Quantity-based or share-based? Discount schemes for the manufacturer when facing two competing retailers

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Received 17 October 2022
Revised 31 December 2022
Accepted 28 January 2023

Abstract

Purpose – The authors investigate the manufacturer's choice of discount schemes in a supply chain with competing retailers.

Design/methodology/approach – Using a game-theoretic model, the authors build two discount frameworks and compare and analyze the effects of different discount schemes on the performance of supply chain members.

Findings – The authors find that the retail price (market demand) in the quantity discount scheme is always higher (lower) than that in the market share discount scheme. The authors also find that the retailers' preference for discount schemes is antithetical to the manufacturer's preference in most cases. However, under certain conditions, there will be a win-win situation where Pareto-optimization occurs between the manufacturer and retailers when they choose the same discount scheme.

Research limitations/implications – On the one hand, the authors assume that the two retailers are symmetrical in market size and operation efficiency. It would be interesting to study the effect of different discount schemes on retailers when the retailers have different market sizes or operating efficiency. On the other hand, the authors study the manufacturer's choice of discount schemes in a supply chain with one common manufacturer and two competing retailers. However, in practice, there exist other supply chain structures. Future research can examine the problem of choices of discount schemes in other different supply chain structures.

Practical implications – This paper help retailers and manufacturers to choose the best discount schemes.

Social implications – This paper suggests that a high discount scale is not always beneficial (detrimental) to retailers (the manufacture).

Originality/value – The authors build two discount schemes (the quantity and the market share) in a supply chain consisting of one manufacturer and two retailers, and the authors focus on the effects of different discount schemes on the competition between two retailers. By comparing the two discount schemes, the authors study which discount scheme is the better choice for the manufacturer when facing competing retailers.

Keywords Supply chain, Quantity discount scheme, Market share discount scheme, Market size

Paper type Research paper

1. Introduction

Over the past few years, due to the COVID-19 pandemic, many industries (e.g. automobile, hotel, tourism and firms) have been hit hard and suffered significant losses, including the remarkable drop in sales and the mass closure of retailers. For example, in the first half of

The authors are grateful to the anonymous referees for their constructive comments and suggestions. The authors also thank the support from the National Science Fund for Distinguished Young People (71925002) and the National Natural Science Foundation of China (71731006).

Modern Supply Chain Research and Applications
Vol. 5 No. 1, 2023
pp. 50-70
Emerald Publishing Limited
2631-3871
DOI 10.1108/MSCRA-10-2022-0025

2020, car sales volumes were 23% lower than those in the first half of 2019 in China, 43% lower in Europe and 21% lower in the U.S. (Liang and Gu, 2021). Therefore, many manufacturers are trying to boost sales by offering wholesale pricing discount schemes for retailers. The quantity discount scheme (QD) has been used widely by manufacturers to motivate their retailers to increase sales in many industries. Under the QD, if a retailer's sale reaches or exceeds a predetermined target quantity, the manufacturer will give him a rebate to the marginal units beyond the target or to the entire purchase quantity. For instance, in 2018, Bavarian motor works (BMW) stimulates its car dealers in China in terms of sales in the third quarter, where the unit rebate was 18 thousand Renminbi (RMB) for cars made in China and 10 thousand RMB for imported cars (Auto, 2018).

Moreover, in practice, a number of manufacturers also use the relative QD to stimulate retailers to increase sales, in which the discount is based on a retailer's relative quantity, i.e. the share of the overall sales. Hereafter, we also call the market share discount mechanism. Academics have devoted much attention to this practice in the last years, trying to provide a good understanding of the profitability of the market share discount scheme (MS) (Mills, 2010; Inderst and Shaffer, 2010; Mantena and Saha, 2021). However, by far, most studies of the MS assume that a retailer sells multiple manufacturers' products. That is, the existing literature focuses on the distribution channel in which multiple manufacturers sell their products through a retailer. In this situation, the manufacturer ignores the competition of retailers. The motivation for using the MS is that the manufacturer desires to keep its competitors out of the market since increasing the manufacturer's share of the retailer's total purchases has to necessarily come at the cost of reducing the competitor's share.

