Comparison study on measurement of rail weld joint between inertial reference method and multi-point chord reference method

Yifan Shi
Track Reasearch Lab, School of Civil Engineering, Southwest Jiaotong University (SWJTU), Chengdu, China
Yuan Wang
Shenzhen EboRail Technology Co., Ltd., Shenzhen, China
Xiaozhou Liu
College of Urban Transportation and Logistics, Shenzhen Technology University, Shenzhen, China, and
Ping Wang
The Hong Kong Polytechnic University, Hung Hom, China

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Abstract

Purpose – Straightness measurement of rail weld joint is of essential importance to railway maintenance. Due to the lack of efficient measurement equipment, there has been limited in-depth research on rail weld joint with a 5-m wavelength range, leaving a significant knowledge gap in this field.

Design/methodology/approach – In this study, the authors used the well-established inertial reference method (IR-method), and the state-of-the-art multi-point chord reference method (MCR-method). Two methods have been applied in different types of rail straightness measurement trollies, respectively. These instruments were tested in a high-speed rail section within a certain region of China. The test results were ultimately validated through using traditional straightedge and feeler gauge methods as reference data to evaluate the rail weld joint straightness within the 5-m wavelength range.

Findings – The research reveals that IR-method and MCR-method produce reasonably similar measurement results for wavelengths below 1 m. However, MCR-method outperforms IR-method in terms of accuracy for wavelengths exceeding 3 m. Furthermore, it was observed that IR-method, while operating at a slower speed, carries the risk of derailing and is incapable of detecting rail weld joints and low joints within the track.

Originality/value – The research compare two methods' measurement effects in a longer wavelength range and demonstrate the superiority of MCR-method.

Keywords Rail weld joint, Inertial reference method, Short-wavelength irregularities, Multi-point chord reference method, 5-m wavelength range

Paper type Research paper

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1. Introduction

The problem of track irregularities represents a primary focus on railway infrastructure maintenance and upkeep throughout the history of railway development (Fan, 2004; Lewis & Olofsson, 2009). Irregularities in rails have a significant impact on the overall structure of the track, resulting in noise, vibrations and wheel-rail impact forces generated by the rolling stock. This phenomenon constitutes a major source of disturbance within the entire vehicle-track coupled system. During the fabrication process of rails, weld joints can introduce geometric discontinuities due to human factors, thereby affecting rail straightness. As train wheelsets continuously traverse these vulnerable areas, the weld joints are prone to issues such as railhead wear, deformation and fatigue cracking. Therefore, the measurement and assessment of rail straightness play a crucial role in gaining a better understanding of the operational condition of the railway track. This information, in turn, aids the railway maintenance department in formulating rational strategies for rail maintenance, ensuring passenger comfort and safety along the routes.

The measurement of rail straightness has seen significant advancements over the years, with various well-established measurement techniques available. The most traditional method involves using a straightedge and feeler gauge, which has been widely applied in domestic railway maintenance for a considerable time. Another method is the guideway rail measurement, where a slider with built-in displacement sensors moves along a 1-m guide rail, measuring the straightness of the rail within the measurement range. Fixed measurement involves placing an instrument equipped with capacitive sensors on the guide rail, enabling the one-time measurement of rail straightness within a 1-m range. The introduction of electronic technology has led to the development of the 1-m electronic straightedge measurement, which replaces manual readings with electronic data recording to determine rail straightness more accurately. In recent years, electronic rail corrugation analyzers have gained widespread use. Italy's MERMEC company has developed a handheld rail corrugation measurement device, while the USA's ENSCO company has designed and developed both onboard rail corrugation detection systems and onboard rail corrugation analyzers. For a long time, China relied on imported equipment to detect short-wavelength irregularities in rails. Professor Stuart Grassie, after extensive research on rail corrugation formation mechanisms, spearheaded the development of the Corrugation Analysis Trolley (CAT) rail corrugation analyzer, which is among the most mature and widely applied devices globally. This analyzer is designed based on the inertial method, primarily utilizing the double integration of acceleration data from onboard sensors to obtain rail corrugation measurement results (Grassie & Kalousek, 1993; Grassie, 1996a, b, 2005, 2012, Grassie, Saxon, & Smith, 1999). In recent years, domestically produced MCR rail corrugation analyzers have emerged as new-generation rail corrugation detection equipment. The MCR analyzer is designed based on the multi-point chord reference system method, which differs from the conventional midpoint chord method as it focuses on establishing a unified model and error analysis system, making it more versatile (Wang, 2021). Both MCR and CAT rail corrugation analyzers employ continuous rail weld joint measurement, leading to improved measurement efficiency compared to traditional methods.

