2-D periodic structures. A numerical study

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Abstract

Purpose – Within the context of 2D square lattices, searching for the existence of band gaps assumes a great interest owing to many possible fields of application: from energy absorption devices to noise and vibration controllers, as well as advanced strategies for the seismic isolation.

Design/methodology/approach – The underlying microstructure may influence the mechanical response of 2D square lattices according to a complex interplay between different factors. A first one is related to the so-called "size-effect". A second one relates, instead, to the mass density distribution.

Findings – It has been observed that lumped masses may induce additional band gaps to appear and may magnify their width. Finally, an additional factor deals with the inner damping characteristics of the constituent materials, which usually are polymer-based.

Originality/value – This study focuses on the first factor from a specific perspective: to investigate the influence of the size effect on the existence and properties of frequency band gaps.

Keywords 2D lattice structures, Frequency band gaps, RUC

Paper type Research paper

1. Introduction

The study of the dynamic properties of a 2D lattice (Zhou *et al.*, 2009; Wang *et al.*, 2009; Martinsson and Movchan, 2003; Phani *et al.*, 2006; Zhen *et al.*, 2008; Gaofeng and Zhifei, 2010) is usually based on few assumptions. Firstly, by virtue of the Bloch theorem, the hypothesis of infinite lattice points allows to restrict the study to the representative unit cell (RUC).

An example is shown in Figure 1, where the symbols $\mathbf{a}_1$ and $\mathbf{a}_2$ denote the generating vectors along the directions of spatial periodicity. In this example, $\mathbf{a}_1$ and $\mathbf{a}_2$ are normal to each other.

The displacement of an arbitrary point P of the infinite 2D lattice is given by:

$$\mathbf{u}(\mathbf{r}) = \mathbf{u}_k(\mathbf{r}) \exp\left(-i\omega t + \mathbf{k} \cdot \mathbf{r}\right) \tag{1}$$

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where the symbol $\mathbf{r}$ indicates the position vector of a generic material point $\mathbf{P}$, the symbol $\mathbf{k}$ denotes the Bloch wave vector, ω is the angular frequency and $\mathbf{u}_k(\mathbf{r})$ is the amplitude. It is important to remark that $\mathbf{u}_k(\mathbf{r})$ and the point lattice exhibit the same spatial periodicity. Finally, the position vector $\mathbf{r}$ is expressed by:

$$\mathbf{r} = \mathbf{r}_0 + n_1\mathbf{a}_1 + n_2\mathbf{a}_2 \tag{2}$$

where (n_1,n_2) is an integer pair and $\mathbf{r}_0$ is the position vector of $\mathbf{O}$, which is the corresponding point of $\mathbf{P}$ within the RUC. Equation (1) becomes:

$$\mathbf{u}(\mathbf{r}) = \mathbf{u}(\mathbf{r}_0)\exp\left(n_1\mathbf{k}\cdot\mathbf{a}_1 + n_2\mathbf{k}\cdot\mathbf{a}_2\right) \tag{3}$$

The periodic boundary condition for the dynamic analysis of the unit cell assumes the following final form:

$$\mathbf{u}(\mathbf{r}) = \mathbf{u}(\mathbf{r}_0)\exp\left[2\pi(n_1k_1 + n_2k_2)\right] \tag{4}$$

Equation (4) is true if the Bloch wave vector is represented by means of a linear combination of the reciprocal space vectors $\mathbf{b}_1$ and $\mathbf{b}_2$:

$$k = k_1b_1 + k_2b_2 \tag{5}$$

2. Numerical experimentation and assumptions

The numerical results presented in this study deal with two RUC configurations: a primary configuration (I) made of four micro-beams [Figure 2(a)] and a different configuration (II) which has been enhanced by means of four auxiliary micro-beams [Figure 2(b)].

We assume that the spatial periodicity vectors $\mathbf{a}_1$ and $\mathbf{a}_2$ are orthogonal to each other with the same norm a. This implies the study is limited to square lattice only. Moreover we assume the RUC configuration can be modelled by interconnecting straight micro-beams with rigid internal connections. Finally, an appropriate micro-scale parameter is introduced to account for the so-called size effect (Mancusi and Feo, 2013; Mancusi *et al.*, 2017; Mindlin, 1963; Lui and Su, 2009; Onck *et al.*, 2000; Andrews *et al.*, 2000; Park and Gao, 2006; Barretta *et al.*, 2017; Ma *et al.*, 2008). The followed approach consists in considering the contribution of couple stresses in addition to classical Cauchy stresses. To this scope, a further constitutive parameter is required. This parameter has with the physical dimension of a scale length (Mindlin, 1963). We want to remark that couple stresses are considered in a

