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Dissecting the stock to flow model for Bitcoin

Thibaut G. Morillon
Department of Finance, Elon University, Elon, North Carolina, USA, and
Ryan G. Chacon
Department of Finance, University of Colorado Colorado Springs, Colorado Springs, Colorado, USA

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Abstract

Purpose – Perhaps the most popular pricing model among Bitcoin enthusiasts is the stock-to-flow (S2F) model. The model gained significant traction after successfully predicting the meteoric rise of Bitcoin prices from late 2020 to early 2021. This paper dissects the S2F model for Bitcoin empirically to determine its viability and investigate whether investors can profit from an S2F-based trading strategy.

Design/methodology/approach – This paper, dissects the S2F model for Bitcoin by putting it through a battery of tests to examine its design, characteristics, robustness and appropriateness.

Findings – Overall, this paper finds the S2F model to be insensitive to differing assumptions in the early stages of the model, alleviating concerns about data mining. This paper produces a dynamic S2F model with no peek-ahead bias and shows evidence that prediction accuracy increases over time. Finally, this paper shows that a dynamic trading strategy that goes long (short) when Bitcoin is undervalued (overvalued) according to S2F is far less profitable than a classic buy-and-hold strategy.

Originality/value – To the best of the authors' knowledge, this is the first paper to analyze the S2F model in an academic setting by providing a rigorous assessment of the model's construction. This paper demonstrates how the model can be implemented realistically without the peek-ahead bias, creating a tool that can be used contemporaneously by investors.

Keywords Bitcoin, Cryptocurrencies, S2F, Stock to flow

Paper type Research paper

1. Introduction

Perhaps no other asset is more polarizing among investors than Bitcoin. The forecasted value of one Bitcoin varies wildly among informed investors, and examples of extremes are abundant. Cathie Wood, chief investor of the ARK Innovation ETF, recently placed a price target of Bitcoin at $500,000 [1]. By contrast, Warren Buffet famously referred to Bitcoin as "rat poison squared." [2] How can informed investors have such stark differences in their opinion of Bitcoin's value?

One great challenge with valuing Bitcoin and other cryptocurrencies is that they are a distinct asset class. They do not produce cash that can be discounted to a present value, they are not a well-established method of payment or currencies, and they are not physical assets with utility like commodities. As such, the fundamental value is extremely hard to quantify, and both the literature and the industry are in their infancy in identifying Bitcoin pricing models. Whether any existing model can reliably be used as a tool by investors, professionals and regulators alike is an important empirical question, especially in the midst

JEL classification – G11, G14, G15, G19

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of Bitcoin's explosion in popularity and the growing regulatory concerns surrounding its adoption.

Papers examining the fundamental value of Bitcoin echo the wide range of opinions observed in the industry. For example, Kristoufek (2013) and Cheah and Fry (2015) suggest the fundamental price of Bitcoin is limited to nonexistent. In contrast, Dwyer (2015), Pagnotta and Buraschi (2018), and Hayes (2019) argue that Bitcoin's network, its technical properties and its cost of production imply a fundamental value that is well beyond zero. In the absence of consensus around a true fundamental model, many resort to technical models. For example, Sun *et al.* (2020) adopt a gradient boosting decision tree (GBDT) to forecast price trends, whereas Liu *et al.* (2021) use the deep learning method stacked denoising autoencoders (SDA) to predict Bitcoin price.

Despite the lack of consensus in the academic literature, perhaps the most popular pricing model among Bitcoin enthusiasts is known as the stock-to-flow (S2F) model. The S2F model was first popularized by crypto analyst "Plan B" in April 2019. The model uses the historical relation between the S2F ratio and Bitcoin prices to forecast future prices, where stock is defined as the total supply of Bitcoin available, and the flow is defined as the increase in available supply. Although the S2F model is adapted from commodity markets, it has special utility in the case of Bitcoin because of Bitcoin's unique supply characteristics. Specifically, the total supply of Bitcoin is fixed at 21 million, and the increase in supply is predictable. Therefore, one can forecast the S2F ratio in time with precision. If the historical relation between the S2F and Bitcoin prices is stable, then one can also forecast Bitcoin prices in the future with accuracy. Although the S2F model is well known in the crypto industry, it has yet to be examined in an academic setting [3]. In this paper, we dissect the S2F model and evaluate it as a Bitcoin pricing model.

The S2F model became popular for two primary reasons: first, when it was initially published in 2019, there appeared to be a strong historical relation between S2F ratios and Bitcoin prices. To observe this relation, Figure 1 plots our replication of the original S2F model prediction against Bitcoin prices. Second, the model predicted the meteoric rise of Bitcoin in late 2020 and early 2021 and has grown increasingly popular as Bitcoin prices seem to follow its prediction with uncanny accuracy. This out-of-sample (OOS) confirmation led to increased confidence in the model's accuracy and contributed to its popularity among crypto investors. However, there are reasons to criticize the economic rationale behind the model and to question whether transposing a model traditionally used for commodities can apply to Bitcoin. Moreover, the model's assumptions and peek-ahead biases are seldom discussed weaknesses.

In this paper, we examine the empirical suitability of the S2F model as a Bitcoin pricing model. We also provide three critiques of the theoretical motivation underpinning the model. First, we point out that the model focuses only on the supply of Bitcoin and is silent regarding demand. The implicit assumption that demand for Bitcoin is constant seems unlikely. Examples of nonconstant demand drivers include institutional adoption, government adoption (e.g. El Salvador), federal reserve policy, dollar strength, liquidity in the financial system, among others. Second, the S2F model implies an ever-increasing Bitcoin price and does not allow any conditions for downward price movement which seems equally unlikely. Third, because the S2F ratio is predictable with precision, there is no information innovation. One should not expect prices to respond to supply changes that are predictable. In equilibrium, the current Bitcoin price should reflect future known changes in supply.

