2) represent the detour distance restriction implying that if demand q is served in the FCS service network, the serving FCS must be within the detour distance restriction. Constraints (3) imply that for demand q to be serviced at FCS i, FCS should be available. Constraints (4) represent the service acceptance restriction implying that if FCS i is available, all demands in Q_i should be serviced in the FCS service network. Constraints (5) state that if FCS i is available, some number of chargers should be deployed; otherwise, no chargers should be deployed. Constraints (6) represent the congestion restriction $\langle \alpha, \beta \rangle$-CR, defined as Prob[$W > \alpha$] $\leq \beta$ for $\forall i \in I$. The derivation for the constraint will be explained in detail later. Constraints (7) represent the budget restriction that the sum of the land acquisition cost and the charger deployment cost should not exceed the total budget. Constraints (8), (9), (10), and (11) represent the binary conditions for variables $\bar{X}_q$, X_{qi}, Y_i, and Z_{ik}, respectively.

For now, we explain the derivation of Constraints (6) and how to determine proper values for $\lambda_{max,k}$. As mentioned, for a M/G/k queuing system in which an FCS is modelled, no exact analytical method has been found to obtain stationary probabilities for waiting times in a queue. To determine such probabilities, we use an approximate formula which seems appropriate for the distribution of charging duration (i.e., service time) at FCSs. Recent statistics, such as the Idaho National Laboratory (2015) and Morrissey et al. (2016), show that C_v^2 for the distribution of charging duration at FCSs is much less than 1. Here, C_v^2 denoted the squared coefficient of variation, and is calculated as the variance divided by the squared mean (i.e., if S is a random variable $C_v^2 = Var(S)/E(S)^2$). For the distribution with $C_v^2 < 1$, the approximate formula proposed by Kimura (1994) is adopted in this paper. Since the author used another

approximate formula by Kimura (1986) to obtain the mean queue length and the mean waiting time in an M/G/k queuing system. The experimental test in Kimura (1994) showed that the formula by Kimura (1986) generally performs well and provides reliable results, especially for distributions with $C_v^2 \leq 2.5$. Therefore, we use the approximate formula proposed in Kimura (1994) to obtain stationary probabilities, which are used to determine the appropriate values for $\lambda_{max,k}$, for the waiting time in a queue at a FCS.

First, the stationary probability of waiting time W being no more than t in a M/G/k queuing system is denoted by $P(W \leq t)$ or P_t. In addition, let $P_t(\cdot)$ be such a probability in the queuing system in the brackets. Also, $EW(\cdot)$ denotes the mean waiting time for the queuing system in the brackets, λ the arrival rate, and ρ the traffic intensity calculated by $\lambda/(k\mu)$. Moreover, let us define the following ratios: $R_G(\rho,k) \equiv EW(M/G/k)/EW(M/M/k)$ and $R_G(\rho,k) \equiv EW(M/D/k)/EW(M/M/k)$. Here, in the Kendall notation, M/M/k denotes a queuing system in which inter-arrival time and service time follow the associated exponential distributions, and the number of servers is k denoting a multiple value. In queuing system M/D/k, the service time distribution is deterministic unlike M/M/k, but the rest is the same as M/M/k. Using these notations, an approximate formula for P_t by Kimura (1994) is presented as follows:

$$P_t \cong 1 - \exp\left\{-\frac{k\mu(1-\rho)t}{R_G(\rho,k)}\right\}\left[\frac{(k\rho)^k}{k!(1-\rho)}\right]\left[\sum_{j=0}^{k-1}\frac{(k\rho)^j}{j!}+\frac{(k\rho)^k}{k!(1-\rho)}\right]^{-1}, t \geq 0 \cdot \tag{12}$$

In this context, Kimura (1994) adopted an approximate formula by Kimura (1986) to determine $R_G(\rho,k)$ as follows.

