Hierarchy grey relational analysis using DEA and AHP

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Abstract

Purpose – This paper aims to apply an integrated data envelopment analysis (DEA) and analytic hierarchy process (AHP) approach to a multi-hierarchy grey relational analysis (GRA) model. Consistent with the most real-life applications, the authors focus on a two-level hierarchy in which the attributes of similar characteristics can be grouped into categories. Nevertheless, the proposed approach can be easily extended to a three-level hierarchy in which attributes might also belong to different sub-categories and further be linked to categories.

Design/methodology/approach – The procedure of incorporating the DEA and AHP methods in a two-level GRA may be broken down into a series of steps. The first three steps are under the heading of attributes and the latter three steps are under the heading of categories as follows: computing the grey relational coefficients of attributes for each alternative using the basic GRA model which further provides the required (output) data for an additive DEA model; computing the priority weights of attributes and categories using the AHP method which provides *a priori* information on the adjustments of attributes and categories in additive DEA models; computing the grey relational grades of attributes in each category for alternatives using an additive DEA model; converting the grey relational grades of attributes to the grey relational coefficients of categories; computing the grey relational grades of categories for alternatives using an additive DEA model; computing the dissimilarity grades of categories for the tied alternatives using an additive DEA exclusion model.

Findings – The proposed approach provides a more reasonable and encompassing measure of performance in a hierarchy GRA, based on which the overall ranking position of alternatives is obtained. A case study of a wastewater treatment technology selection verifies the effectiveness of this approach.

Originality/value – This research is a step forward to overcome the current shortcomings in a hierarchy GRA by extracting the benefits from both the objective and subjective weighting methods.

Keywords Data envelopment analysis, Analytic hierarchy process, Grey relational analysis, Hierarchical structures, Weighing

Paper type Research paper

1. Introduction

Grey relational analysis (GRA) is a multi-attribute decision-making (MADM) tool that provides a single measure of performance for each alternative with respect to a set of incommensurate attributes. Nevertheless, the traditional GRA is only limited to the situations with a single level of attributes, which might not entirely satisfy the need for increasingly complex MADM problems. In real-world applications, there are a great number of MADM activities which not only need to be represented by a set of attributes, but these

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attributes might also belong to different categories constituting a hierarchical structure. Figure 1 illustrates a complex MADM problem into a system of hierarchies in which a set of alternatives lies at the lowest level, and attributes, categories and the overall objective of the decision are on the higher levels of this hierarchy, respectively. For example, the problems of selecting wastewater treatment plants (Zeng *et al.*, 2007), renewable electricity generation technologies (Sarucan *et al.*, 2011), natural gas pipeline operation schemes (Jia *et al.*, 2011), coal-fired power plants (Xu *et al.*, 2011), biomass briquette fuel system schemes (Wang *et al.*, 2015), weapon equipment systems (Guoqing and Lin, 2015), call center sites (Birgun and Gungor, 2014), firms demanding commercial credits (Ertuğ and Girginer, 2015), advertising spokesmen (Hsu and Su, 2008) and stock investments (Li *et al.*, 2010).

These studies use the analytic hierarchy process (AHP) in a multi-level GRA, known as hierarchy GRA. AHP is a subjective data-oriented procedure that determines the relative priorities of attributes based on the formal expressions of decision makers' preferences (Saaty, 1987). The application of AHP not only overcomes the drawback of assigning uniform weights to each attribute by GRA, but also incorporates the effects of attribute (sub) categories in the performance of alternatives. However, since the introduction of AHP in 1980, it has been a target of criticism due to its subjective nature of producing weights (Swim, 2001; Dyer, 1990). Therefore, hierarchy GRA may not result in the best ranking position for each alternative in comparison to all the other alternatives. This flaw can be corrected by integrating data envelopment analysis (DEA) in hierarchy GRA. DEA is an objective data-oriented approach that allows each alternative (known as a decision-making unit in the DEA terminology) to choose its own favorable system of weights to optimize its relative performance (Cooper *et al.*, 2011). This flexibility in selecting the weights, on the other hand, may be undesirable for some decision makers because it may place an alternative in the best ranking position for unlikely weight combinations. By noting the problematic contradiction between objective weights in DEA and subjective weights in AHP, this research is intended to develop an integrated DEA and AHP approach in a multi-level GRA framework. Therefore, it can provide more reasonable and encompassing results for ranking alternatives in GRA. The integration of both the DEA and AHP methods in a single-level GRA can be found in Pakkar (2016a, 2016b). Pakkar (2016a) explores the tradeoff relationship between the objective weights obtained by DEA and the