The rail weld is a key component that connects two sections of the rail, making it a whole. Due to environmental factors and different construction conditions, the welds will also show different morphological differences. Yang analyzed the measured data of 1-m chord measurement of rail joints, and concluded that the irregularity waveforms of joints mainly include harmonic shape, saddle shape, superimposed shape and multi-harmonic shape (Yang, Tao, & Fu, 2017). Currently, rail straightness measurements primarily focus on the 1–3 m range, leaving a gap in research for rail weld joint straightness measurements within a 5-m wavelength range. The traditional research data are measured by 1-m chord of weld irregularity, using 1-m electronic ruler measurement, continuous sampling interval of 5 mm, while the continuous sampling interval of the trolley based on the multi-point chord reference method (MCR-method) is 1 mm, and the measurable wavelength range is 2–3000 mm. In contrast, the joint irregularity can be measured closer to the real welding joint shape (Xu, Cong, Zhao, Wang, & Chen, 2022). Southwest Jiaotong University Cong, proposed a theoretical method for measuring the straightness of rail weld joints with a 3-m wavelength and conducted an outline assessment (Cong *et al.*, 2023). The research categorized rail joint shapes into two main types: “W” and “M”. However, practical line tests for longer wavelength ranges were not conducted. In this study, we addressed this research gap by conducting an analysis and comparison of a series of on-site sampling data. We investigated the 5-m chord range rail straightness data obtained by inertial reference method (IR-method) and MCR-method. See Table 1 and Figure 1.

To validate the results, we combined this analysis with the traditional straightedge and feeler gauge method. Through this measurement data, we aimed to analyze whether these two rail corrugation analyzers could effectively measure the specific characteristics of rail weld joints within the 5-m wavelength range.

2. Test information overview

The testing site is located on a high-speed railway within China. This railway line experiences significant passenger traffic, with numerous trains passing through daily. The high volume of train traffic can lead to various rail defects, making the rail conditions more complex than those typically found on regular passenger routes. The testing section of the railway features an elevated infrastructure structure with CRTSIII plate ballastless track on bridges.

The measurement equipment used includes the domestically produced MCR rail corrugation analyzer and the imported CAT rail corrugation analyzer, both of which are dual-track rail corrugation analyzers, as shown in Plate 1. They are capable of simultaneous measurements on both the left and right rails. The MCR rail corrugation analyzer does not have a pushing speed limit, with a design speed range of 0 km/h–10 km/h, but it can achieve an actual maximum speed of up to 15 km/h. On the other hand, the CAT rail corrugation

Table 1. Comparison of key measurement techniques

Measurement method	Straightedge and feeler gauge	Fixed measurement	Guideway measurement	MCR trolley	CAT trolley
Equipment length	1 m	1.2 m	1.2 m	0.33 m	0.4 m
Equipment weight	2–5 kg	>5 kg	>5 kg	2.3 kg	8 kg
Number of sensors	1	1	100–200	8	1
Sensor type	Manual work	Eddy current displacement	Capacitive displacement	Eddy current displacement	Accelerometer
Measurement principle	Guideway reference method	Guideway reference method	Guideway reference method	Multi-point chord reference system method	Inertial method
Measurement efficiency	Very low	Low	Low	Relatively high	Moderate
Figure number	Figure 1(a)	Figure 1(b)	Figure 1(c)	Figure 1(d)	Figure 1(e)

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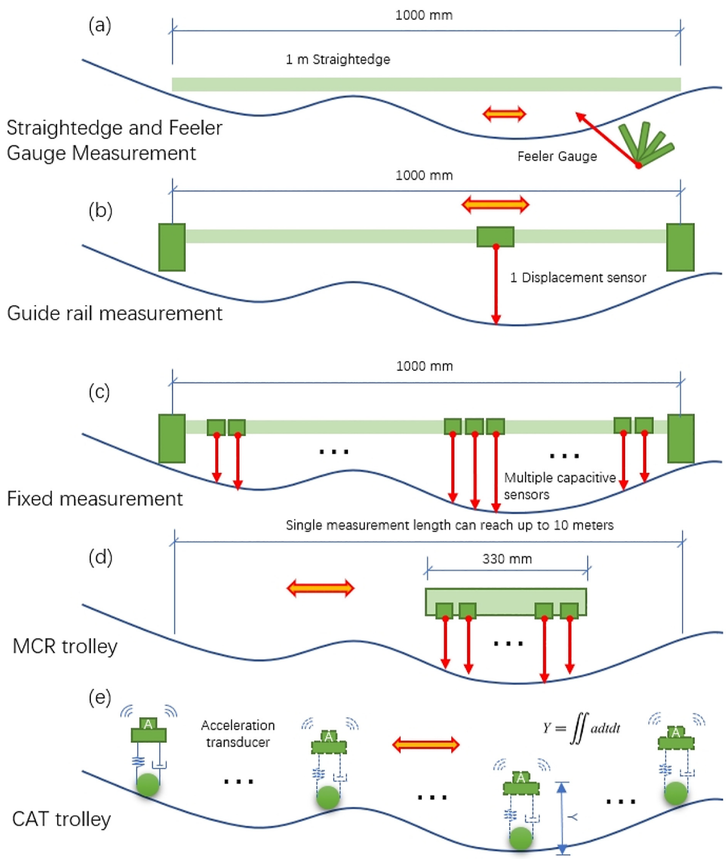