Empirically, we first test the sensitivity of the model's predictions to reasonable changes in assumptions. We find the model to be robust to alternative computations and not

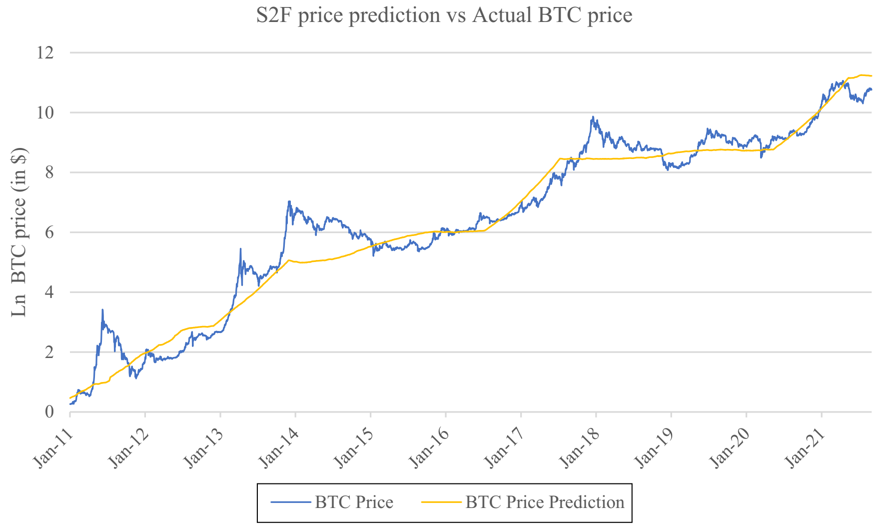


Figure 1.
S2F price prediction vs BTC price

Note: Figure 1 compares (logged) Bitcoin price to the (logged) predicted value of Bitcoin according to the stock to flow model (S2F) over time

sensitive to differing assumptions. Next, we produce a dynamic S2F model to test a trading strategy with no peek-ahead bias, one of the culprits of the original model. We examine the long-term model predictions based on various estimation windows, and although we find the model to be relatively sensitive to these estimation windows, it is increasing in precision over time. Finally, we compare a buy-and-hold strategy to a dynamic trading strategy that goes long when Bitcoin is undervalued according to S2F and short when it is overvalued. Results suggest a buy-and-hold strategy is superior, implying the S2F model has not been useful as a trading indicator.

The contribution of this paper is threefold. First, we contribute to the literature on Bitcoin pricing by investigating a fundamental value model widely popularized by the industry, but that remains to be analyzed through an academic lens. To our knowledge, this is the first paper to analyze the S2F model in an academic setting. Second, we provide a rigorous assessment and demonstrate how the model can be implemented realistically and without the peek-ahead bias, creating a tool that can be used contemporaneously by investors. Finally, our findings should be of interest to retail investors and portfolio managers alike as we show that a simple buy-and-hold strategy is more effective than a basic long–short strategy based on the S2F model.

2. Literature review

General awareness and interest in Bitcoin and the broader cryptocurrency markets have exploded in recent years. Academic interest has followed suit with many publications on the topic. To understand the literature on Bitcoin value, one has to consider Bitcoin's purpose. Despite the increasing number of studies investigating the topic, no consensus has emerged. Early studies describe Bitcoin as a speculative asset, questioning its value as a currency (Yermack, 2015; Corbet *et al.*, 2018). Glaser *et al.* (2014) find that Bitcoin users are mostly uninformed and are primarily interested in cryptocurrencies as an alternative investment rather than as an alternative peer-to-peer transaction system. Several studies corroborate this view, arguing that Bitcoin's volatility resembles that of a speculative asset as opposed to a commodity or a currency (Bouoiyour and Selmi, 2015; Baur *et al.*, 2018; Geuder *et al.*, 2019; Miglietti *et al.*, 2019).

Despite its high volatility, many studies argue that Bitcoin's purpose goes beyond that of a speculative asset. Kristoufek (2015) finds that Bitcoin possesses the characteristics of both a traditional asset and a speculative one. Selgin (2015) argues Bitcoin should be considered as "synthetic commodities money," circumventing the traditional dichotomy between fiat-based money and commodity-based money. Dyhrberg (2016a, 2016b) describes Bitcoin as a hybrid of a commodity and a currency, while Klein *et al.* (2018) concludes that Bitcoin behaves distinctly from any other asset group from an econometrical perspective. Morillon (2021) further argues that Bitcoin can be used as an instrument to short expansionary monetary policies which should be considered when attempting to price Bitcoin.

It is important to note that the Bitcoin network is constantly evolving and has become more efficient over the years (Urquhart, 2016; Nadarajah and Chu, 2017; Tiwari *et al.*, 2018; Kinateder and Papavassiliou, 2019). This dynamism likely at least partly explains academia's scattered opinions as well as the difficulty in deriving a fundamental value model for Bitcoin. The high number of uninformed investors (Glaser *et al.*, 2014) and elevated levels of information asymmetry among market participants likely further reinforce opinion dispersion as the traditional finance literature has demonstrated in various settings (Moeller *et al.*, 2007; Bessler *et al.*, 2011; Choy and Wei, 2012; Yu *et al.*, 2019; Howe and Morillon, 2020; Demir *et al.*, 2021). This divergence of opinion can be seen in some specific subthreads of the literature such as the discussion on Bitcoin's status as a diversification tool and a safe haven. While some studies find diversification benefits and safe haven properties to Bitcoin (Briere *et al.*, 2015; Wu *et al.*, 2019; Bouri *et al.*, 2020; Ghabri *et al.*, 2021), others question these benefits or find contradictory evidence (Shahzad *et al.*, 2020; Baur and Hoang, 2021). Illustrating this lack of consensus, several studies have investigated whether assets including Bitcoin presenting potential safe heavens properties perform adequately during COVID-19 also found contrasting results (Conlon and McGee, 2020; Conlon *et al.*, 2020; Kinateder *et al.*, 2021; Hassan *et al.*, 2021).