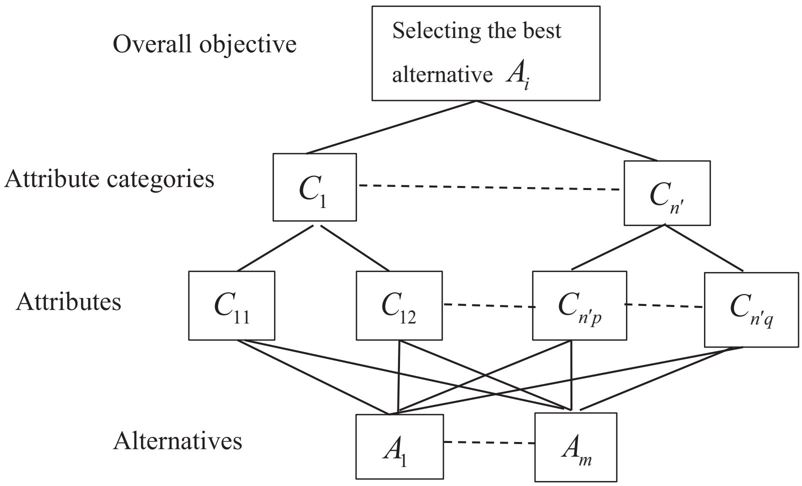

Figure 1. The hierarchical structure of a complex MADM problem

subjective weights obtained by AHP in a GRA methodology. This may result in various ranking positions for each alternative in comparison to the other alternatives. Pakkar (2016b) applies a pair of additive DEA models in a fuzzy multi-attribute GRA methodology to assess the overall performance of alternatives from both the optimistic and pessimistic perspectives. In this approach, the attribute weights obtained by additive DEA models are bounded by AHP. Nonetheless, as mentioned earlier, none of the proposed models consider the hierarchical structures of attributes. Simply treating all the attributes to be at the same level obviously ignores the hierarchical information and further leads up to invalid and unstable measures of performance assessment for alternatives. Therefore, the approach proposed in this research is a step forward to overcome the current shortcomings in a hierarchy GRA by extracting the benefits from both the objective and subjective weighting methods.

2. The proposed approach

As mentioned earlier, we focus on those MADM problems in which the attributes of similar characteristics can be grouped into different categories to construct a two-level hierarchy. The procedure of incorporating DEA and AHP in a two-level GRA may be broken down into the following steps (Figure 2).

2.1 Step 1: Computations at the level of attributes

- Computing the grey relational coefficients of attributes for each alternative using the basic GRA model which further provides the required (output) data for an additive DEA model;
- Computing the priority weights of attributes and categories using the AHP method which provides *a priori* information on the adjustments of attributes and categories in additive DEA models; and
- Computing the grey relational grades of attributes in each category for alternatives using an additive DEA model.

2.2 Step 2: Computations at the category level

- Converting the grey relational grades of attributes to the grey relational coefficients of categories;
- Computing the grey relational grades of categories for alternatives using an additive DEA model; and
- Computing the dissimilarity grades of categories for the tied alternatives using an additive DEA exclusion model.

Note that the idea of the two-level hierarchy is consistent with the most real-world applications. Nevertheless, the proposed approach can be easily extended to a three-level hierarchy in which attributes might also belong to different sub-categories and further be linked to categories (Appendix 1).

2.3 Basic grey relational analysis

Let y_{ik} be the value of attribute C_k ($k = 1, 2, \ldots, n$) for alternative A_i ($i = 1, 2, \ldots, m$) in an MADM problem. The term y_{ik} can be translated into the comparability value r_{ik} by using the following equations:

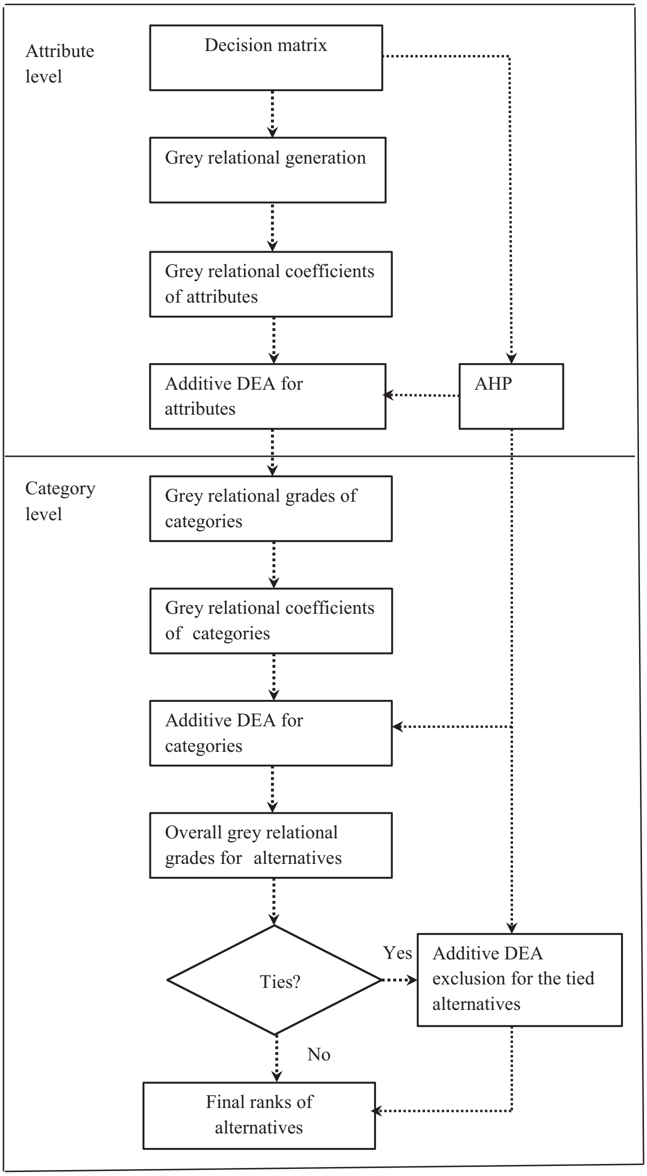

Figure 2.
The flowchart of a two-level hierarchy GRA using DEA and AHP

$$r_{ik} = \frac{y_{ik}}{y_{k(\max)}} \forall i, k \quad \text{for} \quad \text{desirable attributes} \tag{1}$$

$$r_{ik} = \frac{y_{k(\min)}}{y_{ik}} \forall i, k \quad \text{for} \quad \text{undesirable attributes} \tag{2}$$

where $y_{k(\max)} = \max\{y_{1k}, y_{2k}, \ldots, y_{mk}\}$ and $y_{k(\min)} = \min\{y_{1k}, y_{2k}, \ldots, y_{mk}\}$. Note that desirable attributes satisfy the property of "the larger the better" and undesirable attributes satisfy the property of "the smaller the better". To eliminate the scale differences between all attributes, and moreover, ensure that all of them are in the same direction of change, equations (1) and (2) are used. Now, let u_{0k} be the reference value for an ideal alternative, A_0, as follows:

$$u_{0k} = \max\{r_{1k}, r_{2k}, \ldots, r_{mk}\} \forall k \tag{3}$$

Then the ideal alternative, A_0, can be defined as a virtual alternative which is characterized by a reference sequence of the maximum values of all attributes. To measure the degree of similarity of alternative A_i to the ideal alternative A_0, with respect to each attribute, the grey relational coefficient, ξ_{ik} (a distance function), can be calculated as follows:

$$\xi_{ik} = \frac{\min_i \min_k \left| u_{0k} - r_{ik} \right| + \rho \max_i \max_k \left| u_{0k} - r_{ik} \right|}{\left| u_{0k} - r_{ik} \right| + \rho \max_i \max_k \left| u_{0k} - r_{ik} \right|} \tag{4}$$