Nevertheless, in the business market, there always exists more than one retailer in the same region selling products from a manufacturer and we observe that the manufacturer also prefers to give them wholesale pricing discounts based on relative market share than absolute quantity. For example, Sany Heavy Industry, whose excavator sales exceeded 100,000 units in 2021, contracts with its excavator retailers in a region, giving them wholesale pricing discounts based on relative market share. Meanwhile, these retailers compete against each other in the retail market. Therefore, it is important for the manufacturer to study the value of the MS when facing multiple competing retailers. A common wise is that compared with the QD, the MS has a possible advantage of strengthening the competition among retailers and helping the manufacturer extracts higher profits from retailers. Unfortunately, the existing literature lacks in terms of its focus on the test of this conjecture, as well as the study of the value of the MS in the distribution channel with one manufacturer and multiple retailers. These are the focuses of our study. Specifically, we attempt to answer the following research questions: Which discount scheme is the better choice for the manufacturer when facing competing retailers, the quantity discount or market share discount? How does the manufacturer's choice of discount schemes affect the performance of the two retailers and the whole supply chain? How do the discount scale and the product's market size affect the performance of supply chain numbers?

To answer the above research questions, we develop two-stage game models in terms of two discount schemes in a supply chain consisting of one manufacturer and two competing retailers. In each discount scheme, we assume that the manufacturer's base wholesale price is the same for both retailers and the final wholesale price decreases with the retailer's quantity or market share. Then, we figure out the optimal solutions, analyze and compare the optimal solutions of the manufacturer and retailers in two discount schemes. Our analysis yields several notable findings.

Firstly, we find that the retailers' profits decrease in the discount scale while the manufacturer's profit increases in the discount scale in both discount schemes, which is counterintuitive and interesting. The reason could be that a high discount scale implies a high weight of performance-based, making retailers' competition much fiercer. It suggests that a high discount scale is not always beneficial (detrimental) to retailers (the manufacturer).

Secondly, we find that the retail price (market demand) in the QD is always higher (lower) than that in the MS. This is because, in the MS, the manufacturer can use the discount scale as a tool to intensify the market competition between two retailers and control their selling prices to improve sales. This implies that compared to the QD, the MS is beneficial to consumers.

Thirdly, we find that whether or not selling through the market share discount mechanism can be more beneficial to the manufacturer depends on the discount scale and the product's market size. If the product's market size is large, or the product's market size is small and the discount scale is high, the profit of the manufacturer in the MS is higher than that in the QD, while the results are reversed for retailers. That is, when the product's market size is large, or the product's market size is small and the discount scale is high, two retailers tend to select a QD. Interestingly, we find that there exist two different strategies of Pareto improvement, i.e. MS or QD. Specifically, if the product's market size is moderate (small) and the discount scale is relatively low (high), the strategy of Pareto improvement is the market share (quantity) discount scheme.

The rest of the paper is organized as follows. In section 2, we review the literature related to our paper. In section 3, we introduce the model setting and describe the equilibrium solutions in section 4. Section 5 compares the performance of supply chain members in different discount schemes. In section 6, we conclude the paper and provide future research directions. All proofs of theorems and propositions are given in Appendix.

2. Literature and review

Our paper contributes to the literature on two research streams, including the QD and the MS in operations management.

2.1 The quantity discount scheme in the supply chain

The QD has been extensively studied by scholars based on different economic rationales. Some scholars study how to achieve price discrimination by using the QD (Buchanan, 1952; Gabor, 1955; Dolan, 1987). Some scholars regard the QD as an efficiency-increasing scheme to shift the inventory holding cost to buyers or to increase the logistics system efficiency in a distribution channel (Jadidi *et al.*, 2021; Jackson and Munson, 2016; Mansini *et al.*, 2012; Sawik, 2010). And some scholars study channel coordination by QD, which is the most widely studied. For example, Jeuland and Shugan (1983) are the first to study how to apply the QD to coordinate the supply chain. Ingene and Parry (1995) introduce competing retailers to the quantity discount literature. Chen and Roma (2010) study the retailers' decision of whether to purchase together to obtain lower wholesale prices under the QD. Yan *et al.* (2017) extend the research of Chen and Roma (2010) to asymmetric retailers. Ahmadi *et al.* (2018) study the effect of group purchasing on the healthcare supply chain under the QD. Heydari and Momeni (2021) examine the two non-competing retailers' coalition advantages and challenges in a supply chain where the wholesaler offers an all-unit QD. Wu and Li (2021) explore the inherent law of the quantity-discount-contract coordinating the supply chain with stochastic market demand and price and the risk-averse supplier. Kwon *et al.* (2022) introduce a new the quadratic quantity discount contract and study its unique properties by comparing it with linear quantity discount and wholesale price contracts.