$$R_G(\rho,k) \cong \frac{1+C_v^2}{2C_v^2+(1-C_v^2)/R_D(\rho,k)} \quad \text{for } k > 1; \quad R_G(\rho,k) = 0.5 \quad \text{for } k = 1, \tag{13}$$

$$R_D(\rho,k) \cong \frac{1}{2}\left\{1+\frac{\theta}{8(1+\theta)}\left(\sqrt{\frac{9+\theta}{1-\theta}}-2\right)\left(\frac{\rho}{1-\rho}\right)\xi(a(\rho),\theta)\eta(\rho,b(\theta))\right\}, \tag{14}$$

where $\theta = k-1/k+1$, $\xi(x,\theta) = \sqrt{1-\exp\{-(1-\theta)x/\theta\}}$ for any value x, $a(\rho) = 25.6/\{\eta(\rho,25.6/2.2)(1-\rho)/\rho\}^2$, $\eta(\rho,y) = 1-\exp\{-\rho y/1-\rho\}$ for any value y, and $b(\theta) = \theta/[\{\theta/8(1+\theta)\}(\sqrt{9+\theta/1-\theta}-2)\xi(2.2,\theta)]$.

From the definition of P_t, $\langle\alpha, \beta\rangle$-CR can be represented as $P_\alpha \geq 1-\beta$. The total arrival rate for FCS $i \in I$ is the same as the left-hand side in Constraint (6), which has the same meaning as λ in the above queuing analysis. Given k, μ, and α, P_α is clearly a monotonically decreasing function of λ, which can be understood intuitively. Thus, each $\lambda_{max,k}$ is the value of λ satisfying $P_\alpha = 1-\beta$, which can be obtained utilizing a search method, such as a binary search or the solution-finding functionality in the MS Excel. This calculation is done in the pre-processing step for the proposed models.

4. Case study

4.1 Data gathering and parameter setting

This section presents the case study where the proposed model is applied. In the case study, the proposed model is used to locate public FCSs in Jeju Island, South Korea. Most trips on this island can be completed via single-stop refuelling since the longest coastal road on the island is only 176 km long. Jeju Island features two cities, Jeju city in the north and Seogwipo in the south. Jeju city and Seogwipo are composed of 26 and 17 administrative subdivisions, respectively. Except for two small island subdivisions of Jeju city, the administrative office locations for the remaining 41 subdivisions on the main island were considered as demand nodes (i.e., $|N| = 41$). We considered all ordered pairs of subdivisions with repetition as demands (i.e., $|Q| = 41 \times 41 = 1681$). As candidate locations for FCSs, 82 locations were selected, which included joint locations of main roads, rest areas along main roads, and main public parking lots (i.e., $|I| = 82$). The locations for demand and FCS candidate nodes on the road network are shown in Fig. 1, and subdivisions represented by demand nodes are shown in Table 1.

The land acquisition cost f_i was set to $100,000 or $50,000 according to whether an FCS candidate node i is located in a downtown area or in a suburban area, and the charger deployment unit cost g was set to $40,000 USD. Since 2-4 chargers are generally deployed at each FCS on the island, such a deployment was reflected by assuming $K_i = \{2,3,4\}$ for $\forall i \in I$. To determine values for $\lambda_{max,k}$ in Constraint (7), we used an approximate approach in which the

solution-finding functionality in MS Excel was used as a search method of $\lambda_{max,k}$ values, and the approximate formula (12) for P_α is used. From this, we computed $\lambda_{max,k}$ for all combinations of $k = \{2,3,4\}$ and $(\alpha, \beta) \in \{(10min, 10\%),(15min,10\%)\}$, as shown in Table 2. Meanwhile, using Google Maps, we obtained the distance between all nodes on the real road network.
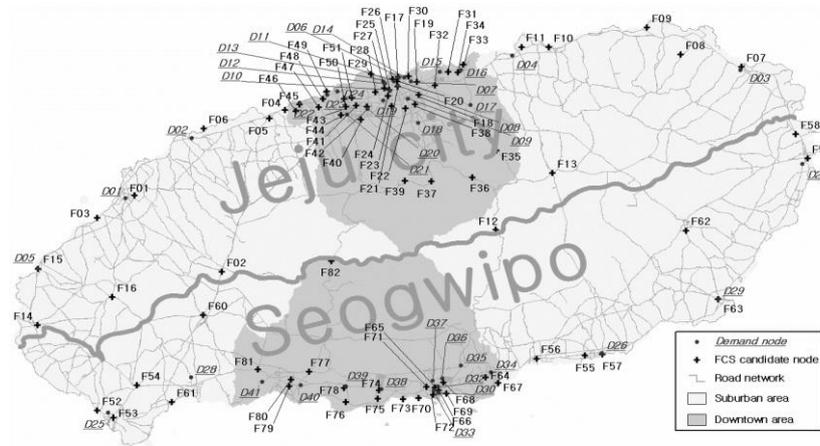
Figure 1. Demand and FCS candidate nodes in Jeju island.

