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Nano-beams under torsion: a stress-driven nonlocal approach

Raffaele Barretta
Department of Structures for Engineering and Architecture, University of Naples Federico II, Naples, Italy

Luciano Feo
Department of Civil Engineering, University of Salerno, Fisciano, Italy

Raimondo Luciano
Department of Civil and Mechanical Engineering, University of Cassino and Southern Lazio, Cassino, Italy

Francesco Marotti de Sciarra
Department of Structures for Engineering and Architecture, University of Naples Federico II, Naples, Italy, and

Rosa Penna
Department of Civil Engineering, University of Salerno, Fisciano, Italy

Abstract

Purpose – This study aims to model scale effects in nano-beams under torsion.

Design/methodology/approach – The elastostatic problem of a nano-beam is formulated by a novel stress-driven nonlocal approach.

Findings – Unlike the standard strain-driven nonlocal methodology, the proposed stress-driven nonlocal model is mathematically and mechanically consistent. The contributed results are useful for the design of modern devices at nanoscale.

Originality/value – The innovative stress-driven integral nonlocal model, recently proposed in literature for inflected nano-beams, is formulated in the present submission to study size-dependent torsional behavior of nano-beams.

Keywords Carbon nanotubes, Torsion, Integral elasticity, Nano-beams

Paper type Research paper

1. Introduction

Micro- and nano-devices are structures whose characteristic size (thickness, diameter, etc.) is in the order of the micron and the sub-micron. Such elements are diffusely adopted as sensors and actuators (Li *et al.*, 2008) and are scale-dependent (Kahrobaiyan *et al.*, 2011;

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Tajalli *et al.*, 2009; Lam *et al.*, 2003; McFarland and Colton, 2005). Size effects were observed during micro-torsion tests on thin copper wires (Fleck *et al.*, 1994).

Beam-like components under torsional loading conditions and/or prescribed torsional displacements are used in several micro- and nano-electromechanical systems, such as microscanners (Arslan *et al.*, 2010), micromirrors (Zhang *et al.*, 2001; Huang *et al.*, 2006), micro-gyroscopes (Maenaka *et al.*, 2005) and springs (Papadakis *et al.*, 2004).

A consistent modeling of nano-structures is hence crucial for the design and optimization of modern nano-systems.

Nowadays, methodologies of nonlocal continuum mechanics are widely exploited for the analysis of nano-structures (Peddieson *et al.*, 2003; Reddy, 2007; Wang and Liew, 2007; Aydogdu, 2009; Civalek and Demir, 2011; Thai and Vo, 2012; Rafiee and Moghadam, 2014; Sedighi, 2014; Sedighi *et al.*, 2015; Barretta *et al.*, 2016b, 2016a; Feo and Penna, 2016a, 2016b).

They are mainly based on nonlocal theory by Eringen (1983).

Material parameters in nonlocal models, in addition to the classical elastic constants, are introduced to capture size effects.

Evaluation of constitutive parameters can be performed by micro-bending, micro-torsion and micro/nano indentation tests (Fleck *et al.*, 1994; Paliwal *et al.*, 2012; Brcic *et al.*, 2013; Song *et al.*, 2014).

However, the strain-driven integral elastic model proposed by ERINGEN cannot be adopted for nonlocal structural problems at nanoscale. This issue has been discussed in detail in Romano *et al.* (2017). A mechanically consistent stress-driven integral elastic model for inflected nano-beams has been recently developed in Romano and Barretta (2017a, 2017b). Comparisons between strain-driven and stress-driven nonlocal formulations have been carried out in Romano and Barretta (2017a, 2017b). Free vibrations of BERNOULLI-EULER nano-beams have been investigated in Apuzzo *et al.* (2017).

The motivation of the present paper is in applying the new stress-driven integral elastic theory to torsion of nano-beams.

The plan is the following. Basic equations governing the elastic equilibrium problem of a nano-beam under torsion, formulated according to the new stress-driven integral constitutive model, are provided in Section 2. Size-effects are computed for cantilever and doubly clamped nano-beams under uniform distributions of couples per unit length in Section 3. Concluding remarks are given in Section 4.

2. Stress-driven integral elastic model for circular nano-beams under torsion

Let us consider a circular beam at nanoscale of length L subjected to a distribution of torsional couples per unit length m in the interval [0,L] and concentrated couples $\mathcal{M}$ at the end cross-sections.

The abscissa along the nano-beam axis will be denoted by x.

The geometric torsional curvature, kinematically compatible with the torsional rotation field θ, is given by:

$$\chi = \frac{d\theta}{dx}. \tag{1}$$

Equilibrium equations write as:

$$\begin{cases} \dfrac{dM}{dx} = -m\,, & \text{in} \quad [0,\mathrm{L}]\,, \\ M = \mathcal{M}\,, & \text{at} \quad \{\,0,\mathrm{L}\,\} \end{cases} \tag{2}$$

with M twisting moment.

The proposed stress-driven nonlocal model for twisted nano-beams is defined by the following convolution:

$$\chi_{el}(x) = \int_0^L \phi_\lambda(x-\xi)\, C(\xi)\, M(\xi)\, d\xi, \tag{3}$$

with χ_{el} torsional elastic curvature, C local elastic compliance and ϕ_λ kernel function depending on a dimensionless nonlocal parameter $\lambda > 0$.