All the above research on the QD is focused on applying the scheme in new or existing models to examine the rationales. Different from them, in this paper, we build two discount schemes (the quantity and the market share) in a supply chain consisting of one manufacturer and two retailers, and we focus on the effects of different discount schemes on the competition between two retailers. By comparing the two discount schemes, we study which discount scheme is the better choice for the manufacturer when facing competing retailers.

2.2 The market share discount scheme in operations management

Numerous scholars have analyzed the effect of the MS on the performance of firms. Most of these scholars focus on exclusionary effects. For example, Tom *et al.* (1999) study the economics of market share discount to motivate deals exclusively or nearly exclusively. Spector (2005) reviews the procompetitive and anti-competitive motives for the market share discount and finds that, compared with simple predatory pricing strategies; the market share discount can achieve exclusion at a lower cost and be more credible. Chen and Shaffer (2014) find that an incumbent seller can use the market share scheme to reduce the probability of entry by a rival. Mills (2010) studies the trade-off between the exclusionary effect and the effect of inducing selling effort. He finds that although market share discount may have exclusionary effects, it also can induce downstream selling effort. Calzolari and Denicolò (2020) use the discount-attribution test to assess the competitive effects of loyalty discounts. Chen and Shaffer (2019) compares exclusive dealing and market share contracts in a model of naked exclusion and find that market share contracts are better at maximizing a seller's benefit from foreclosure

Moreover, a few scholars study other reasons for employing the market share discount. For example, Inderst and Shaffer (2010) find that compared with own-supplier contracts, market share contracts can dampen simultaneously intra- and Interbrand competition. Vassallo (2012) finds that eliminates the double marginalization problem, thereby maximizing the joint profit in the supply chain while simultaneously increasing consumer surplus. Mantena and Saha (2021) study the impact of the market share contract on demand allocation, prices and welfare in a setting where a single central B2B buyer procures multiple units of a product on behalf of a set of users with heterogeneous preferences.

The differences between our paper and the above literature are twofold as follows. On the one hand, the above literature focuses on the distribution channel in which a retailer buys substitutable products from different manufacturers. Therefore, the market share in the above literature is interpreted as a manufacturer's share of the retailer's overall purchases. Conversely, we focus on the competition of downstream retailers and consider a distribution channel in which a manufacturer sells a product through two competing retailers. In this situation, the market share is interpreted as a retailer's share of the manufacturer's total quantity. On the other hand, given the extensive exploration of market exclusion in the literature, we do not focus on this issue. Indeed, our paper focuses on the comparison between the MS and the QD. Specifically, we try to study the effects of different discount schemes on competing retailers and the choice of manufacturer between two discount schemes.

3. The mode

Consider a distribution channel (see Figure 1) where an upstream manufacturer can sell a homogeneous good to end consumers through two homogenous and competing downstream retailers. The firms are risk-neutral and maximize profits. Without loss of generality, we assume that two homogenous retailers incur the same operating cost c in purchasing and selling per unit of the product and the manufacturer's production cost is normalized to zero, which is also adopted in the literature of Chen and Roma (2010), Xiao *et al.* (2020) and Deng *et al.* (2021).

Both homogenous retailers are indexed by $i \in \{1, 2\}$ and $j = 3 - i$. Retailer i's demand, q_i, decreases with its own price p_i and increases with the opponent's price p_j. We consider a linear demand function

$$q_i = A - p_i + \theta\left(p_j - p_i\right) \tag{1}$$

where A is the market base and θ ($\theta \in [0, 1]$) measures substitutability between two retailers and reflects their competition intensity. There is no competition when $\theta = 0$. The above

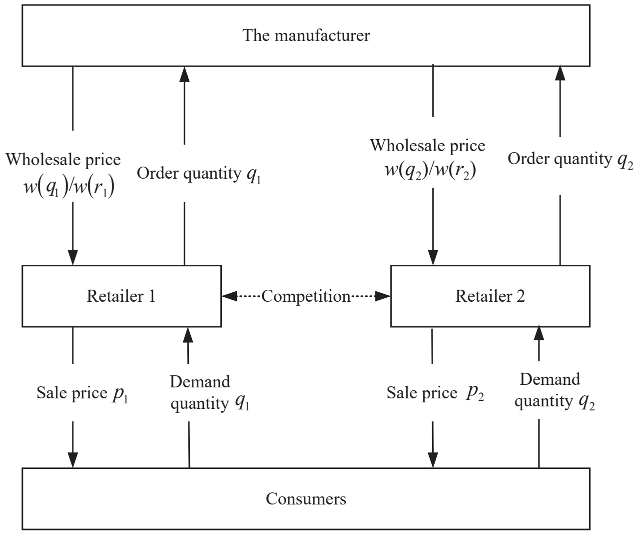


Figure 1.
The supply chain structure

demand function is common in economic and marketing literature (see Li *et al.*, 2021; Tian *et al.*, 2018; Abhishek *et al.*, 2016).