Table 1. Subdivision represented by each demand node, and its outgoing and incoming rates.

Subdivision	Demand node	Outgoing rate	Incoming rate	Subdivision	Demand node	Outgoing rate	Incoming rate
Hallim-eup	D01	0.1115	0.1387	Oedo-dong	D22	0.0416	0.0825
Aewol-eup	D02	0.1429	0.1781	Iho-dong	D23	0.0157	0.0161
Gujwa-eup	D03	0.0934	0.1281	Dodu-dong	D24	0.0327	0.0145
Jocheon-eup	D04	0.1000	0.1074	Daejeong-eup	D25	0.1070	0.1612
Hangyeong-myeon	D05	0.0421	0.0735	Namwon-eup	D26	0.0928	0.1443
Ildo 1-dong	D06	0.0419	0.0139	Seongsan-eup	D27	0.1011	0.1352
Ildo 2-dong	D07	0.0992	0.1305	Andeok-myeon	D28	0.0626	0.0738
Ido 1-dong	D08	0.0701	0.0283	Pyoseon-myeon	D29	0.0904	0.0980
Ido 2-dong	D09	0.2473	0.1816	Songsan-dong	D30	0.0298	0.0346
Samdo 1-dong	D10	0.0561	0.0520	Jeongbang-dong	D31	0.0376	0.0215
Samdo 2-dong	D11	0.0477	0.0310	Jungang-dong	D32	0.0723	0.0311
Yongdam 1-dong	D12	0.0336	0.0291	Cheonji-dong	D33	0.0483	0.0292
Yongdam 2-dong	D13	0.0795	0.0578	Hyodon-dong	D34	0.0235	0.0380
Geonip-dong	D14	0.0489	0.0358	Yeongcheon-dong	D35	0.0636	0.0367
Hwabuk-dong	D15	0.0887	0.1041	Donghong-dong	D36	0.1419	0.1667
Samyang-dong	D16	0.0332	0.0859	Seohong-dong	D37	0.0582	0.0750
Bonggae-dong	D17	0.0189	0.0158	Daeryun-dong	D38	0.0782	0.0993
Ara-dong	D18	0.0970	0.0992	Daecheon-dong	D39	0.0290	0.0621
Ora-dong	D19	0.0602	0.0392	Jungmun-dong	D40	0.0636	0.0731
Yeon-dong	D20	0.3007	0.1533	Yerae-dong-dong	D41	0.0596	0.0288
Nohyeong-dong	D21	0.2422	0.1996				
						3.3046	3.3046

We considered the estimated number of passenger EVs for non-business purposes on the island at the end of 2017. The EV penetration rate of the island is the highest at 1% in Korea in September, 2016. Passenger EVs for non-business purposes account for over 80% of all EVs in operation, with a total of 3,608 EVs in operation on Jeju Island. They accounted for 3,128 of the total EVs. The number of passenger EVs is expected to exceed 12140 at the end of 2017, corresponding to the expected EV penetration rate of 4% estimated by the Jeju Special Self-Governing Province (2016).

Due to confidentiality issue, we could not obtain the average rate λ_q of charging requests and the average service rate μ at FCSs for Jeju island. Instead, this study considers the data which Idaho National Laboratory (2015) gathered from 100 FCSs regionally distributed in the United States for 2013 and published. This includes the charging duration distribution across weekdays in a probability mass function shown in Table 3. We assumed that the random service time variable S follows the probability mass function in this case study. From the Table, we calculated μ=0.048 and $C_v^2 = 0.0332$ for service time at FCSs. In addition, data from the Idaho National Laboratory (2015) shows that the peak time on weekdays is approximately 4:00 pm- 6:45 pm, and about 25% of charging events that happen on weekdays occur during this period (that is, the probability of fast charging during the peak time on weekdays is about 25%). Moreover, we observe that the average electricity used per charging event at FCSs was 8.4 kWh. We used this information in order

to estimate demand.