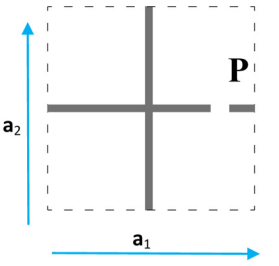

Figure 1.
Two-dimensional square lattice material (example of a RUC)

simplified manner, according to the so-called modified couple stress theory, assuming that only the symmetric part of the rotation gradient contributes to the strain energy density.

Geometric and mechanical information of the reference unit cell considered in this study are listed in the following Table I.

The following hypotheses are also taken into account:

- primary micro-beams are composed of aluminium: E = 9.00 × 104 N/mm^2; v = 0.23; G = 3.66 × 10^4 N/mm^2; l = 6.58 μm; ρ = 2.70 × 10^{-6} kg/mm^3;
- auxiliary micro-beams are composed of aluminium (i), as above, or epoxy resin (ii): E = 1440 N/mm^2; v = 0.38; G = 522 N/mm^2; l = 17.6 μm; ρ =1.10 × 10^{-6} kg/mm^3.

Numerical results are relative to the following two choices:

(1) the micro-scale characteristic length l is zero (i.e. the size effect is discarded); and

(2) the size effect is accounted for.

It is worth remarking that the present analysis represents a generalization of the parametric analysis discussed by Mancusi *et al.* (2017).

Ten finite elements over each micro-beams are considered for the numerical analysis. The convergence rate and the accuracy of the numerical solutions have been adequately assessed (Mancusi and Feo, 2013; Mancusi *et al.*, 2017).

The dimensionless frequencies which are detected by means of an eigenvalue problem, are studied over the boundary of the irreducible part of the first Brillouin zone and are presented in a non-dimensional form as $\tilde{\omega} = \omega/\omega_1$, where:

$$\omega_1 = \frac{\pi^2}{a^2}\sqrt{\frac{EI}{\rho A}} \tag{6}$$

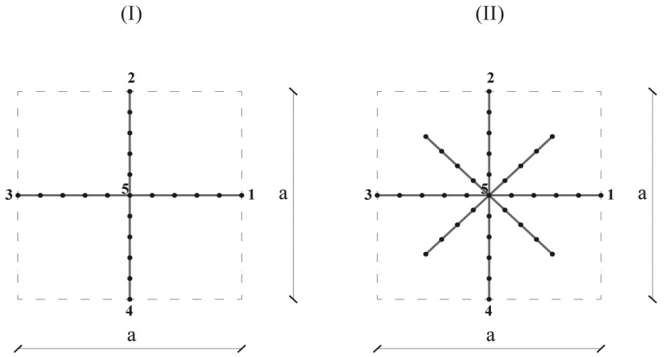

Figure 2.
2D square lattice configurations (a denoting the lattice constant): a first configuration made of four micro-beams (a) and a second one which has been enhanced by means of four auxiliary micro-beams (b)

#	RUC	a [mm]	l_1 [mm]	A [mm^2/mm]	A_s [mm^2/mm]	I [mm^4/mm]	l_2 [mm]
1	I	1.0×10^{-1}	5.0×10^{-2}	5.0×10^{-3}	4.17×10^{-3}	1.04×10^{-8}	0.00
2	II	1.0×10^{-1}	5.0×10^{-2}	5.0×10^{-3}	4.17×10^{-3}	1.04×10^{-8}	3.54×10^{-2}

Table I.
Geometry and mechanical parameters (I/II as in in Figure 2)

In Table II, the main results concerning the lowest natural frequencies (not dimensional) are presented. They refer to the points **O**, **A** and **B** indicated in Figure 3.

A comparison emerges between two different hypotheses concerning the constituent material of the auxiliary micro-beams:

(1) aluminium (i.e. the same material as the primary micro-beams); or

(2) epoxy resin.

Moreover, the influence of the microstructure length is also considered according to the assumptions previously summarized by acronyms "a" and "b".

In Table III, the comparison is finally extended to the position and the width of the first band gaps. The row indicates as "lower band" denotes the number of the frequency band immediately below the considered gap.