To promote sales, the manufacturer may implement the absolute QD between itself and retailers, which has been widely utilized as an incentive scheme in the supply chain (e.g. Cachon and Kök, 2010; Raju and Zhang, 2005). Following Ingene and Parry (1995) and Raju and Zhang (2005),in the QD, the manufacturer's final unit wholesale price decreases with the increase of a retailer's order quantity. We assume that the quantity discount function is

$$w(q_i) = w_q + \frac{d}{q_i} \tag{2}$$

where $w_q > 0$ is the base minimum wholesale price, which is decided by the manufacturer as a decision variable in this paper. From (2), we find the manufacturer's final wholesale price consists of two parts: the first term is a base wholesale price, representing the minimum wholesale price of the manufacturer. And the second term is the function $w(q_i)$ for quantity q_i, implying that the final wholesale price $w(q_i)$ decreases with q_i.

Moreover, the manufacturer is able to stimulate market sales by using the relative quantity (market share) discount scheme (MS). In this paper, the MS is interpreted that the manufacturer's final unit wholesale price decreases in a retailer's share of the manufacturer's total quantity. Different from the absolute QD, in the MS, the final wholesale price is the function of retailer *i*'s share of overall sales instead of the function for the absolute quantity. In this case, the unit wholesale price $w(r_i)$is

$$w(r_i) = w_r + \frac{d}{r_i} \tag{3}$$

where r_i is the market share of retailer i, i.e. $r_i = \frac{q_i}{q_i + q_{3-i}}$ $d > 0$ is the discount scale under the two discount schemes, reflecting how quickly the wholesale price decreases with the order quantity or market share.

The event sequence is as follows: the manufacturer, as a leader of the supply chain, first decides the base wholesale price w_q or w_r, and afterward the two competing retailers, as the followers of the supply chain, decide their selling price p_i simultaneously and independently.

To ensure the market demand is non-negative, we assume that $A - c > 0$ in this paper. Moreover, The QD and the MS are indexed by *QD* and *MS*, and we use the superscript *QD** and *MS** denote the optimal solutions in the above two discount schemes, respectively.

We summarize the notations that will be used in this paper in Table 1.

4. Equilibrium

In this section, we derive the equilibrium prices, quantities and profits of firms in two discount schemes. One is the QD, where the manufacturer's final wholesale price given to retailer i depends on the base wholesale price w_q and the retailer i's selling quantity q_i. The other is the MS, in which the manufacturer's final wholesale price given to retailer i is decided by the base wholesale price w_r and the retailer i's market share r_i.

4.1 Quantity discount scheme

We start with the scenario where the manufacturer offers to sell its product to two competing retailers through a QD. Under QD, The manufacturer's and the retailers' profit functions are

$$\Pi_M = \sum_{i=1}^{2} w(q_i)q_i \tag{4}$$

$$\Pi_i = (p_i - c - w(q_i))q_i \tag{5}$$

We analyze the problem by backward induction. At the final stage, given w_q, the two competing retailers independently and simultaneously determine their price p_i by maximizing (5). Denote p_i^{QD*} $(i = 1, 2)$ as the selling price of retailer i in equilibrium in QD, which is characterized in the following Theorem.

Theorem 1. Given w_q, there exists a unique pure-strategy Nash equilibrium (p_1^{QD*}, p_2^{QD*}) for two retailers, which is given by

Notation	Explanation
θ	Coefficient of substitutability between two retailers
A	The product's market size
c	The two homogenous retailers' operating cost
d	The discount scale in two discount schemes
$w(q_i)$	The manufacturer's wholesale price in the quantity discount scheme
r_i	$r_i = \frac{q_i}{q_i+q_j}$, the market share of retailer i
$w(r_i)$	The manufacturer's wholesale price in the market share discount scheme
q_i	The retailer i's selling quantity, where $i = 1, 2$
Π_i, Π_M and Π_T	The retailer i's profit, manufacturer's profit and supply chain's profit, respectively
Decision variables	
p_i	The retailer i's price, where $i = 1, 2$
w_q	The manufacturer's base wholesale price in the quantity discount scheme
w_r	The manufacturer's base wholesale price in the market share discount scheme