Table 2. $\lambda_{max,k}$ for $k \in \{2,3,4\}$ and $(\alpha, \beta) \in \{(10min, 10\%),(15min,10\%)\}$

K	(α, β)	
	(10min,10%)	(15min,10%)
2	0.0384	0.0452
3	0.0756	0.0860
4	0.1163	0.1293

Table 3. Probability mass function of service time.

S [min]	2.4	7.4	12.4	17.4	22.4	27.4	32.4	37.4	42.4	47.4	Otherwise
Probability [%]	14.01%	6.52%	11.17%	15.19%	16.76%	13.42%	11.15%	4.89%	3.42%	3.48%	0.00%

To estimate demand, we conducted the following procedure. First, henceforth, λ_q and λ_{ij} are used interchangeably, where $O_q = i$ and $D_q = j$. Each λ_{ij} was set as follows:

$$\lambda_{ij} = chgE_{ij}^{FCS,PT} \times \frac{1}{E(C)} \times \frac{1}{nMin^{PT}} , \qquad (15)$$

where $chgE_{ij}^{FCS,PT}$, $E(C)$ and $nMin^{PT}$ represent the average amount of electricity used to charge EVs at FCSs during peak times on weekdays, the average amount of electricity required per charging event, and the number of minutes during the peak time, respectively. $E(C)$ was set to 8.4 kWh, and $nMin^{PT}$ was set to 165 min (corresponding to the peak time from 16:00- 18:45). The rest, $chgE_{ij}^{FCS,PT}$, was estimated as:

$$chgE_{ij}^{FCS,PT} = cnsE_{ij} \times ratio^{FCS} \times ratio^{PT} , \qquad (16)$$

where $cnsE_{ij}$, $ratio^{FCS}$ and $ratio^{PT}$ represent the average electricity consumed to charge EVs on a weekday demand, the average electricity usage ratio of the public FCS service network to all charging facilities, and the average ratio of charging electricity usage of the peak time to the whole weekday, respectively. $ratio^{PT}$ was set to 0.25, and $ratio^{FCS}$ was set as 0.2 as recommended by Schroeder and Traber (2012). $cnsE_{ij}$ was set as:

$$cnsE_{ij} = n_{ij} \times (d_{ji} + d_{ij} + \Delta) \times \frac{1}{effE} , \qquad (17)$$

where n_{ij}, Δ and $effE$ represent the estimated number of EVs for the corresponding demand, the adjusted distance such that $\sum_{i \in N} \sum_{j \in N} n_{ij}(d_{ji} + d_{ij} + \Delta)$ implies an estimate of the total EV driving distance on a weekday for the demands, and the average electricity efficiency in units of km/kWh, respectively. n_{ij} was estimated as:

$$n_{ij} = n_j \times ratio_i^{EMP} \quad \text{and} \quad n_j = n_{tot} \times ratio_j^{PP} , \qquad (18)$$

where n_j, $ratio_i^{EMP}$, n_{tot} and $ratio_j^{PP}$ represent the number of non-business purpose passenger EVs of people residing in subdivision j, the ratio of the number of employees working in subdivision i to the total employees in all 41 subdivisions, the total number of non-business-purpose passenger EVs on Jeju Island, and the ratio of the population residing in subdivision j to that in all 41 subdivisions, respectively. n_{tot} was set to 12,140 (the 2017 non-business EV estimate), and $ratio_j^{PP}$ and $ratio_i^{EMP}$ were calculated using statistics provided by the Korean Statistical Information Service (website: http://kosis.kr). After reallocating 1 EV to demands with less than 1 EV and rounding off the number of non-business passenger EVs for each demand, the total number was 12,089. The average EV driving distance d_{WD} per weekday was reported as 53.0 km by the Jeju Special Self-Governing Province (2015). Δ was set to 7.4 km, which was obtained using the following EV driving distance conservation equation:

$$\sum_{i \in N} \sum_{j \in N} n_{ij} d_{WD} = \sum_{i \in N} \sum_{j \in N} n_{ij} (d_{ji} + d_{ij} + \Delta) \quad . \tag{19}$$

effE was set to 6.99 km/kWh, which is the market share weighted electricity efficiency for the top 3 EVs based on the local market share. The sum of the market shares for the 3 EVs exceeds 75%. The subdivisions, outgoing rate ($\sum_{j \in N} \lambda_{ij}$), and incoming rate ($\sum_{j \in N} \lambda_{ji}$) for demand node i are reported in Table 1. Due to (18), these rates are proportional to $ratio_i^{EMP}$ and $ratio_i^{PP}$.