Despite this lack of consensus, papers have investigated both technical and fundamental models in an attempt to forecast Bitcoin prices. Examples of technical models include Mensi *et al.* (2019) who use a FIGARCH model to forecast Bitcoin and Ethereum returns, Sun *et al.* (2020) who adopt a GBDT to forecast price trends, Demir *et al.* (2021) who use a nonlinear autoregressive distributed lag model to investigate the impact of Bitcoin on Altcoin prices and Liu *et al.* (2021) who use deep learning method SDA to predict Bitcoin price.

Other papers focus on fundamental drivers in an attempt to value Bitcoin. Ciaian *et al.* (2016) propose a model derived from a gold-standard-related supply and demand equilibrium. Dwyer (2015) argues that Bitcoin's value is derived from its innovative solution to the double- spending problem as well as the scarcity of its supply. Pagnotta and Buraschi (2018) propose a model where price equilibrium is determined by the demand for Bitcoin and the supply of the hash rate. Relatedly, several recent studies have investigated the existence of a causal relation between volume and Bitcoin returns (Balcilar *et al.*, 2017; Bouri *et al.*, 2019; Fousekis and Tzaferi, 2021; Fousekis and Grigoriadis, 2021). Also, Hayes (2019) shows that Bitcoin's production cost implies a fundamental value that is well above zero and observes that Bitcoin prices converge toward that equilibrium post bubbles. Our paper most closely relates to this strand of the literature. Our work is unique in that rather than proposing a new pricing model, we evaluate a model popularized by the industry through an academic lens.

3. The stock to flow model: theoretical framework and criticisms

The S2F model was originally proposed by crypto analyst *Plan B* in April 2019 [4]. Inspired by commodities markets, the S2F model is a two-stage least squares model (2SLS) that allows to price any commodity as a function of its S2F ratio. The S2F ratio of a commodity is obtained by dividing the total supply of said commodity (the stock) by its annual production (the flow) as follows:

$$S2F\,ratio = \frac{Stock}{Flow} = \frac{1}{Supply\ \ growth\ \ rate} \tag{1}$$

The first stage of the model requires a training sample on which we regress the log value of the market capitalization of a commodity on the log value of its S2F ratio:

$$Ln(Market\ \ Capitalization)_t \ = \ \beta_0 + \beta_1{*}Ln\ \ (S2F)_t \tag{2}$$

The coefficients $\hat{\beta}_0$ and $\hat{\beta}_1$ obtained in the first stage are then used to calibrate the second stage of the model and predict future prices. To understand the second stage of the model [see equation (3)], it is important to realize that the left-hand term is equal to the price of Bitcoin at time $t+1$:

$$\hat{Price}_{\mathbf{t}+1} = \frac{(Market\ \ capitalization)_{\mathbf{t}+1}}{Total\ \ Supply_{t+1}} = \frac{e^{\left(Ln(Market\ \ capitalization)_{\mathbf{t}+1}\right)}}{Total\ \ Supply_{t+1}}$$

With that observation in mind, it follows that the second stage of the model (2SLS) is computed as:

$$\frac{e^{\left(Ln(Market\ \ capitalization)_{t+1}\right)}}{Total\ \ Supply_{t+1}} = e^{\left(\hat{\beta}_0\ \ +\ \ \hat{\beta}_1{*}\ \ Ln(S2F)_{t+1}\right)} \tag{3}$$

Intuitively, the S2F model prices the value of a commodity based on the growth rate of its supply. As such, Bitcoin's predictable supply properties allow the S2F model to price Bitcoin at any point in the future. This is possible because Bitcoin's supply grows at a predetermined rate built in its source code. As such, the future growth rate of Bitcoin's supply is known, allowing one to compute its S2F ratio and by extension forecast its price at any point in the future. This makes the S2F model a very unique valuation tool on two accounts:

1. it does not require financial data to price the underlying asset; and
2. it can price an underlying asset using contemporaneous or future known data as opposed to lagged values.

The specificities detailed above make the S2F model a unique tool that has become a mainstay among cryptocurrency investors. We recognize its merit as a simple model that is reminiscent of valuation tools used in the commodities market, which is intuitive for an asset frequently dubbed as "e-gold." Moreover, it allows investors to price Bitcoin based on its supply properties.

Although the main focus of this paper is to test the properties of the empirical model, we propose three criticisms of the theoretical intuition that we believe should be acknowledged, despite the model's simplicity and attractiveness.

The first criticism pertains to the assumption that the price of Bitcoin is driven solely by the growth rate of its supply, with no regard for demand. The absence of consideration for demand for an asset such as Bitcoin raises questions. Given the infancy of the asset and its wild price swings, it seems likely that demand is a dominant driver of pricing. Critical pricing implications of Bitcoin such as institutional and corporate adoption, regulatory acceptance, tax implications, monetary policy and many other pricing drivers are absent from the model.

It is worth noting that the S2F model originates from the commodity markets and that this first criticism can be extended to other commodities. However, the underlying assumption of a stable demand is more reasonable with most mature commodities whose utility is well understood. At least at this stage of Bitcoin's maturity as an asset, a model assuming stable demand is questionable.