Figure 1. Schematic diagram of mainstream measurement techniques

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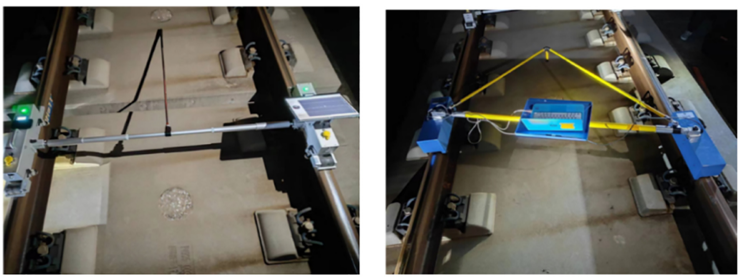

(a) **(b)**

Plate 1. (a) MCR double-track rail corrugation instruments (b) CAT double-track rail corrugation instruments

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analyzer has a pushing speed limit of 2.9 km/h–4.3 km/h, and speeds slower than this range or stopping can result in abnormal waveforms.

The principle of the CAT rail corrugation analyzer is primarily based on the inertial method. It calculates the track short-wavelength irregularity value Y by performing a double integration of the detected values from the built-in accelerometers in the measurement trolley. This approach is used to assess the rail corrugation conditions (Yu, 2008; Xu, Xu, Femg, Wang, & Sun, 2021; Zhang, 2022), as shown in Formula (1). The use of inertial sensors for measuring rail irregularities is also one of the mainstream measurement methods (Wang, 2014; Li, Molodova, Núñez, & Dollevoet, 2015; Liu, 2015).

$$Y = \iint adtdt \tag{1}$$

However, the inertial method is significantly influenced by the speed of the measuring instrument, and it involves integration, which can be sensitive to saturation when dealing with a large amount of low-frequency signals. Moreover, the measurement accuracy can be affected by other forms of rail defects (Yu, 2008). In contrast, the chord reference system method, which is one of the earliest and most widely used methods for rail irregularity detection, offers several advantages. It is cost-effective, and its practical engineering applications are extensive (Nielsen, Berggren, Lölgen, & Müller, 2013; Haigermoser, Luber, Rauh, & Gräfe, 2015; Wang *et al.*, 2018). The measurement and calculation in chord reference systems are independent of speed, and they generally provide higher accuracy. However, in the conventional midpoint chord method, when the chord length used for measurement is 2k times the wavelength of the rail irregularity (where $k = 0, 1, \ldots$), the amplitude gain response becomes zero. When such data are fed into the model recursively, errors gradually accumulate, causing the measurement results to deviate from the actual values and significantly affecting measurement accuracy (Wang, Xu, Zhou, Li, & Chen, 2012; Wang, Xu, Chen, Xiao, & Wang, 2015; Liu, 2016; Xu, Wang, Wang, & Xiao, 2016). These chord methods typically refer to single-point chord reference systems or simplified multi-point chord reference systems (Chen, Xu, Zhou, & Chen, 2011; Mao, Xu, & Zhou, 2013; Yin, Zhu, Wang, Wu, & Jin, 2017).

The principle of the MCR rail corrugation analyzer is primarily based on the “multi-point chord reference system method” proposed by Wang Yuan from Southwest Jiaotong University. This method serves as a unified approach for all multi-point measurement systems that use chords as their reference. It establishes measurement and inversion models using a linear system approach in two steps. In the first step, the measurement model is established. It involves dividing a chord into N measurement points and treating the N vector deviations as a chord measurement vector. Linear equations are formulated by relating chord measurements to their corresponding geometric relationships, resulting in a measurement matrix. Taking into account the random errors associated with each measurement point during each measurement, an error matrix is introduced to establish the measurement model, as shown in Figure 2. The second step involves the inversion model, which uses the least squares method to reverse-engineer the geometric position of the track. This process essentially reverses the measurement model, using the acquired chord measurements to deduce the actual rail irregularities. This method achieves the precision required for engineering applications and exhibits favorable transfer function properties (Wang, 2021).