Denoting by μ is the local shear modulus, the torsional elastic compliance C is the inverse of the local elastic stiffness:

$$K := \mu J, \tag{4}$$

with J polar moment of inertia about the center of the circular cross-section.

The kernel function fulfills symmetry, positivity and limit impulsivity:

$$\begin{cases} \phi_\lambda(x-\xi) = \phi_\lambda(\xi - x) \geq 0, \\ \lim\limits_{\lambda \to 0} \phi_\lambda(x) = \delta(x), \end{cases} \tag{5}$$

where δ is the DIRAC unit impulse at $0 \in \mathcal{R}$ and the limit being intended in terms of distributions:

$$\lim_{\lambda \to 0} \int_{-\infty}^{+\infty} \phi_\lambda(x-\xi) \cdot f(\xi)\, d\xi = f(x), \quad \forall f \in C^0(\mathcal{R};\mathcal{R}), \tag{6}$$

Hereafter, we assume the following special form of the kernel:

$$\phi_\lambda(x) := \frac{1}{2\,L_c} \exp\left(-\frac{|x|}{L_c}\right), \tag{7}$$

where the length characteristic L_c, expressing the amplitude of the range of nonlocal action, is defined by $L_c := \lambda L$. It can be proven that the output of the stress-driven integral convolution Equation (3), described by the special kernel Equation (7), provides the unique solution of the constitutive differential equation:

$$\frac{\chi_{el}(x)}{L_c^2} - \frac{d^2 \chi_{el}}{dx^2}(x) = \frac{C \cdot M}{L_c^2}(x), \tag{8}$$

With the constitutive boundary conditions:

$$\begin{cases} \dfrac{d\chi_{el}}{dx}(0) = \dfrac{1}{L_c} \cdot \chi_{el}(0), \\ -\dfrac{d\chi_{el}}{dx}(L) = \dfrac{1}{L_c} \cdot \chi_{el}(L), \end{cases} \tag{9}$$

Geometric and elastic torsional curvature fields are assumed to be coincident in the sequel $\chi = \chi_{el}$, since a purely elastic constitutive behavior is considered.

3. Examples

The stress-driven nonlocal model illustrated in Section 2 is adopted hereafter to examine the size-dependent structural behavior of nanocantilever and doubly clamped nano-beams of length L subjected to a uniform distribution m of couples per unit length.

Twist elastic rotation fields are obtained by substituting Equation (1) in Equations (8) and (9) and prescribing differential and boundary conditions of equilibrium on the torsional moment M and kinematic boundary conditions on θ.

To this end, let us now introduce the following dimensionless parameters:

$$\begin{cases} \xi = \dfrac{x}{L}, \\ \theta^{*}(\xi) = \dfrac{k}{m L^{2}} \theta(\xi). \end{cases} \quad (10)$$

Torsional rotations θ^* versus ξ of both the nano-beams are displayed in Figures 1 and 2 for the following values of the nonlocal parameter:

$$\lambda \in \{0, 0.1, 0.2, 0.3, 0.4, 0.5\}, \quad (11)$$

4. Closing remarks

The outcomes of the present paper may be summarized as follows:

- Size-dependent behavior of nano-beams under torsion has been investigated by an innovative stress-driven nonlocal elastic model;

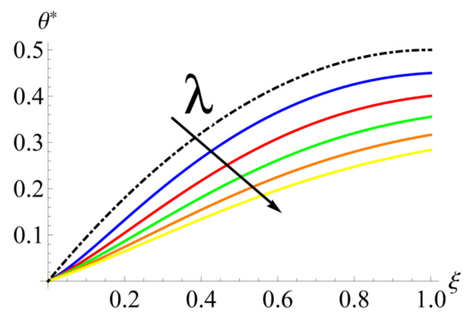

Figure 1. Nanocantilever under a uniform distribution m of couples: torsional rotation θ^* vs ξ for increasing values of λ

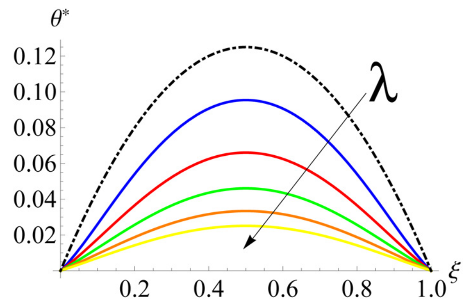

Figure 2. Doubly clamped nano-beam under a uniform distribution m of couples: torsional rotation θ^* vs ξ for increasing values of λ

- Unlike the Eringen strain-driven nonlocal integral elastic model which cannot be applied to nano-structures of technical interest, the stress-driven theory is mathematically consistent and useful for nano-electromechanical system applications;
- The proposed nonlocal strategy has been illustrated with reference to nano-cantilever and doubly clamped nano-beams subjected to a uniform distribution of torsional couples per unit length; and
- As shown in Figures 1 and 2, the stress-driven model provides an elastic stiffness increasing with the nonlocal parameter λ.

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Corresponding author
Raffaele Barretta can be contacted at: rabarret@unina.it