4.2 Solution analysis

The effects of the following factors are investigated: budget h, the maximum detour distance $\bar{d}$, and the congestion restriction parameter pair (α, β). We set the rest of the parameters consistent with the previous sections and optimally solved all the problem instances using a well-known integer programming solve, CPLEX 12.6.3.0.

The impacts of the maximum allowable budget h are first investigated. Optimal solutions of [PLCFDC] were obtained for various budgets (h = $1.0M, $1.5M, $2.0M, $2.5M, and $3.0M) with $\bar{d}$=1.0 km and (α, β) = (15 min, 10%), as shown in Table 4. According to the values of (α, β), $\lambda_{max,k}$ for $\forall k \in K = \{2, 3, 4\}$ was set to 0.0452, 0.0860 and 0.1293, respectively, as in Table 2. Optimal solutions for such problem instances is shown in Table 4.

Table 4. Optimal solutions of the problem [PLCFDC] for various budgets with $\bar{d}$=1.0 km and (α, β)=(15 min, 10%).

Budget	Total number of FCSs (suburban, downtown) [2 chargers, 3 cg., 4 cg.]	Number of FCSs opened also at the next budget level	Coverage [request/min]	Coverage growth from the previous budget level	Locations (number of chargers)
$1.0M	5(5, 0) [0, 2, 3]	3	0.5575		F07(4),F11(3),F56(4),F62(3),F63(4)
$1.5M	7(7, 0) [0, 0, 7]	3	0.8740	0.3165	F07(4),F11(4),F13(4),F53(4),F54(4), F56(4),F57(4)
$2.0M	9(7, 2) [0, 0, 9]	5	1.1630	0.2890	F01(4),F04(4),F05(4),F13(4),F50(4), F51(4),F56(4),F57(4),F63(4)
$2.5M	12(11, 1) [0, 2, 10]	8	1.4650	0.3020	F01(4),F04(4),F05(4),F07(4),F11(3), F13(4),F45(4),F53(4),F54(3),F55(4), F57(4),F62(4)
$3.0M	14(13, 1) [0, 0, 14]		1.7802	0.3152	F01(4),F02(4),F04(4),F05(4),F09(4), F13(4),F15(4),F44(4),F53(4),F54(4), F56(4),F57(4),F60(4),F62(4)