where $|u_{0k} - r_{ik}|$ represents the absolute deviation of each alternative from the ideal alternative with respect to a particular attribute. Obviously, ξ_{ik} decreases when $|u_{0k} - r_{ik}|$ increases and ξ_{ik} increases when $|u_{0k} - r_{ik}|$ decreases. $\min_i \min_k |u_{0k} - r_{ik}|$ and $\max_i \max_k |u_{0k} - r_{ik}|$ are the minimum and maximum absolute deviations among all alternatives with respect to all attributes. $\rho \in [0, 1]$ is the distinguishing coefficient, which adjusts the range of the grey relational coefficient. The smaller the ρ is, the greater is its distinguishing power. Generally it is taken as 0.5. To find an aggregated measure of similarity of alternative A_i to the ideal alternative A_0, over all the attributes, the grey relational grade, Γ_i, can be computed as follows:

$$\Gamma_i = \sum_{k=1}^{n} w_k \xi_{ik} \tag{5}$$

Where w_k is the weight of attribute C_k and $\sum_{k=1}^{n} w_k = 1$. In practice, expert judgments using AHP are often used to obtain the weights of attributes. When such information is unavailable, equal weights seem to be a norm. In the next section, we show how the hierarchical structures of attributes can be incorporated in a traditional GRA method to constitute a two-level hierarchy in GRA.

2.4 Two-level grey relational analysis

The computational structure of a two-level GRA is illustrated in Figure 3. Suppose y_{ijk} is the value of attribute C_{jk} $(k = p, p + 1, \ldots, q)$ in category $C'_j (j = 1, 2, \ldots, n')$ for alternative A_i $(i = 1, 2, \ldots, m)$ while $1 \leq p \leq q \leq n$. Using equations (1)-(5), the grey relational grade of attributes in category C'_j for alternative A_i, denoted as Γ_{ij}, can be computed as follows:

$$\Gamma_{ij} = \sum_{k=p}^{q} w_{jk}\xi_{ijk} \tag{6}$$

where ξ_{ijk} and w_{jk} are the grey relational coefficient and the weight of attribute C_{jk} in category C'_j for alternative A_i. Again, using equations 3-(5) on Γ_{ij}, the grey relational grade of categories for alternative A_i, denoted as Γ'_i, is obtained as follows:

$$\Gamma'_i = \sum_{j=1}^{n'} w_j \xi_{ij} \tag{7}$$

2.5 The analytic hierarchy process

The AHP procedure for computing the priority weights of attributes and their categories may be broken down into the following steps:

Step 1: A decision maker makes a pairwise comparison matrix of different attributes of each category, denoted by B with the entries of $b_{jkk'}$ $(k = k' = p, p+1, \ldots, q)$ while $1 \leq p \leq q \leq n$. The comparative importance of attributes is provided by the decision maker using a rating scale. Saaty (1987) recommends using a 1-9 scale. In a similar way, a pairwise comparison matrix can be made to compare the importance of each category. This matrix is denoted by D with the entries of $d_{jj'}$ $(j = j' = 1, 2, \ldots, n')$.

Step 2: The AHP method obtains the priority weights of attributes of each category by computing the eigenvector of matrix B (equation 8), $W_j = (w_{jp}, w_{jp+1}, \ldots, w_{jq})^T$, which is related to the largest eigenvalue, $\gamma_{\max}$:

$$BW_j = \gamma_{\max} W_j \tag{8}$$

In a similar way, the priority weights of each category are obtained by computing the eigenvector of matrix D (equation 9), $W = (w_1, w_2, \ldots, w_{n'})^T$, which is related to the largest eigenvalue, $\gamma_{\max}$:

$$DW = \gamma_{\max} W \tag{9}$$

To determine whether the inconsistency in a comparison matrix is reasonable, the random consistency ratio, $C.R$, can be computed by the following equation:

$$C.R = \frac{\gamma_{\max} - N}{(N-1)R.I} \tag{10}$$

where $R.I$ is the average random consistency index and N is the size of a comparison matrix.

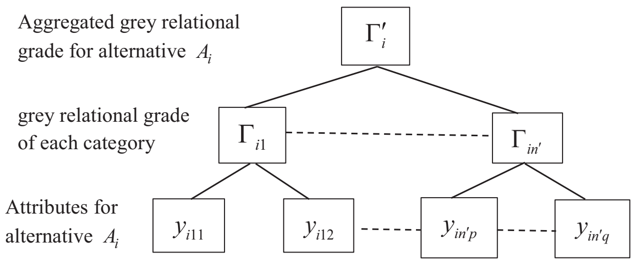

Figure 3.
The computational structure of a two-level GRA

2.6 Additive DEA models

To compute the grey relational grade of attributes in a particular category for each alternative, an additive DEA model can be developed in which all the grey relational coefficients, ξ_{ijk}, are treated as outputs. This model is similar to the additive model in Cooper *et al.* (1999) without explicit inputs as follows:

$$
\begin{aligned}
& P_{oj} = \max \sum_{\substack{k=p \\ 1 \le p \le q}}^{q} w_{jk} s_{jk} \\
& \text{s.t.} \sum_{i=1}^{m} \lambda_{ij} \xi_{ijk} - s_{jk} = \xi_{ojk} \quad \forall k, \\
& \sum_{i=1}^{m} \lambda_{ij} = 1 \\
& s_{jk}, \lambda_{ij} \ge 0,
\end{aligned}
\tag{11}
$$

while $0 \le P_{oj} \le 1$, and $1 - P_{oj}$ indicates the grey relational grade, Γ_{oj} ($o = 1,2,\ldots, m$, $j = 1, 2, \ldots, n'$), of attributes in category C'_j for alternative under assessment A_o (known as a decision-making unit in the DEA terminology). S_{jk} is the slack variable of attribute C_{jk} ($k = p, p+1,\ldots, q$) in category C'_j, expressing the difference between the performance of a composite alternative and the performance of the assessed alternative with respect to each attribute. In other words, S_{jk} identifies a shortfall in the attribute value of C_{jk} of category C'_j for alternative A_o. Obviously, when $P_{oj} = 0$, alternative A_o is considered as the best alternative in comparison with all the other alternatives in category C'_j. w_{jk} is the priority weight of attribute C_{jk} of category C'_j which is defined out of the internal mechanism of DEA using AHP, and λ_{ij} is the weight of alternative A_i ($i = 1, 2, \ldots, m$) in category C'_j. The convexity constraint in Model (11) meets the assumption of *variable returns-to-scale* frontier for an additive model. Similarly, we can develop a model to obtain the grey relational grade of categories for each alternative as follows:

$$
\begin{aligned}
& P_o = \max \sum_{j=1}^{n'} w_j s_j \\
& \text{s.t.} \sum_{i=1}^{m} \lambda_i \xi_{ij} - s_j = \xi_{oj} \quad \forall j, \\
& \sum_{i=1}^{m} \lambda_i = 1 \\
& s_j, \lambda_i \ge 0,
\end{aligned}
\tag{12}
$$

while $0 \le P_o \le 1$ and $1 - P_o$ indicates the grey relational grade, $\Gamma'_o (o = 1, 2, \ldots, m)$, of categories for alternative under assessment A_o S_j is the slack variable of category C'_j. w_j is the priority weight of category C'_j, obtained by AHP, and λ_i is the weight of alternative A_i ($i = 1, 2, \ldots, m$). One should notice that the additive DEA models bounded by AHP does not necessarily yield results that are different from those obtained from the original additive DEA models (Charnes *et al.*, 1985). In particular, it does not increase the power of discrimination between the considerable number of alternatives which form the best practice-frontier. The alternatives on this frontier are usually ranked in the first place by obtaining the grey relational grades of 1. To eliminate the ties that occur for the best alternatives, we propose model (13) that is similar to the additive DEA exclusion (or super-efficiency) model in Du *et al* (2010) without explicit inputs:

$$
\begin{aligned}
&\alpha_o = \min \sum_{j=1}^{n'} w_j t_j \\
&s.t. \sum_{i=1, i \neq o}^{m} \lambda_i \xi_{ij} \geq \xi_{oj} - t_j \quad \forall j, \\
&\sum_{i=1, i \neq o}^{m} \lambda_i = 1 \\
&t_j, \lambda_i \geq 0,
\end{aligned} \tag{13}
$$

After removing alternative A_o from the best practice frontier of model (12), we need to decrease the grey relational coefficients of categories for alternative A_o to reach the frontier constructed by the remaining alternatives. Note that the value of objective function, α_o, can be considered as a dissimilarity grade between alternative A_o and the remaining alternatives. t_j is a slack variable representing a decrease in the grey relational coefficient of category C'_j for alternative A_o to reach the frontier.