Our second and third criticisms below are specific to Bitcoin as they stem from its unique supply properties. The nature of Bitcoin's supply (predictable, scheduled and disinflationary) marks a rupture with that of traditional commodities. The S2F model's only input is Bitcoin's supply growth rate. Bitcoin's decreasing supply growth converging toward zero mechanically implies a rising price that should approach infinity once the last Bitcoin is mined [see equation (1)]. An ever-rising price at a constant pace seems to be an unreasonable long-term assumption. Moreover, a consequence of that framework is that the S2F model does not allow for downside price movement.

Finally, because S2F ratios are predictable with precision, why would changes lead to movement in Bitcoin prices? In equilibrium, there is no new information produced when Bitcoin's supply constricts as it was already known with certainty today. Given there is no new information, we should not expect supply changes to lead to price increases. With these three criticisms in mind, we proceed to test the model empirically.

4. Data and empirical method

We collect Bitcoin price and supply information from Cryptoquant [5] and proceed to compute S2F ratios following equation (1). First, we test the S2F model's reliability. There are several choices one needs to make to compute the model.

The window used for the training sample is the first hurdle. There is no obvious starting point because early Bitcoin price data is not reliable as the asset was not widely traded following inception. The appropriate size of the training sample and whether the model succeeds at predicting OOS prices over extended periods remain empirical questions. The second hurdle comes in the form of the periodicity of the data considered. Bitcoin is a very volatile asset, and therefore it is logical to wonder whether considering the data on a daily, weekly or monthly basis influences predictions. Finally, the smoothing method used to compute the S2F ratio (defined as the number of days/observations used as the basis to compute a mean S2F ratio) also impacts the volatility of estimates.

To compute S2F ratios, we look at the number of newly created Bitcoins every day and divide it by the existing supply annualized. New Bitcoins are created whenever a new block is mined, as a reward to the miner. Although the number of mined blocks is predictable over longer time windows, the day-to-day volatility is significant. The new supply created is the product of the number of new blocks mined multiplied by the reward per block. Consequently, S2F ratios are predictable over the longer term but vary day to day as a direct function of the newly created supply. Moreover, the reward miners receive with each block mined is divided by two after every "halving event" which occurs every 210,000 blocks (roughly every four years), meaning the supply growth generated for each block mined decreases significantly after every halving, as displayed in Panel A of Figure 2.

In this original paper, Plan B uses monthly data ranging from December 2009 to February 2019, but does not describe the frequency of the computation nor the smoothing method used [6]. As part of our battery of tests, we compute the S2F model using different smoothing methods ranging from 1 to 365 days. 365 days is the most commonly used method in most graphical representations. Panel B of Figure 2 graphically displays the impact of S2F ratio smoothing on price predictions. In the absence of smoothing, S2F ratios jump abruptly following every halving, while the increase is more gradual with longer smoothing periods.

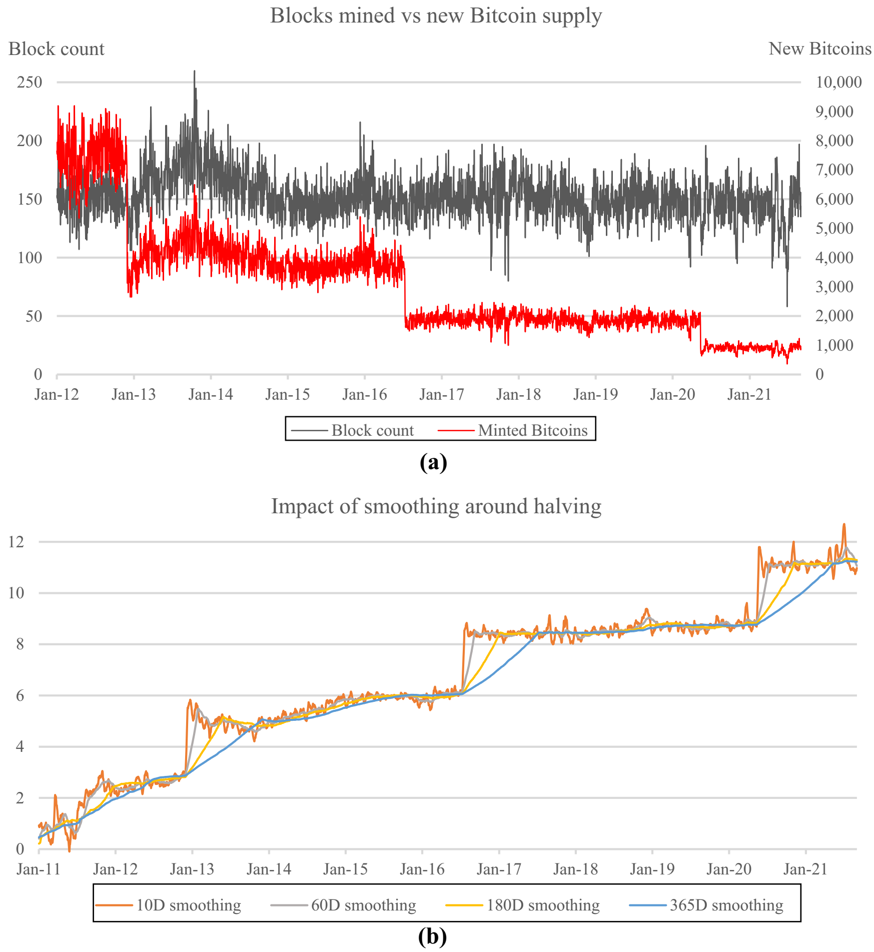


Figure 2.
(A) Block mining and new supply and (B) smoothing S2F ratios

Notes: Panel A contrasts the number of daily new blocks mined to the daily newly created supply of Bitcoin. Panel B reports S2F model price predictions using different smoothing methods

To make sure each model is treated equally and not impacted by the smoothing method used, we use a year worth of data from December 2009 to December 2010 to get the necessary 365 days to smooth S2F ratios for the longest period. We then use the remaining available observations in our regressions starting with January 2011.