3. Final remarks

It has been observed that the position and width of the first band gaps are influenced by the magnitude of the micro-scale parameter l. In general, a forward shift of the first band gaps is related to the size effect. Moreover, flat bands also appear in the low-frequency region if auxiliary micro-beams are present. Finally a not-uniform mass density distribution over the RUC may influence the band structure of the 2D periodic material.

Points		1	2	3	4	5	6	7	8	9	10
1.a	O	0.0	0.0	4.2	4.2	6.1	9.3	18.8	18.8	23.9	29.4
	A	0.8	2.4	3.3	7.0	11.5	13.2	14.5	20.0	25.5	32.9
	B	1.7	2.2	8.9	11.6	11.6	12.7	17.0	17.0	28.9	34.7
1.b	O	0.0	0.0	10.8	10.8	15.9	21.7	37.1	37.1	52.1	56.4
	A	2.0	6.6	7.5	15.9	17.8	25.7	30.4	36.6	51.6	53.2
	B	4.2	5.7	14.1	14.1	22.1	27.4	30.6	30.9	48.2	48.3
2.a.i	O	0.0	0.0	2.6	3.7	4.0	4.3	4.3	4.8	7.9	9.3
	A	0.6	2.2	3.0	3.3	4.2	4.3	4.3	8.4	10.3	11.9
	B	1.5	2.2	3.0	4.2	4.3	4.3	9.7	9.8	10.7	12.7
2.a.ii	O	0.0	0.0	1.1	1.1	1.1	1.1	4.0	4.0	5.8	6.3
	A	0.6	1.1	1.1	1.1	1.2	2.3	3.3	6.2	6.3	6.3
	B	1.0	1.1	1.1	1.1	1.7	2.2	6.1	6.3	6.3	6.3
2.b.i	O	0.0	0.0	7.0	9.6	10.2	11.4	11.4	12.7	19.5	21.7
	A	1.5	6.0	6.8	8.0	10.1	11.4	11.4	14.5	20.4	26.4
	B	3.7	5.7	7.7	8.8	11.4	11.4	11.5	13.8	25.0	27.3
2.b.ii	O	0.0	0.0	1.2	1.2	1.2	1.2	6.9	6.9	6.9	7.0
	A	1.2	1.2	1.2	1.2	2.0	6.2	6.8	6.9	6.9	7.1
	B	1.2	1.2	1.2	1.2	4.2	5.7	6.9	6.9	6.9	6.9

Table II.
First natural frequencies ω_1

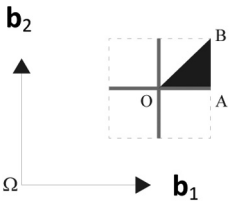

Figure 3.
Irreducible part of the first Brillouin zone

		1	2	3	4	5	6	7	8	9	10
Position	1.a	3.300	13.860	21.936	29.166	35.260	45.858	66.796	74.461	89.245	105.955
Gap		0.046	1.347	3.856	0.472	1.133	7.388	1.283	0.672	5.260	0.910
Lower band		2	6	8	9	10	12	16	18	20	22
Position	1.b	7.263	15.914	28.868	42.659	64.808	74.250	91.520	104.320	118.630	156.655
Gap		0.280	0.044	2.999	11.055	5.240	3.774	7.614	8.200	8.280	10.170
Lower band		2	4	6	8	12	14	16	18	20	26
Position	2.a.i	3.184	13.449	20.105	24.745	26.916	28.006	35.162	45.711	51.342	67.185
Gap		0.115	1.439	2.704	3.840	0.501	1.205	0.942	3.948	7.314	0.505
Lower band		3	10	12	13	15	16	18	22	24	28
Position	2.a.ii	1.068	1.075	3.240	14.644	16.943	22.136	29.207	35.240	44.295	46.572
Gap		0.001	0.013	0.061	0.213	0.012	3.828	0.390	1.094	4.155	0.049
Lower band		2	4	6	18	22	24	25	30	36	38
Position	2.b.i	6.551	7.743	12.121	17.030	28.808	36.918	44.041	47.837	49.800	51.654
Gap		0.139	0.085	1.228	5.022	3.019	0.571	7.288	0.303	2.460	1.002
Lower band		2	3	7	8	10	12	14	16	17	18
Position	2.b.ii	1.205	1.225	6.639	7.299	11.587	12.471	28.859	36.909	42.145	47.197
Gap		0.001	0.040	0.115	0.166	0.843	0.925	2.992	0.157	9.421	0.053
Lower band		2	4	6	10	12	14	22	28	32	34

Table III.
Position and width for the lowest gaps

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