3. Case study

In this section, we present the application of the proposed approach to assess the performance of four wastewater treatment technology alternatives: anaerobic/anoxic/oxic (A_1), triple oxidation ditch (A_2), anaerobic single oxidation ditch (A_3) and

Goal	Categories	Attributes	Alternatives 1	2	3	4
Wastewater treatment technology selection	C'_1 economic category	C_{11} capital cost ($\times 10^4$ RMB)	13,762	12,080	12,375	11,870
		C_{12} O&M cost ($\times 10^4$ RMB)	7,612	8,747	8,126	8,233
		C_{13} land area ($\times 10^4$ m^2)	9.88	11.78	11.93	9
	C'_2 technical category	C_{24} removal efficiency of nitrous and phosphorous pollutants	G (0.7)	M (0.5)	E (0.9)	M (0.5)
		C_{25} sludge disposal effect	P (0.3)	G (0.7)	G (0.7)	P (0.3)
		C_{26} stability of plant operation	G (0.7)	E (0.9)	E (0.9)	G (0.7)
		C_{27} maturity of technology	E (0.9)	G (0.7)	G (0.7)	P (0.3)
	C'_3 administrative category	C_{38} professional skills required for operation and maintenance	M (0.5)	E (0.9)	G (0.7)	M (0.5)

Table I. Data for wastewater treatment technology selection

Linguistic values	Quantity
Excellent (E)	0.9
Good (G)	0.7
Moderate (M)	0.5
Poor (P)	0.3
Very Poor (VP)	0.1

Table II. Linguistic values scale

sequencing batch reactor (A_4) with respect to eight attributes which are grouped into three attribute categories. Table I presents the required data as adopted from Zeng *et al.* (2007). Note that some attributes are provided by the numerical values and some are by the quantification of the linguistic values of experienced decision makers based on Table II (Zeng *et al.*, 2007). Capital cost, operation and maintenance (O & M) cost and land area are undesirable attributes while the other attributes are desirable. These data are turned into the comparability sequence by using equations (2) and (3) as presented in Table III. Using Equation (4), all grey relational coefficients for attributes are

Table III. Results of grey relational generation for wastewater treatment technology selection

Categories	Attributes	Alternative technologies A_0	A_1	A_2	A_3	A_4
C'_1	C_{11}	1	0.86	0.98	0.96	1
	C_{12}	1	1	0.87	0.94	0.93
	C_{13}	1	0.91	0.76	0.75	1
C'_2	C_{24}	1	0.78	0.56	1	0.56
	C_{25}	1	0.43	1	1	0.43
	C_{26}	1	0.78	1	1	0.78
	C_{27}	1	1	0.78	0.78	0.33
C'_3	C_{38}	1	0.56	1	0.78	0.56

Table IV. Results of grey relational coefficients for attributes

Categories	Attributes	Alternative technologies A_1	A_2	A_3	A_4
C'_1	C_{11}	0.705	0.944	0.893	1
	C_{12}	1	0.72	0.848	0.827
	C_{13}	0.788	0.583	0.573	1
C'_2	C_{24}	0.604	0.432	1	0.432
	C_{25}	0.37	1	1	0.37
	C_{26}	0.604	1	1	0.604
	C_{27}	1	0.604	0.604	0.333
C'_3	C_{38}	0.432	1	0.604	0.432

Table V. The priority weights of attributes and categories obtained by AHP

Goal	Categories	Weights	Attributes	Weights
Prioritizing attributes and categories	C'_1 economic category	0.6371	C_{11} capital cost	0.6371
			C_{12} O&M cost	0.1052
			C_{13} land area	0.2581
	C'_2 technical category	0.2581	C_{24} removal efficiency of nitrous and phosphorous pollutants	0.2271
			C_{25} sludge disposal effect	0.1904
			C_{26} stability of plant operation	0.2483
			C_{27} maturity of technology	0.3345
	C'_3 administrative category	0.1052	C_{38} professional skills required for operation and maintenance	1

computed to provide the required (output) data for the additive DEA model (11) as shown in Table IV.

Note that grey relational coefficients depend on the distinguishing coefficient ρ, which here is 0.50. Table V depicts the hierarchical structure of attributes for wastewater treatment technologies and the corresponding priority weights in the AHP model as constructed by Zeng *et al.* (2007).