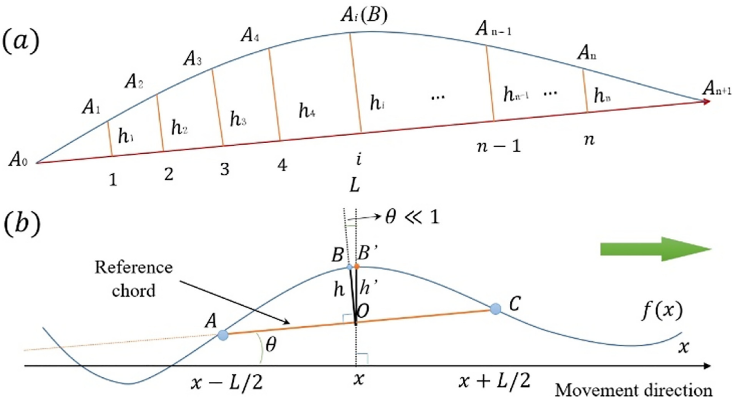

Figure 2. The principle diagram of multi-point chord reference system

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3. Comparison of overall test results

3.1 Comparison of raw waveform results

Both MCR-method and IR-method were simultaneously used to measure the railway line in the upward direction. The total length of the measurement was 1 kilometer. Both MCR and CAT analyzers are dual-track rail corrugation analyzers, with a sampling interval of 1 millimeter. The measurement positions were all at the centerline of the rail's top surface. The raw measurement data from MCR and CAT are compared in Figure 3. From Figure 3, it can be observed that the raw measurement results of MCR-method and IR-method are roughly similar in terms of amplitude.

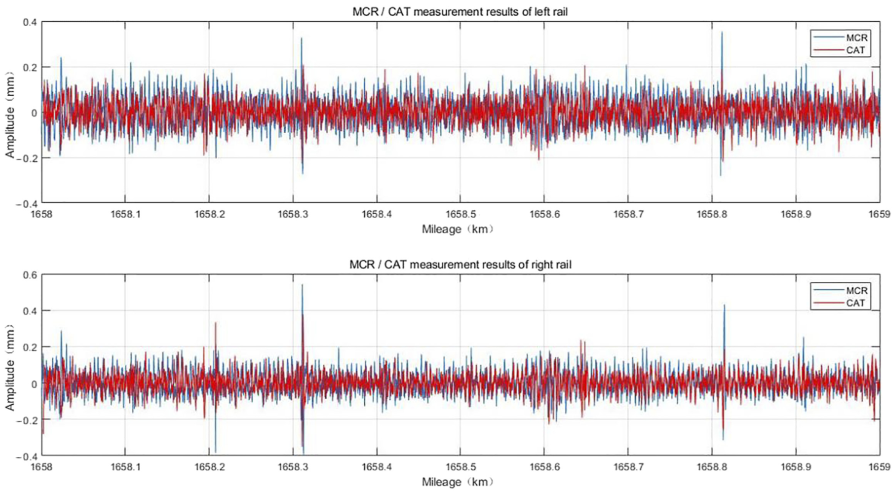

Figure 3. Comparison of MCR/IR method measurement results

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3.2 Comparison of power spectral density (PSD)

The power spectral density (PSD) of the measurement results from both MCR-method and IR-method, filtered within the range of 0.1 m to 1 m, were calculated. The results are shown in Figure 4.

From Figure 4, it can be observed that the PSD values of the measurement results from MCR-method and IR-method are quite close, with only a slight difference in PSD values at the primary frequency.

3.3 Comparison of filtered results

The measurement results from both MCR-method and IR-method were subjected to bandpass filtering. The comparative results for different wavelength ranges, along with local magnification effects, are shown in Figure 5.

Due to the CAT rail corrugation analyzer's susceptibility to false waveforms caused by speed reduction (or stopping), as evident in Figure 5(c), it can be observed that at position K1658 + 640, two large waveforms appear due to two instances of deceleration during actual measurements. However, false waves exceeding 0.2 mm are not present in the field, and this interference needs to be excluded during data analysis. Comparing Figure 5(d) and Figure 5(f), it can be noted that within the 0.01–0.5 m wavelength range, particularly in the case of short waves, the alignment between MCR-method and IR-method result is high. This reflects that the phase synchronization of the measurement results from these two different devices is consistent, which is a prerequisite for the subsequent comparison of measurement results. However, within the 0.01–0.3 m wavelength range, the alignment between MCR-method and IR-method result noticeably decreases, indicating a significant difference between the two devices in measuring long waves. Consequently, this study adopts the third measurement method, the "flat ruler and feeler gauge", to evaluate the accuracy of MCR-method and IR-method within the long-wave range.