Table 1. Notations

$$\left(p_1^{QD*}, p_2^{QD*}\right) = \left(\frac{A + (w_q + c)(1+\theta)}{\theta + 2}, \frac{A + (w_q + c)(1+\theta)}{\theta + 2}\right).$$

It is clear that the selling price p_1^{QD*} of retailer 1 is equal to the selling price p_2^{QD*} of retailer 2 because of the symmetry of two retailers. Moreover, Theorem 1 shows that two retailers' selling prices increase in both the market size A and the operating cost c, which is common and intuitive. Because when the market size is large, the competition between two retailers erodes, resulting in both retailers improving their prices to gain more profits, while a high operating cost makes the retailer increase its selling price to offset the cost.

Now we study what base wholesale price will be set by the manufacturer anticipating the retailers' price p_i^{QD*}. By putting p_1^{QD*} and p_2^{QD*} into Π_M, the manufacturer's profit function becomes

$$\Pi_M = \frac{-2(\theta+1)w_q^2 + 2(A-c)(\theta+1)w_q + 2d(\theta+2)}{\theta+2} \tag{6}$$

The following Theorem characterizes the manufacturer's optimal pricing decision in QD.

Theorem 2. In the QD, the optimal base wholesale price set by the manufacturer is

$$w_q^{QD*} = \frac{A-c}{2}.$$

Theorem 2 shows that the manufacturer's base wholesale price increases in the market size A while decreases in the retailer's operating cost c. Because a large market size helps the manufacturer improve its price to gain more profits. While when the operating cost is high, the manufacturer should decrease its wholesale price to incentive retailers to sell products.

From Theorems 1 and 2, we also find that the quantity discount scale has no influence on the manufacturer's and retailers' pricing decisions, implying that the QD cannot be used as a tool to adjust the pricing decisions and the consumers also cannot benefit from the QD.

Denote q_i^{QD*}, w_i^{QD*}, Π_M^{QD*}, Π_i^{QD*}, and Π_T^{QD*} as the retailer i's optimal selling quantity, the manufacturer's optimal final wholesale price, the manufacturer's optimal profit, the retailer i's optimal profit, and the corresponding supply chain profit under QD, which are shown in Table 2.

4.2 Market share discount scheme

We now consider the scenario where the manufacturer offers to sell its product to two competing retailers through a MS. Under MS, the manufacturer's final wholesale price $w(r_i)$ depends on the base wholesale price w_r and the retailer i's market share r_i. The manufacturer's and the retailers' profit functions are

$$\Pi_M = \sum_{i=1}^{2} w(r_i)q_i \tag{7}$$

$$\Pi_i = (p_i - c - w(r_i))q_i \tag{8}$$

Analogous to the solving process under QD, we solve the problem by backward induction. At the final stage, given w_r, the retailers independently and simultaneously determine their price

Table 2. Equilibrium solutions under QD and MS

	QD	MS
p_i^*	$\frac{(3+\theta)A+(1+\theta)c}{2(\theta+2)}$	$\frac{(\theta+3)A+(\theta+1)c-(2\theta+1)d}{2(\theta+2)}$
q_i^*	$\frac{(A-c)(1+\theta)}{2(2+\theta)}$	$\frac{(1+\theta)(A-c)+(2\theta+1)d}{2(2+\theta)}$
Π_i^*	$\frac{(\theta+1)(A-c)^2}{4(\theta+2)^2}-d$	$\frac{((\theta+1)(A-c)+(2\theta+1)d)((\theta+1)(A-c)-(2\theta+1)(2\theta+3)d)}{4(\theta+2)^2(\theta+1)}$
$w_q^*(w_r^*)$	$\frac{A-c}{2}$	$\frac{(1+\theta)(A-c)-(2\theta+3)d}{2(1+\theta)}$
w_i^*	$\frac{A-c}{2}+\frac{2(\theta+2)d}{(1+\theta)(A-c)}$	$\frac{(\theta+1)(A-c)+(2\theta+1)d}{2(\theta+1)}$
Π_M^*	$=\frac{(\theta+1)(A-c)^2}{2(2+\theta)}+2d$	$\frac{((1+\theta)(A-c)+(2\theta+1)d)^2}{2(\theta+1)(2+\theta)}$
Π_T^*	$\frac{(\theta+3)(\theta+1)(A-c)^2}{2(\theta+2)^2}$	$\frac{(\theta+1)(\theta+3)(A-c)^2+2(2\theta+1)(A-c)d-(2\theta+1)^2d^2}{2(\theta+2)^2}$

p_i by maximizing (8). Denote p_i^{MS*} $(i=1,2)$ as the selling price of retailer i in equilibrium in MS, which is characterized in the following Theorem.