From the results of Table 4, we could made several interesting observations. First, FCSs with 4 chargers, rather than 2 chargers, and suburban FCSs, rather than urban FCSs appeared more in optimal solutions. For example, in the case of h = $1.0M, 3 FCSs with 4 chargers were selected, but no FCS with 2 chargers were chosen. Moreover, all FCSs units were located in suburban areas. Similarly, in the case of h = $3.0M, 4 chargers were deployed in all selected FCSs, and 13 suburban FCSs were identified, with only a single FCS chosen in a downtown area. This is due to the FCS opening cost for a suburban area is cheaper than in a downtown area, and FCSs with 4 chargers have the highest capacity per unit charger, which can be observed in Table 2. Second, a substantial portion of selected FCSs at a specific budget level were also selected at the next budget level. For example, 5 FCSs were selected for h = $1.0M, and among these, 3 FCSs were also selected at h = $1.5M. For h = $2.5M, 12 FCSs were constructed and 8 ones among them were opened at h = $3.0M. Based on this observation, the proposed model may make robust location decisions for different budgets. This is a desirable property for infrastructure building projects in which investments are made in several stages. In this case, budget for the initial investment is generally certain, but subsequent investment budgets are often uncertain. Therefore, the proposed model may be applied to such cases. Third, as the budget level increase, the resulting coverage becomes larger. However, coverage growth remained almost constant in the considered ranges for budget. This implies that the resulting coverage can be estimated easily at least for some budget level within the considered ranges for budget. Therefore, this observation can be used for efficient budgeting.
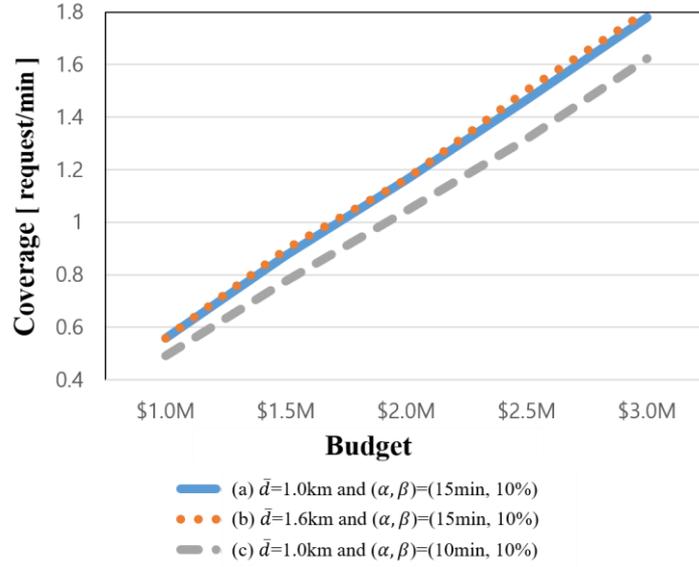
Figure 2. Optimal coverages for different budgets and the following settings: (a) $\bar{d}$=1.0 km and (α, β)=(15min, 10%); (b) $\bar{d}$=1.6 km and (α, β)=(1min, 10%); (c) $\bar{d}$=1.0 km and (α, β)=(10min, 10%).

The resulting coverages shown in Table 4 were compared with cases in which the maximum detour distance is larger than 1.0 km and the congestion restriction is tighter. Fig. 2 shows the resulting coverage trends at various budget levels and the following three cases: (a) the reference case of $\bar{d}$=1.0 km and (α, β)=(15 min, 10%); (b) $\bar{d}$=1.6 km and (α, β)=(15 min, 10%); and (c) $\bar{d}$=1.0 km and (α, β)=(10 min, 10%). From the results, the following observations were made. First, within the budget range considered in this study, linear trend lines can be seen for cases (a)-(c). As aforementioned, this can be used to estimate the coverage for a specific budget level or, conversely, the budget level needed to ensure desired coverage. Second, the trend lines for cases (a) and (b) were similar. This observation implies that the maximum detour distance seems to have little impact on coverage. This is very interesting in that the case of the typical maximal coverage problem with the service acceptance restriction (i.e., Constraints (4)) relaxed in the proposed model, the trend line for case (b), in which a relatively large detour is allowed, would be above that for case (a). This is obvious since the feasible region of case (b) includes that of case (a) in such a problem. Thus, these results may occur because the service acceptance restriction breaks the inclusion relation of the feasible regions between the two cases. Third, the trend line of case (c) is under that of the case (a). This is due to the congestion restriction in case (c) is tighter than in case (a), and hence, tight capacities or small values of $\lambda_{max,k}$ in Constraints (6) are considered in case (c).

Table 5. Optimal locations of the problem [PLCFDC] for various budgets with the two cases, (a) $\bar{d}$=1.6 km and ($\boldsymbol{\alpha}$, $\boldsymbol{\beta}$)=(15min, 10%) and (b) $\bar{d}$=1.0 km and ($\boldsymbol{\alpha}$, $\boldsymbol{\beta}$)=(10min, 10%), and locational comparison with the base case with $\bar{d}$=1.0 km and ($\boldsymbol{\alpha}$, $\boldsymbol{\beta}$)=(15 min, 10%)

Budget	(a)			(b)		
	Locations	Total number of FCSs	Number of FCSs opened also for the base case	Locations	Total number of FCSs	Number of FCSs opened also for the base case
$1.0M	F07,F10,F56,F62,F63	5	4	F07,F10,F55,F62,F63	5	3
$1.5M	F07,F11,F13,F53,F54, F56,F57	7	7	F07,F11,F53,F54,F56, F62,F63	7	5
$2.0M	F01,F04,F05,F06,F10, F11,F13,F15,F55,F57	10	5	F07,F11,F13,F15,F31, F35,F56,F57,F62	9	3
$2.5M	F01,F04,F05,F06,F07, F11,F13,F53,F54,F56, F57,F63	12	9	F02,F07,F11,F13,F53, F54,F56,F57,F60,F62, F63,F79	12	7
$3.0M	F01,F02,F04,F05,F06, F07,F11,F35,F53,F54, F56,F57,F60,F62	14	10	F01,F02,F04,F05,F07, F11,F13,F15,F53,F54, F55,F57,F60,F77	14	10