From Figure 6, it can be observed that in various wavelength ranges, the majority of measurement results from MCR-method and IR-method exhibit good alignment in terms of waveform and amplitude. In contrast to the results before filtering, it can be observed that

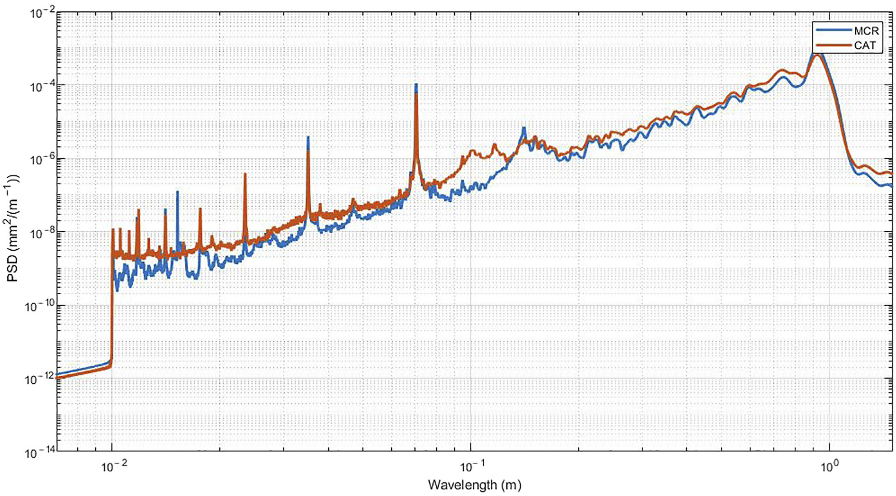

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Figure 4. MCR/IR method measurement results power spectral density (PSD)

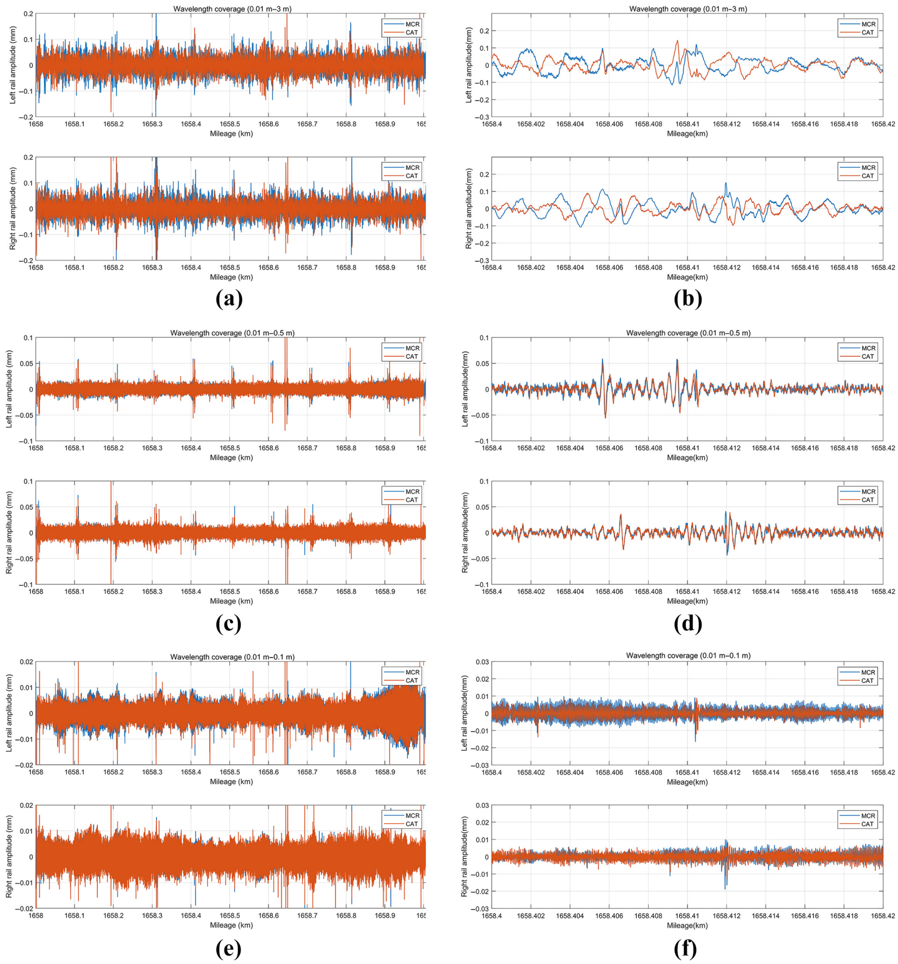

Figure 5.
Comparative results in various wavelength ranges (a) 0.01 m–3 m; (b) Local magnification comparative results in the 0.01 m–3 m; (c) 0.01 m–0.5 m; (d) Local magnification comparative results in the 0.01 m–0.5 m; (e) 0.01 m–3 m; (f) Local magnification comparative results in the 0.01 m–3 m

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after filtering, false waveforms still exist near milepost K1658 + 650 and have not disappeared due to the filtering process.