Theorem 3. Given w_r, there exists a unique pure-strategy Nash equilibrium (p_1^{MS*}, p_2^{MS*}) for two retailers, which is given by

$$\left(p_1^{MS*}, p_2^{MS*}\right) = \left(\frac{A+(\theta+1)(c+w_r)+d}{\theta+2}, \frac{A+(\theta+1)(c+w_r)+d}{\theta+2}\right).$$

Similar to Theorem 1, Theorem 3 shows intuitively that the two retailers' selling prices increase in both the market size A and the operating cost c in MS. The reasons are the same as those in Theorem 1. However, different from Theorem 1, we find from Theorem 3 that the discount scale has an influence on the retailers' pricing decisions in MS. Specifically, two retailers' selling prices decrease in the discount scale, which implies that the consumers can benefit from the MS.

Now we characterize the equilibrium of the manufacturer's base wholesale price in MS. By putting p_1^{MS*} and p_2^{MS*} into (7), the manufacturer's profit function becomes

$$\Pi_M = \frac{-2(\theta+1)w_r^2 + 2((A-c)(\theta+1)-(3+2\theta)b)w_r + 4((A-c)(\theta+1)-b)b}{2+\theta} \quad (9)$$

The following Theorem characterizes the manufacturer's optimal pricing decision.

Theorem 4. In the MS, the optimal base wholesale price set by the manufacturer is

$$w_r^{MS*} = \frac{(1+\theta)(A-c)-(2\theta+3)d}{2(1+\theta)}.$$

Theorem 4 shows that the manufacturer's base wholesale price increases in the product's market size A while decreases in the retailer's operating cost c in MS, which are the same as the results in Theorem 2. Moreover, Theorem 4 shows that the manufacturer's base wholesale price decreases in the discount scale, implying that there exists a substitution relation between the base wholesale price and the discount scale in MS.

Denote q_i^{MS*}, w_i^{MS*}, Π_M^{MS*}, Π_i^{MS*}, and Π_T^{MS*} as the retailer i's optimal selling quantity, the manufacturer's optimal final wholesale price, the manufacturer's optimal profit, retailer i's optimal profit and the corresponding supply chain profit under MS, which also are shown in Table 2.

5. Comparison and analyzation of QD and MS

On the basis of equilibrium results in Sections 4.1 and 4.2, in this section, we first study the effects of competition intensity and discount scale on the profits of supply chain members. Then, we compare the equilibrium solutions of two schemes (QD and MS), so as to provide theoretical references for firms choosing discount scheme in practice.

Proposition 1. $\frac{\partial \Pi_i^{j*}}{\partial \theta} < 0, \frac{\partial \Pi_M^{j*}}{\partial \theta} > 0$, where $i = 1, 2$, and $j \in \{MS, QD\}$.

Proposition 1 shows the effect of competition intensity on the retailers' and the manufacturer's profits in two discount schemes. The retailers' profits decrease in the competition intensity while the manufacturer's profit increases in the competition intensity under both QD and MS, which is consistent in much literature (Chen and Roma, 2010; Liang and Gu, 2021). The reason is that a higher competition level leads to the lower retail price and higher demand, which results in lower retailers' profits and higher manufacturer's profit.

Figure 2 depicts intuitively the results shown in Proposition 1. Moreover, we can find from Figure 2 that the sensitivities of retailers' and the manufacturer's profits to the competition intensity are different in the two discount schemes. Specifically, in the MS, two retailers' profits drop faster while the manufacturer's profit rises faster than those in QD, implying that the profits of two retailers and the manufacturer are more sensitive to the competition intensity in MS. Therefore, when the competition between downstream retailers is fierce, the manufacturer prefers to adopt MS while two retailers prefer to QD.

Proposition 2. $\frac{\partial \Pi_i^{j*}}{\partial d} < 0, \frac{\partial \Pi_M^{j*}}{\partial d} > 0$, where $i = 1, 2$, and $j \in \{MS, QD\}$.