In addition, we investigated whether the proposed model makes robust location decisions for the maximum detour distance $\bar{d}$ and the congestion restriction parameter pair (α, β). Considering the case with $\bar{d}$=1.0 km and (α, β)=(15 min, 10%), for which the location decision is shown in Table 4 as the reference case, the location decision for the base case were compared with those for the case with $\bar{d}$=1.6 km and (α, β)=(15 min, 10%), and the case in which $\bar{d}$=1.0 km and (α, β)=(10 min, 10%). Specifically, we examined how many FCSs are opened identically for the same budget levels in each of the associated case pairs (that is, the base case and the case with $\bar{d}$=1.6 km and (α, β)=(15 min, 10%), or the base case and the case with $\bar{d}$=1.0 km and (α, β)=(10 min, 10%)). The results along with location decisions for each of the comparison cases were reported in Table 5.

As shown in Table 5, a substantial portion of open FCSs for budget levels in case (a) with a long detour allowed and in case (b) with tight congestion restriction tended to be opened also for the base case. For example, 5 FCSs were selected for h=\$1.0M in case (a) and 4 of these FCSs were selected for the same budget level in the base case. Moreover, for h=\$3.0M in case (b), 14 FCSs were selected and among these, 10 FCSs were also selected in the base case. Based on these results, although the estimation for the maximum detour distance or the congestion restriction parameter pair (α, β) deviates from real values, the proposed model may give a solution substantially similar to the solution associated with real values. That is, like the robustness of the proposed model against budget levels as discussed prior, the model seems to make robust location decisions also for the maximum detour distance and the congestion restriction parameter pair (α, β). The robustness property for the maximum detour distance is very desirable when considering the fact that an estimation work often gives a wrong answer. In addition, the robustness property for the tightness of the congestion restriction (i.e., for different values for (α, β)) is also helpful since the number of EVs to be considered inevitably changes and some tight congestion restriction given the current number of EVs corresponds to some loose congestion restriction given the number of EVs to be increased in the future.

Based on the above observations, various managerial implications are drawn as follows. First, there might be no significant difference of the resulting coverages for the case involving short detours permitted and that with longer detours permitted. This observation is interesting in that the typical maximal coverage problem without the service acceptance restriction would yield the opposite result. Second, the optimal location decision made by the proposed model seems robust in conjunction with the maximum detour distance as well as the tightness of the congestion restriction. Although planners carefully estimate the maximum detour distance, there is often error in these calculations. Thus, the robustness of the location decision against these parameters is desirable. Moreover, the location decision robust against the tightness of the congestion restriction is valuable since some tight congestion restriction given the current number of EVs corresponds to some loose congestion restriction in the future due to the increase in EVs.

5. Conclusions

A design problem for an FCS service network was considered. Specifically, the problem involves locating FCSs and deploying chargers subject to a budget restriction and a congestion restriction in order to maximize the demand coverage. In the considered stochastic environment, an FCS is modelled as a M/G/k queuing system. Since an exact analytic method has not been established for the queuing system, we used an effective approximate approach to derive the congestion restriction. This work is unique in that related studies have generally focused on deterministic problems. We formulated the proposed problem into an integer programming model and conducted a real-life case study. Various results from the case study are discussed and from the results, several managerial implications are drawn.

In future research, the proposed problem involving non-split demand could be extended to include split demand. That is, the proposed problem assumed that each demand is serviced at a single FCS with the shortest detour distance, but it may be more realistic for a large portion of each demand to be serviced at FCSs with relatively short detour distances and a small portion at FCSs with long detour distances. Moreover, to perform a case study using the proposed model, a well-known commercial solver was used, but it took more than 10 hours for some problem instances. Thus, an efficient heuristic approach is needed for problem instances involving more nodes (demand or FCS candidate nodes) or for larger budget levels.

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