4. Comparison of rail weld joint position test results

On the testing site, sampling was conducted using a 1-m steel ruler and feeler gauge to measure the rail weld joint position, as illustrated in Plate 2. Within the mileage range of K1658 + 642–646, there is another rail weld joint. Although this location does not correspond to the periodic 100-m rail weld joint positions, there are clear signs of welding scars on the outer side of the left track at this location, as depicted in Plate 3.

The waveforms within a 5-m range at the rail weld joint positions were extracted separately for MCR-method and IR-method, and the amplitudes at the rail weld joint positions were evaluated based on a 1-m chord. The results are presented in Table 2 and Figure 7. From

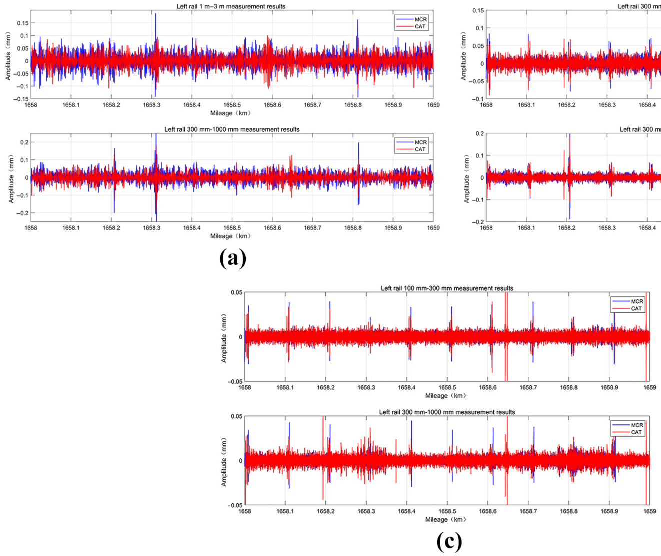

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Figure 6.
Filtering results (a) 1 m–3 m; (b) 300 mm–1000 mm; (c) 100 mm–300 mm

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Plate 2.
On-site 1-m steel ruler + feeler weld

these figures and tables, it can be observed that IR-method is unable to recognize the rail weld joint at all, while MCR-method at the rail weld joint positions are close to the results obtained using the steel ruler and feeler gauge, with errors of approximately 0.05 mm.

It can be seen from the above Figure 7 that the average error of the MCR-method in the actual line is 0.021 mm, and the variance is 0.025 mm. The average error of the measured error of the inertial method is 0.22 mm, and the variance is 2.33 mm. The difference is very significant.

Plate 3. The obvious welding scar position on the measurement outside of the left rail in the range of K1658 + 642–646

Source(s): Authors' own work

Table 2. 1-m steel ruler + feeler measurement results (comparison of weld amplitude after synchronized position)

Serial number	Mileage	Category	Left/ right rail	Feeler result/ mm	MCR method result/mm	IR method result/mm	Figure number
1	K1658 + 705–715	convex	right	0.06	0.06	−0.07	Figure 8(a)
2	K1658 + 305–315	convex	right	0.35	0.33	−0.20	Figure 8(b)
3	K1658 + 642–646	concave	left	−0.1	−0.14	0.09	Figure 9
4	K1658 + 905–915	convex	left	0.1	0.08	0.08	
5	K1658 + 905–915	convex	right	0.05	0.07	0.15	
6	K1658 + 805–815	convex	left	0.25	0.24	−0.08	
7	K1658 + 805–815	convex	right	0.29	0.29	−0.10	
8	K1658 + 705–715	convex	left	0.05	0.1	−0.07	
9	K1658 + 305–315	convex	left	0.24	0.27	0.07	

Source(s): Authors' own work

The measurement results at the rail weld joint positions for MCR-method and IR-method are shown in Figures 8–9.