One of the goals of this paper is to present a version of the S2F model that can be used contemporaneously by investors. Indeed, many graphical representations of the original S2F model are deceptive because they plot "predicted" Bitcoin prices over time periods comprised in the training sample. In other words, these appealing graphs can be misleading because they suffer from an ex post conditioning bias.

To correct this bias, we propose two different methods to compute the S2F model. The first method consists in using rolling four-year windows (from year $t-3$ to t) to train the sample in the first stage of the model [see equation (2)] and then to use the obtained coefficient estimates in the second stage of the model to predict prices in year $t+1$ [see equation (3)]. Because halving occurs roughly every four years, using a four-year window allows us to capture a "full cycle." Moreover, by excluding observations older than four years, we mechanically put an emphasis on most recent data, which might be desirable because Bitcoin's supply grew very quickly in its early years. The second method involves simply using all the data available at time t to train the sample and then to predict prices in year $t+1$. Each incremental data point is added to the training sample over time when available while older data is not discarded.

It is important to understand how to interpret first-stage coefficients as they are used to calibrate the second stage of the model. The intercept coefficient sets the baseline for the model prediction. A higher (lower) intercept increases (decreases) the baseline of the second-stage predictions. The coefficient of the log values of S2F ratios impacts the upward drift of the second-stage predictions. Because Bitcoin's supply growth is ever decreasing, its S2F ratio is conversely ever increasing. Higher S2F coefficients imply higher predicted Bitcoin price. Finally, the smoothing method used to compute S2F ratios determines how significant projected price increases are following a halving event (after which annualized S2F ratios double). Longer smoothing periods translate to longer, less extreme adjustments following a halving.

To estimate the accuracy of our contemporaneously implementable S2F model, we follow Welch and Goyal (2008) and Pabuçcu *et al.* (2020) to contrast the OOS statistics of our S2F model to that of the in-sample of version of the S2F model. Our OOS statistics are computed as follows:

$$R^2 = 1 - \frac{SSR}{SST}, \quad \overline{R^2} = 1 - \frac{(1-R^2)-(n-1)}{n-k-1}, \quad \Delta\overline{R^2} = \overline{R^2}_E - \overline{R^2}_B$$
$$MSE = \frac{1}{n}\sum_{t=1}^{n}\left(Y_t - \hat{Y}_t\right)^2, \quad RMSE = \sqrt{MSE}, \quad \Delta RMSE = \sqrt{MSE_E} - \sqrt{MSE_B}$$
$$MAE = \frac{1}{n}\sum_{t=1}^{n}|Y_t - \hat{Y}_t|, \quad \Delta MAE = MAE_E - MAE_B$$
$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{Y_t - \hat{Y}_t}{Y_t}\right|, \quad \Delta MAPE = MAPE_E - MAPE_B \tag{4}$$

We compute the differences in adjusted r square ($\overline{R}^2$), root mean squared error (RMSE), mean absolute error (MAE) and mean absolute percentage error (MAPE), where n is the number of observations and k is the number of parameters. Subscript E denotes the S2F model free from the peek-ahead bias and subscript B denotes the original S2F model. These OOS statistics allow us to determine the magnitude of the change in accuracy when using a contemporaneous S2F model.

5. Empirical results

We begin by recreating the original model using differing assumptions to test the robustness of the estimates [7]. Table 1 reports the results of the first-stage model used to calibrate the price predictions of the second-stage model. Panels A–C report results of the first stage using 1, 10 and 365-day smoothing methods to compute S2F ratios. From left to right, we report first-stage results computed using daily, weekly, monthly, and yearly data. Strikingly, all estimates are very close and consistent with one another regardless of the computation method, data frequency or smoothing method used. Particularly, *Ln*(S2F) coefficients are concentrated in a narrow range between 14.06 and 14.53. This range gets even narrower (from 14.19 to 14.43) if we remove annual computations with a very low number of observations. This is an important observation as *Ln*(S2F) coefficient estimates are included in an exponential term in the second stage of the model used to predict prices. Consequently, estimates fitting in a narrow range, independently of initial computational choices and assumptions, should reinforce confidence in the predictions of the model. Goodness of fit measures confirm this assessment, as R^2 values are high ranging from the high 1980s to the mid-1990s. To conclude, these first findings suggest the relation between S2F and Bitcoin prices is not the result of data mining.

Next, we investigate the ability of the S2F model to predict Bitcoin prices after removing the peek-ahead bias. To justify the accuracy of the model, S2F proponents commonly display graphs showing how accurate it has been at predicting Bitcoin price since inception. However, these "predictions" include the sample from which the first-stage model was

	Daily	Weekly	Monthly	Yearly
Panel A: 1-day S2F ratio (no smoothing)				
Intercept	14.39*** (0.05)	14.19*** (0.14)	14.27*** (0.28)	14.06*** (0.62)
Ln(S2F)	3.27*** (0.02)	3.35*** (0.06)	3.32*** (0.12)	3.43*** (0.26)
N	3,012	431	99	8
Adjusted R^2	0.874	0.883	0.892	0.960
Panel B: 10-day smoothed S2F ratio				
Intercept	14.33*** (0.05)	14.21*** (0.14)	14.29*** (0.28)	14.09*** (0.61)
Ln(S2F)	3.30*** (0.02)	3.35*** (0.06)	3.32*** (0.12)	3.42*** (0.26)
N	3,012	431	99	8
Adjusted R^2	0.886	0.886	0.892	0.962
Panel C: 365-day smoothed S2F ratio				
Intercept	14.42*** (0.04)	14.32*** (0.10)	14.43*** (0.19)	14.53*** (0.53)
Ln(S2F)	3.43*** (0.02)	3.48*** (0.05)	3.43*** (0.09)	3.40*** (0.24)
N	3,012	431	99	8
Adjusted R^2	0.941	0.932	0.943	0.967