Proposition 2 shows the effect of the discount scale d on the profits of retailers and the manufacturer in two discount schemes. We can find interestingly that the retailers' profits decrease in the discount scale d while the manufacturer's profit increases in the discount scale d in QD and MS. Intuitively, a higher discount scale benefits retailers and hurts the manufacturer. However, in our paper, we discover that the results reverse. The reasons are as follows. In QD, the selling price p_i and the market demand q_i remain constant with the d, while the manufacturer's wholesale price w_i increases with the d. Then the manufacturer (retailers) gets (get) a higher (lower) profit(s) with a higher value of d. But in MS, the retail price (market demand) decreases (increases) in d, and the benefits of increased market demand cannot offset the loss caused by the decrease of the retail price. Then a higher d hurts two competing

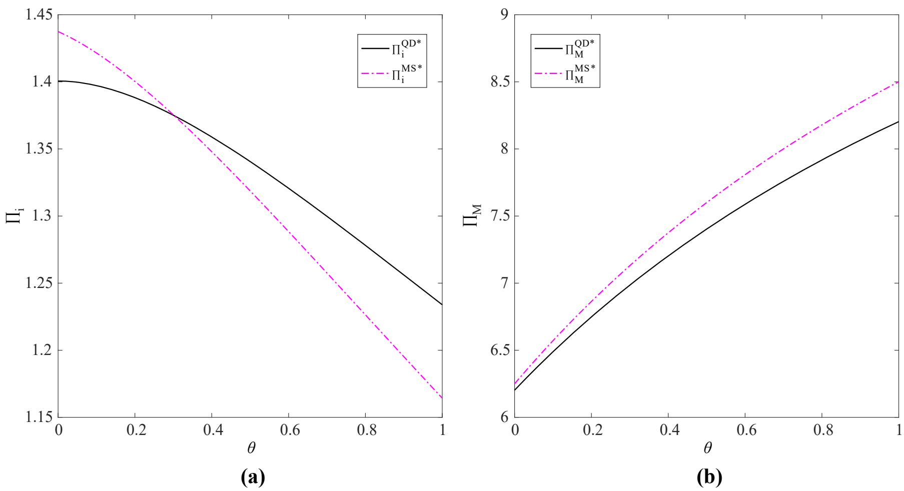


Figure 2.
The effect of competition intensity on profits of supply chain members ($A = 5$, $c = 0.1$, $d = 0.1$)

retailers. However, for the manufacturer, the wholesale price also increases in d, which leads to the profit of the manufacturer increases in d.

Figure 3 illustrates the above results. Furthermore, Figure 3 also depicts the sensitivities of the manufacturer's and retailers' profits to the discount scale. Although two retailers' profits decrease in the discount scale in two discount schemes, we find that the profits of two retailers decrease in the discount scale at a slower speed in QD compared with in MS. Similarly, the manufacturer's profit increases in the discount scale at a slower speed in QD compared with in MS. This reveals that ceteris paribus, the manufacturers should choose the discount scheme according to the discount scale. The manufacturers who provide discounts with high discount scale in practice should adopt MS rather than QD to gain more profits.

Proposition 3. (i) $w_q^{QD*} > w_r^{MS*}$; (ii) $p_i^{QD*} > p_i^{MS*}$; (iii) $q_i^{QD*} < q_i^{MS*}$.

Proposition 3 and Figure 4 depict the relationships of the optimal base wholesale price, retail prices and order quantities under two discount schemes. Proposition 3(i) shows that the optimal base wholesale price in QD is always higher than that in MS. The reason is that in MS, the manufacturer can use the discount scale to regulate the wholesale price, which is not implemented in QD (Figure 4(a)). In other words, there exists a substitution relation between the base wholesale price and the discount scale in MS. The manufacturer can offset the loss of decreasing the base wholesale price by increasing the discount scale because the final wholesale price and profit increase with the increase of the discount scale.From Proposition 3(ii) and (iii), we find that the selling price in QD is always higher than that in MS, while the market demand in QD is always lower than that in MS, implying that MS strengthens the competition between retailers. This is because the manufacturer can use the discount scale as a tool to control selling prices of its downstream retailers to improve sales in MS than in QD. As shown in Figure 4(b), when the discount scale is zero, the selling prices of retailers in QD are equal to that in MS. While with the increase of the discount scale, the selling prices of retailers in QD (MS) remain constant (decrease), resulting in that retailers' selling prices in MS lower than that in QD. Obviously, the decrease of selling price leads to the increase of order quantity in MS (Figure 4(c)).