From Figure 8, it can be observed that although there is a significant difference in measurement results between the two types of rail corrugation analyzers at the rail weld joint positions, there are many similarities in waveforms at the same mileage positions away from the rail weld joints. Within a relatively small range of mileage changes, the relative amplitude

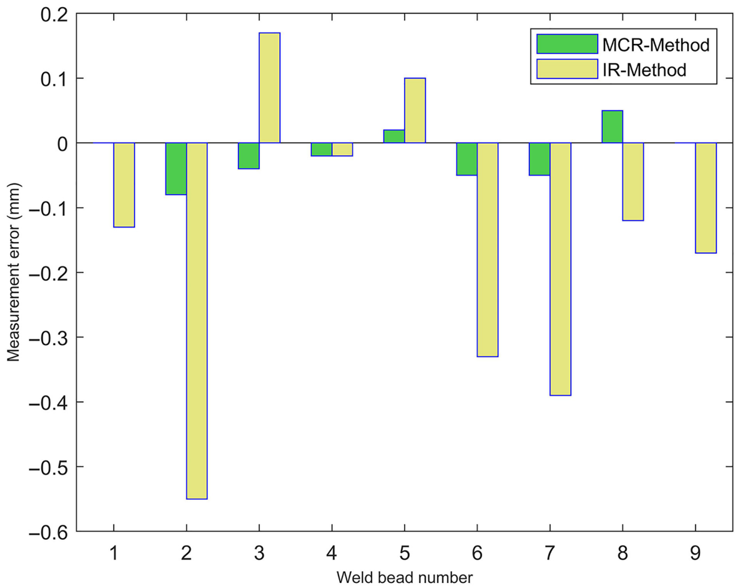

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Figure 7.
Comparison of measurement error from MCR-method/IR-method

changes are very close. This phenomenon indicates that during the comparative testing of MCR-method and IR-method, the phase is essentially synchronized, and the testing conditions are fair. The results can effectively reflect the differences between MCR-method and IR-method. After ensuring the fairness of the test results, further analysis can be conducted on the remaining data.

Based on the actual on-site testing conditions, it can be determined that the position of rail weld joint 3 is a low joint. However, as shown in Figure 9, the IR-method results do not reflect the actual condition of this rail weld joint, while the MCR-method results clearly show a downward concave waveform at rail weld joint. In the measurement results of the MCR-method, there is an abnormal downward spike on the left side of the bottom of the waveform, and after analysis, it was determined that this phenomenon occurred because both the MCR and CAT suddenly stopped at this point when measuring the low joint, resulting in abnormal results.

From Figures 8–9, it can be observed that the measurement results of the MCR-method mostly exhibit a phenomenon where the waveform is either above or below the 1-m chord (indicating low joints) at the rail weld joint positions. This allows for better identification of the rail weld joint positions. In contrast, the measurement results from the IR-method show waveforms fluctuating above and below the 1-m chord, making it difficult to identify the rail weld joints. The data show significant errors in both positive and negative directions, and the amplitudes differ significantly from the results obtained using the steel ruler and feeler gauge method.