For the attributes and categories shown in Table V, four comparison matrices need to be elicited from the decision maker–three for computing the weights of attributes with respect

Categories	C_1'	C_2'	C_3'	Weights
C_1'	1	3	5	0.6371
C_2'	1/3	1	3	0.2581
C_3'	1/5	1/3	1	0.1052

Notes: $\gamma_{\max}$ = 3.0379; $C.R$ = 0.0327

Table VI. Pairwise comparison matrix at category level

	Alternative technologies				
Categories	A_0	A_1	A_2	A_3	A_4
C_1'	1	1	0.845	0.851	1
C_2'	1	1	0.871	1	0.562
C_3'	1	0.432	1	0.604	0.432

Table VII. Grey relational grades with respect to each category

	Alternative technologies			
Categories	A_1	A_2	A_3	A_4
C_1'	1.000	0.648	0.656	1.000
C_2'	1.000	0.688	1.000	0.393
C_3'	0.333	1.000	0.418	0.333

Table VIII. Grey relational coefficients with respect to each category

Alternatives	P_o	$\Gamma'_o = 1 - P_o$	Rank
A_1	0	1	1
A_2	0	1	1
A_3	0	1	1
A_4	0.157	0.843	4

Table IX. Overall grey relational grades for alternatives

Alternatives	α values	Rank
A_1	0.219	2
A_2	0.656	3
A_3	0.009	1

Table X. Dissimilarity grades for alternatives using additive DEA exclusion to each category and one for estimating the priority weights of categories with respect to the problem goal. Table VI shows the results of the pairwise comparison matrix at the category level with respect to the goal which further are used in the additive DEA model (12) and the additive DEA exclusion model (13).

Obtaining the results of grey relational coefficients and the priority weights of attributes, the additive DEA model (11) can be run. Table VII shows the results of running model (11) that computes the grey relational grade of attributes in each category for the alternative under assessment.

Again, using equation (4), the grey relational grades of each category are turned into the grey relational coefficients for that category as shown in Table VIII.

The overall grey relational grade for the alternative under assessment is obtained from the additive DEA model (12) as shown in Table IX. Since alternatives A_1, A_2 and A_3 are placed in the best ranking positions, the additive DEA exclusion model (13) is run to create a unique rank order among these alternatives. As indicated in Table X, the three alternatives A_1, A_2 and A_3 are ranked 2, 3 and 1, based on the minimum grade of dissimilarity, respectively. Therefore, the anaerobic single oxidation ditch (A_3) is selected as the optimal alternative for the studied municipal wastewater treatment technologies.

4. Conclusions

In many MADM cases, it makes sense to group attributes hierarchically, while different weights may be assigned to different attributes and their own categories to reflect their relative priorities. The standard GRA model is not able to reflect such hierarchical structures, as they assume that all the attributes use the same weights. To cope with this problem, scholars have adopted the application of AHP in GRA, known as hierarchy GRA, where attributes are constructed hierarchically and different weights can be used at different levels. However, the subjective process of producing weights in AHP may not place each alternative in its best light in comparison with all the other alternatives. To overcome this issue, we integrate the two variants of DEA models in hierarchy GRA. Since we use both the DEA and AHP methods in a multi-level GRA framework, more reasonable and encompassing results can be provided for assessing the performance of alternatives. Finally, the usefulness of the proposed approach is demonstrated using a real case study of the hierarchy system of wastewater treatment technology selection.