Notes: Table 1 reports the intercept and S2F coefficient of the first stage of the S2F model using different smoothing methodologies and data frequency. From left to right, each column reports the coefficients using daily, weekly, monthly and yearly data, respectively. Panel A reports the coefficient of a specification without smoothing. Panel B reports the coefficient of a specification using a 10-day smoothing period. Panel C reports the coefficient of a specification using a 365-day smoothing period. ***, ** and * denote significance of coefficients at the 1%, 5% and 10% levels, respectively

Table 1.
S2F first-stage computation

created, suffering of an implied peak-ahead bias. To test the S2F model's performance under these conditions, we use a year worth of data from December 2009 to December 2010 to get the necessary 365 days to smooth S2F ratios, then use the next four years (2011–2014) as our training sample to predict daily Bitcoin prices for the following year. As such, 2015 is the first year with available predictions. We compute the predictive model using both the rolling four-year window and a model that uses all previous data at a given point in time as described in Section 4.

Table 2 presents the S2F model betas, the S2F forecast at the end of the following year, the actual price at the end of the year, and the model's prediction of prices as of August 31, 2021, for each model. Panel A reports results using a four-year rolling window, while Panel B reports results using all available data at a given point in time. The primary takeaway from these results is the model's sensitivity to the estimation window. For example, the first row of Panel A shows that using only data from 2011 to 2014 would produce a price target on August 31, 2021 of $1,211,642.84, whereas the model using data from 2017 to 2020 produces a price target on August 31st, 2021 of $128,451.92. Prediction accuracy increases over time and appears to stabilize as we move down row by row over time, especially for model using all available data. The largest discrepancies in estimates come from the oldest estimation windows, a consequence of Bitcoin's predetermined sharp decrease in supply growth over the first few years following inception.

Figure 3 depicts this decrease, which is particularly obvious on the logarithmic scale in Panel B. As such, there is reason to believe that predictions will become more stable over time as supply growth, the key driver of the S2F model, decreases and becomes steadier.

Figure 4 presents the model forecast of the S2F models without the peek-ahead bias and contrasts them to that of the original S2F model. One can graphically see the difference removing the peek-ahead bias makes and the fact that this difference fades away over time.

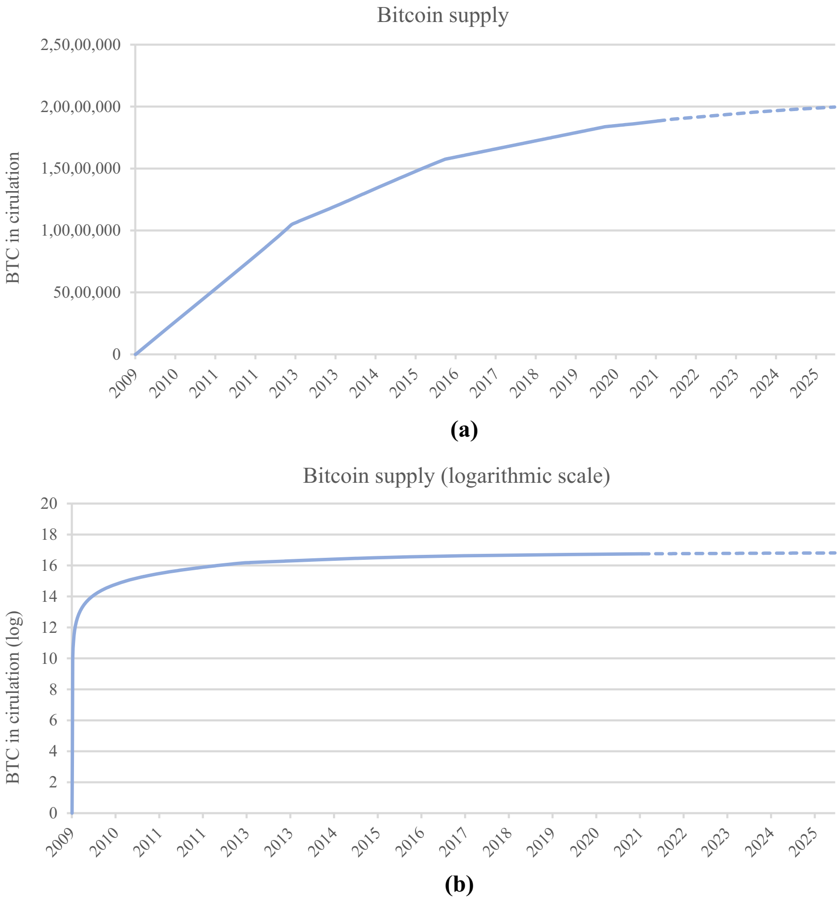
To confirm this visual observation, in Table 3 we report estimates of forecast errors of each of the S2F models. Following Welch and Goyal (2008) and Pabuçcu *et al.* (2020), we report the differences in $\overline{R}^2$, MSE, RMSE, MAE and MAPE. We report yearly forecast errors as well as a full sample error estimate in the last row. Unsurprisingly, we observe that the original S2F model suffering from peek-ahead bias produces more accurate estimates on almost every instance for every year as forecast error estimates are much lower and $\overline{R}^2$ is higher with the exception of 2018 where the S2F model without peek-ahead bias using all available data slightly outperformed. These results confirm that the original S2F model likely overstates predictability. However, this is not to say that removing the peek-ahead bias renders the model unusable. In fact, it seems the models without the peek-ahead bias are becoming increasingly accurate over the years as every gap seems to narrow, particularly over the last two years. This is further confirmation that additional data intuitively helps refine the model which is a good sign for investors interested in capitalizing on the S2F predictions.