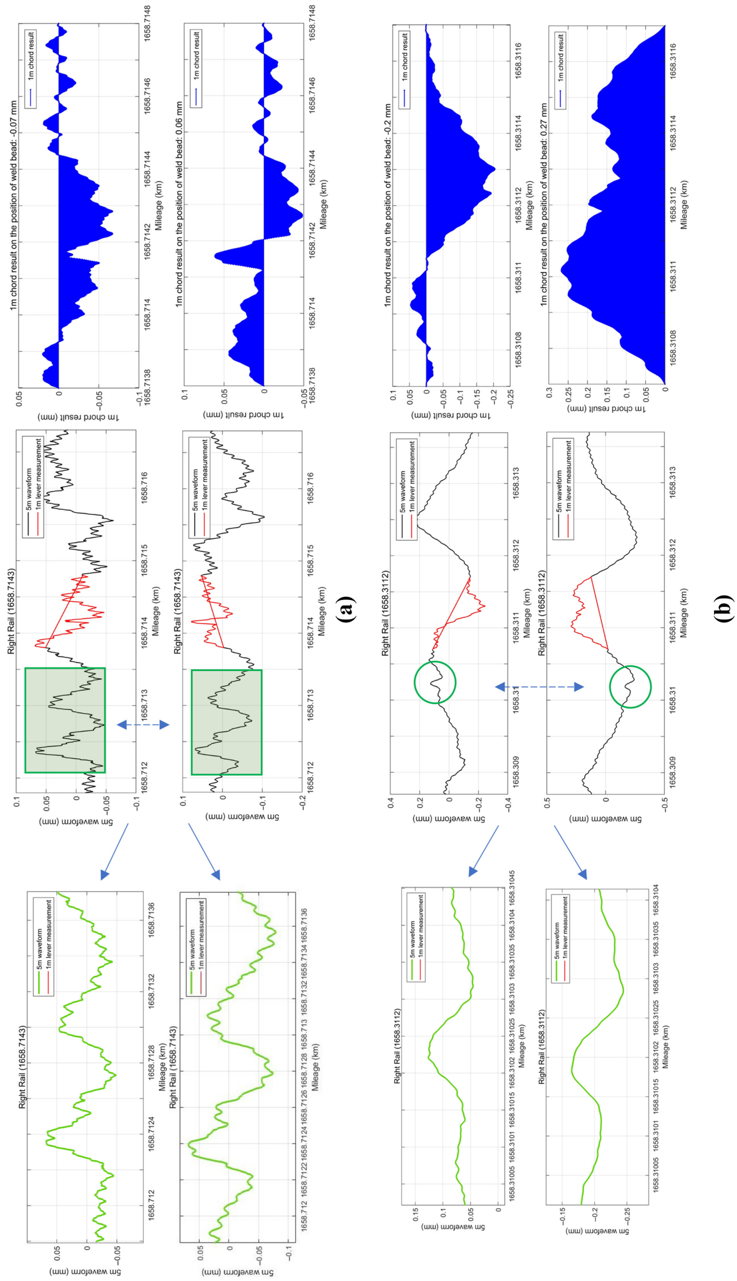

Figure 8.
Rail weld joint 1 and 2 comparison between MCR-method/IR-method results and enlarged view of partial similar regions

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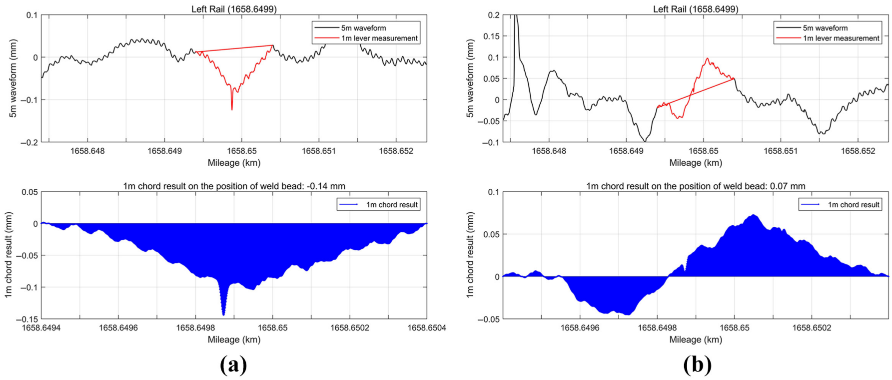

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Figure 9. Comparison of MCR/IR-method results in weld 3 (a) is the result of MCR-method measurement; (b) is the result of IR-method measurement

5. Conclusion

Through the comparative test of a certain section of a high-speed uplink in China, combined with the verification of the flat ruler + feeler, the following main conclusions can be drawn (see Table 3).

Hence, it is evident that the MCR-method performs better than the IR-method in the measurement of wavelengths greater than 3 m. It can adapt to more complex track conditions, such as measuring rail weld joint positions and low joints. According to the actual line measurement results, the average error and the variance of the average error are roughly less than 10% of the IR-method, and the advantage is obvious. Additionally, it offers convenient operation and higher measurement efficiency, making it a more reasonable instrument for assessing rail smoothness. Adopting the MCR-method can significantly enhance the efficiency of railway maintenance personnel and improve inspection reliability.

Furthermore, the test results reflect that domestic advanced rail corrugation measurement instruments have reached a high level of technological sophistication. In comparison to traditional inertia-based methods and standard chord methods, the MCR-

Criteria	IR-method	MCR-method
Operational difficulty	Requires manual balance control, prone to derailment	Operates more stably; less prone to derailment
Measurement efficiency	Slower working speed (2.9 km/h-4.3 km/h); speed variations can lead to abnormal results; appearing false waveform and affecting data analysis	Faster working speed (up to 15 km/h); less affected by speed variations
Measurement content	Unable to detect welds and low joints in the track; significant differences in measurement amplitudes; sometimes opposite directions; the measurement amplitude of the aluminothermic welding position is obviously small	Capable of identifying welds and low joints in the track; measurement results corroborated with measurements using a straightedge and caliper; more reliable; the error is basically maintained at about 0.05 mm

Source(s): Authors' own work

Table 3. A comparison of the advantages and disadvantages of MCR/IR-method in actual testing

method may be a more suitable approach for manufacturing high-precision rail corrugation measurement instruments. This method can be applied to the measurement of rail weld joint flatness within a 5-m wavelength range, and it yields good results. It fills the current gap in measuring the flatness of long-wavelength rail weld joints and greatly aids in understanding the morphology of rail weld joint positions during the service life of railway tracks.

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Corresponding author
Yifan Shi can be contacted at: yifan001213@126.com