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Appendix 1

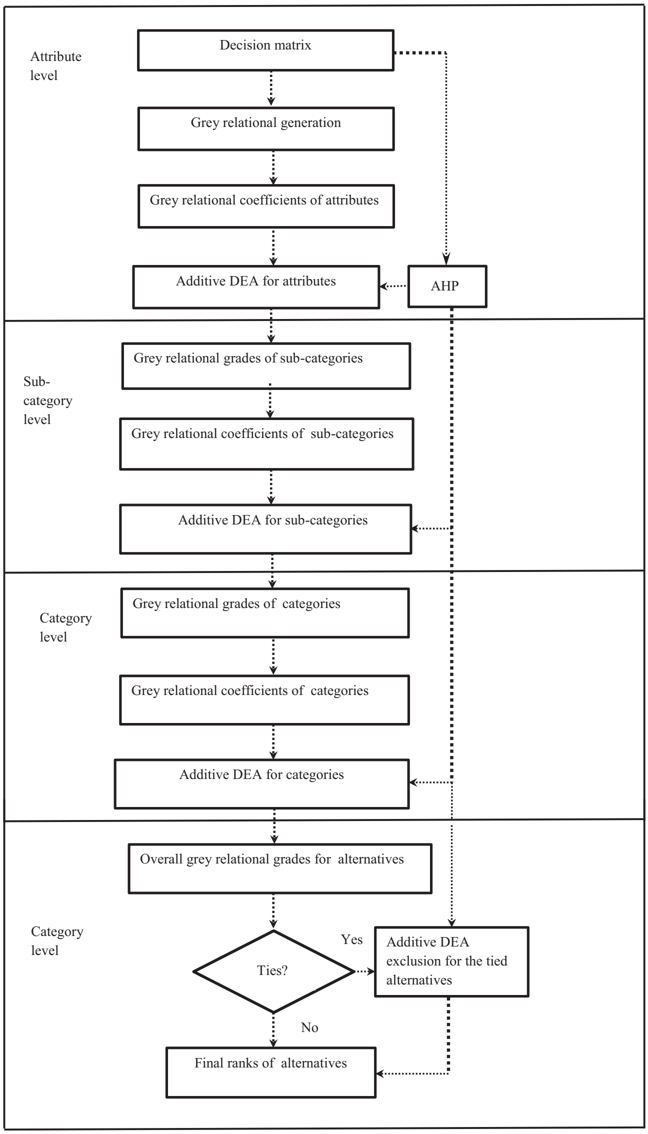

Figure A1.
The flowchart of a three-level hierarchy GRA using DEA and AHP

Appendix 2. The glossary of modeling symbols

y_{jk} is the value of attribute C_k (k = 1,2, . . . , n) for alternative A_i (i = 1, 2, . . . , m).
r_{ik} is the comparability value of attribute C_k for alternative A_i.
$y_{k(\max)}$ is the maximum value of attribute C_k.
$y_{k(\min)}$ is the minimum value of attribute C_k in a basic GRA model.
u_{ok} is the reference value for a virtual ideal alternative, A_o.
ξ_{ik} is the grey relational coefficient of attribute C_k for alternative A_i.
ρ is the distinguishing coefficient.
Γ_i is the grey relational grade for alternative A_i in a basic GRA model.
w_k is the weight of attribute C_k in a basic GRA model.
y_{ijk} is the value of attribute C_{jk} ($k = p, p+1, \ldots q$) in category $C'_j (j = 1, 2, \ldots, n')$ for alternative A_i while $1 \leq p \leq q \leq n$.
ξ_{ijk} is the grey relational coefficient of attribute C_{jk} in category C'_j for alternative A_i.
w_{jk} is the weight of attribute C_{jk} in category C'_j, obtained by AHP.
Γ_{ij} is the grey relational grade of attributes in category C'_j for alternative A_i.
ξ_{ij} is the grey relational coefficient of category C'_j for alternative A_i.
w_j is the weight of category C'_j obtained by AHP.
Γ'_i is the grey relational grade of categories for alternative A_i.
$b_{jkk'}$ is the $k - k'$ ($k = k' = p, p+1, \ldots q$) element of the pairwise comparison matrix for attributes, denoted by B, with respect to category C'_j.
d_{jj}' is the $j - j'$ ($j = j'$ = 1, 2, . . . , n') element of the pairwise comparison matrix for categories, denoted by D, with respect to the problem goal.
$\gamma_{\max}$ is the largest eigenvalue.
$R.I$ is the average random consistency index.
N is the size of a comparison matrix.
$C.R$ is the random consistency ratio.
$1 - P_{oj}$ is the grey relational grade, Γ_{oj} (o = 1, 2, . . . , m, j = 1, 2, . . . , n'), of attributes in category C'_j for alternative under assessment A_o.
S_{jk} is the slack variable of attribute C_{jk} ($k = p, p+1, \ldots q$) in category C'_j.
λ_{ij} is the weight of alternative A_i in category C'_j.
$1 - P_o$ is the grey relational grade, $\Gamma'_o (o = 1, 2, \ldots, m)$, of categories for alternative under assessment A_o.
S_j is the slack variable of category C'_j.
λ_i is the weight of alternative A_i (i = 1, 2, . . . , m).
α_o is a dissimilarity grade between alternative A_o and the remaining alternatives in the additive DEA exclusion (or super-efficiency) model.
t_j is a slack variable of category C'_j in the additive DEA exclusion model.

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