Finally, we examine how a long–short trading strategy based on the S2F model would compare to a buy-and-hold strategy. Specifically, the strategy goes long when Bitcoin is undervalued according to S2F and short when Bitcoin is overvalued according to S2F. The results are presented in Table 4. The cumulative returns from 2015 to 2021 for each strategy are 14,859.5%, 71.8% and 369.7% for buy and hold, S2F using rolling betas and S2F using all available data, respectively. Moreover, we note that the buy-and-hold strategy is more profitable every year. Much of the difference appears to be caused by sudden price spikes during periods investors are shorting Bitcoin, as displayed in Figure 5. This finding raises questions regarding the implementation of such a strategy and the associated risks. Our

Table 2.
S2F forecasts over time

Training period ($t-4$ to t)	Intercept	*Ln*(S2F)	S2F at the end of $t+1$	Price prediction end $t+1$	Actual price $t+1$	*Prediction today (8/31/2021)*
Panel A: Rolling four-year S2F						
2011–2014	13.470	4.266	11.07	$1,337.60	$429.69	$1,211,642.84
2012–2015	13.599	3.954	15.42	$2,498.43	$973.37	$390,337.48
2013–2016	16.224	2.705	23.99	$3,591.30	$13,782.83	$34,171.31
2014–2017	17.633	2.148	25.62	$2,768.21	$3,728.57	$14,617.63
2015–2018	13.519	3.701	26.75	$7,845.33	$7,184.22	$128,926.20
2016–2019	14.078	3.506	41.23	$32,126.06	$28,937.49	$102,254.75
2017–2020	12.991	3.830	N/A	N/A	N/A	$128,451.92
Panel B: S2F with all available data						
2011–2014	13.470	4.266	11.07	$1,337.60	$429.69	$1,211,642.84
2011–2015	14.009	3.756	15.42	$2,187.82	$973.37	$263,142.23
2011–2016	14.160	3.624	23.99	$8,454.57	$13,782.83	$179,756.10
2011–2017	14.463	3.405	25.62	$6,853.76	$3,728.57	$100,083.91
2011–2018	14.371	3.469	26.75	$8,578.98	$7,184.22	$118,027.02
2011–2019	14.432	3.428	41.23	$34,259.88	$28,937.49	$106,250.59
2011–2020	14.473	3.403	N/A	N/A	N/A	$100,061.51

Notes: Table 2 reports the coefficients of the S2F second-stage model using multiple training periods, as well as price predictions at the end of the next year vs actual Bitcoin price. Panel A reports the results using *four-year rolling* window. Panel B reports the results using *all available data*



Notes: : (a) Bitcoin supply growth; (b) Bitcoin supply growth on a logarithmic scale
Panel A shows the evolution of Bitcoin's supply over time (in number of BTC).
Panel B shows the evolution of Bitcoin's supply on a logarithmic scale

Figure 3.
Bitcoin supply

findings imply that investors are better off buying and holding Bitcoin rather than trading around S2F valuation with a basic, easy to implement, long–short strategy. Evidence suggests that the model does not improve a trader's returns and therefore should not be used to trade Bitcoin. The theoretical issues mentioned earlier such as the absence of a demand-related component in the model, the nature of Bitcoin's supply and the predictability of S2F ratios likely explain this phenomenon. Therefore, it is not surprising the model does not actually improve trading performance.

6. Conclusion

The S2F model is frequently touted to support ever-increasing price targets for Bitcoin. We provide a thorough empirical examination of the S2F model. We find

(a)

(b)

Notes: (a) S2F model using four-year rolling betas; (b) S2F model using all available data Figure 4 compares the price prediction of the original S2F model with the peek-ahead bias (orange) to the prediction of the S2F model without peek-ahead bias (green) and the actual Bitcoin price (blue). Panels A and B display S2F predictions using a four-year rolling window (all available data)

Figure 4.
S2F estimates without peek-ahead bias

evidence that the model is robust to alternative computation methods and that a relation between the S2F ratio and Bitcoin prices is undeniable. Moreover, the accuracy of our peek-ahead bias free S2F model seems to improve over time as additional data is incorporated. However, its usefulness to investors remains a question. We show evidence that when accounting for peek-ahead bias a simple buy-and-hold strategy would have been significantly more profitable for investors than a basic trading strategy that goes long (short) when Bitcoin is overvalued (undervalued) according to S2F.

Table 3. Forecast error per year

	Original S2F					Rolling four years					Forecast error difference (Δ)				
	$\overline{R}^2$	MSE	RMSE	MAE	MAPE	$\overline{R}^2$	MSE	RMSE	MAE	MAPE	$\Delta\overline{R}^2$	ΔMSE	ΔRMSE	ΔMAE	ΔMAPE
Panel A: Original S2F vs rolling four-year S2F forecast error															
2015	98.48	9.45	30.74	30.74	4.84	68.85	194.02	139.29	139.29	24.72	29.63	(184.58)	(108.56)	(108.56)	(19.87)
2016	98.76	3.94	19.85	19.85	2.22	87.94	38.39	61.96	61.96	9.06	10.82	(34.45)	(42.11)	(42.11)	(6.84)
2017	59.40	29.81	54.59	54.59	5.85	56.43	31.99	56.56	56.56	5.38	2.97	(2.18)	(1.96)	(1.96)	0.47
2018	62.01	27.62	52.56	52.56	5.19	(62.60)	118.22	108.73	108.73	11.51	124.61	(90.59)	(56.17)	(56.17)	(6.32)
2019	79.40	14.76	38.42	38.42	4.00	79.30	14.83	38.51	38.51	3.52	0.10	(0.07)	(0.09)	(0.09)	0.48
2020	96.24	5.56	23.57	23.57	2.13	95.24	7.03	26.51	26.51	2.39	0.99	(1.47)	(2.93)	(2.93)	(0.26)
2021	96.59	23.01	47.97	47.97	3.84	89.27	72.31	85.03	85.03	6.64	7.32	(49.30)	(37.07)	(37.07)	(2.80)
Full	93.96	15.96	39.95	39.95	4.02	74.30	67.86	82.38	82.38	9.15	19.66	(51.90)	(42.43)	(42.43)	(5.13)
Panel B: Original S2F vs all available data S2F forecast error															
2015	98.48	9.45	30.74	30.74	4.84	68.85	194.02	139.29	139.29	24.72	0.30	(1.85)	(1.09)	(1.09)	(0.20)
2016	98.76	3.94	19.85	19.85	2.22	90.97	28.76	53.63	53.63	7.76	7.79	(24.82)	(33.78)	(33.78)	(5.53)
2017	59.40	29.81	54.59	54.59	5.85	(3.93)	76.30	87.35	87.35	10.67	63.33	(46.49)	(32.76)	(32.76)	(4.81)
2018	62.01	27.62	52.56	52.56	5.19	77.38	16.45	40.56	40.56	3.55	(15.37)	11.17	12.00	12.00	1.63
2019	79.40	14.76	38.42	38.42	4.00	74.71	18.12	42.57	42.57	3.79	4.69	(3.36)	(4.15)	(4.15)	0.21
2020	96.24	5.56	23.57	23.57	2.13	93.78	9.19	30.32	30.32	2.68	2.46	(3.64)	(6.75)	(6.75)	(0.55)
2021*	96.59	23.01	47.97	47.97	3.84	93.51	43.75	66.14	66.14	4.86	3.08	(20.74)	(18.18)	(18.18)	(1.03)
Full	93.96	15.96	39.95	39.95	4.02	78.88	55.77	74.68	74.68	8.46	15.08	(39.81)	(34.73)	(34.73)	(4.44)

Notes: This table compares statistics on forecast error for the original S2F model vs the *rolling four-year model* (Panel A) and *model using all available data* (Panel B). All numbers are in per cent. $\overline{R}^2$ is the adjusted r square. MSE is the mean square error. RMSE is the root mean square error. MAE is the mean absolute error. MAPE is the mean absolute percentage error. $\Delta\overline{R}^2$, ΔMSE, ΔRMSE, ΔMAE and ΔMAPE are the differences between the original S2F model and the *rolling four year* (Panel A) and *model using all available data* (Panel B)

Table 4.
S2F trading strategy

Year	End of year price	Buy-and-hold cumulative returns (%)	Rolling four-year cumulative S2F returns (%)	All available data cumulative S2F returns (%)
2015	\$430.16	36.6	36.6	36.6
2016	\$973.37	209.0	144.4	152.0
2017	\$13,771.18	4,272.4	−4.9	889.8
2018	\$3,762.18	1,094.5	74.4	608.9
2019	\$7,179.44	2,179.5	8.9	430.4
2020	\$28,949.51	9,091.5	58.5	838.4
2021 (until 31 August)	\$47,116.49	14,859.5	71.8	369.7

Notes: Table 4 compares the results of an active trading strategy using the *rolling four-year* S2F model and *all available data* model to a simple buy-and-hold strategy over time.

Figure 5.
Cumulative returns of S2F-based trading strategies

Notes: Figure 5 plots the cumulative returns of a simple buy and hold strategy (blue) versus the cumulative returns of a long-short peek-ahead bias free S2F trading strategy (Rolling 4-year in yellow, Continuous in green)

Future studies should investigate whether more complex strategies can allow investors to take advantage of the S2F predictions, as well as the impact of potential regulation on these strategies. Our paper provides a step toward understanding the price behavior of Bitcoin. One challenge in regulating Bitcoin and other crypto-related technologies is the lack of understanding of the assets. Lawmakers seeking to regulate cryptocurrencies benefit from a deeper understanding of what drives Bitcoin pricing.

Notes

1. Available at: www.barrons.com/articles/cathie-wood-wants-elon-musk-back-bitcoin-bull-51622210411.
2. Available at: https://fortune.com/2018/05/07/warren-buffett-bitcoin-rat-poison/.

3. Evidence that S2F is an important model in the Bitcoin space is the publication of the S2F values on many cryptocurrency investment websites including buybitcoinworldwide and lookintobitcoin.
4. The original article can be found here.
5. available at: https://cryptoquant.com/
6. Bitcoin's genesis block was mined on January 3, 2009 but the earliest price estimates started between October and December 2009.
7. In the original setting, Plan B explains using monthly data ranging from December 2009 to February 2019. To ensure all models are on equal footing, we use a year worth of data from December 2009 to December 2010 to get the necessary 365 days to smooth S2F ratios, then run the models using data from January 2011 to February 2019 (with the exception of the yearly model where we stop in December 2018).

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Corresponding author
Thibaut G. Morillon can be contacted at: tmorillon@elon.edu