Proposition 4.

(1) If $A \geq \widehat{A}$, or $c < A < \widehat{A}$ and $d > d_R$, $\Pi_i^{QD*} > \Pi_i^{MS*}$;

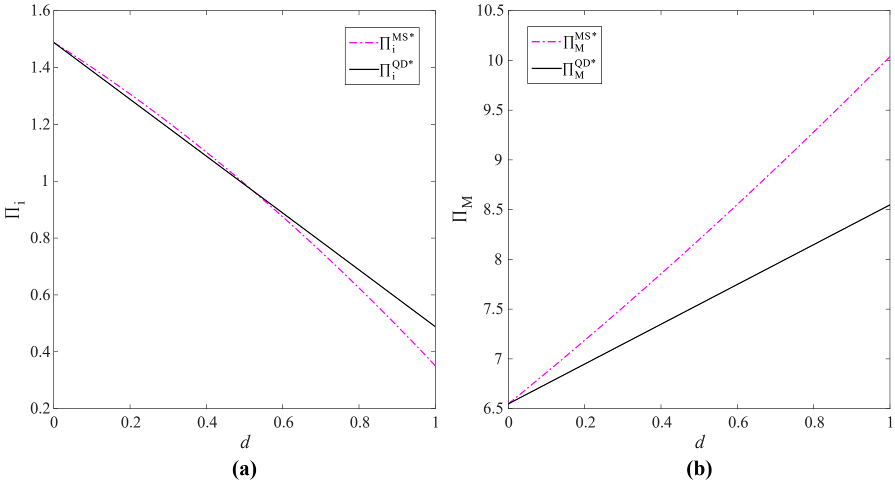
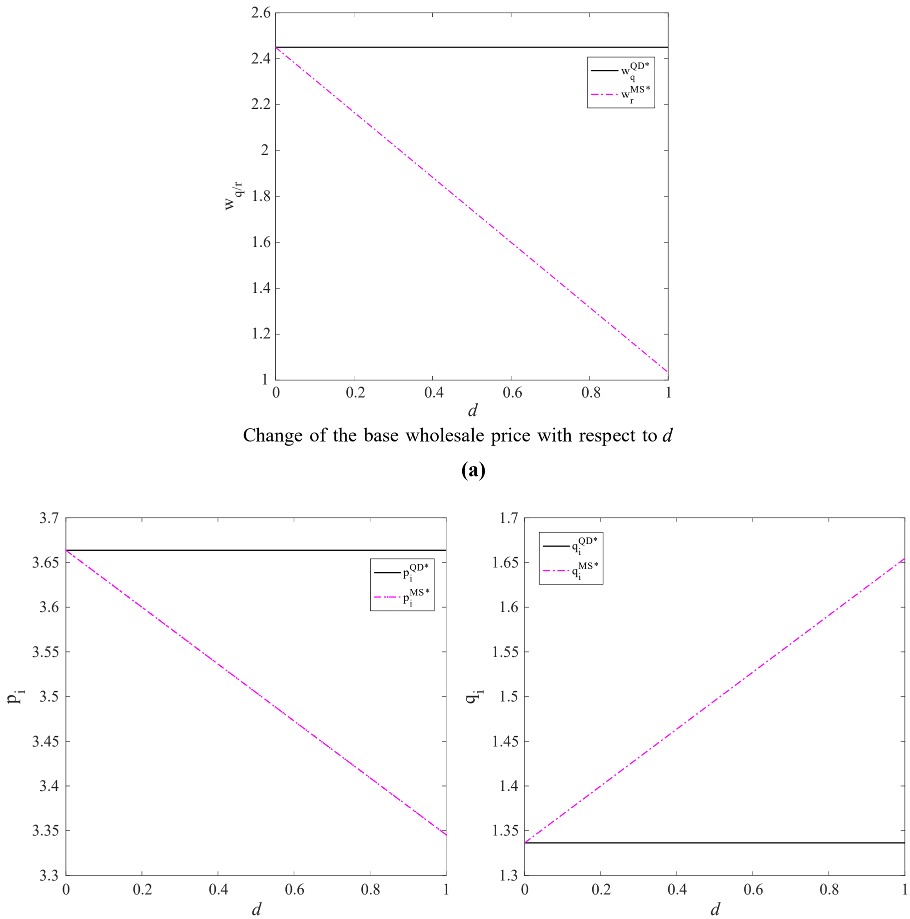


Figure 3. The effect of discount scale on profits of supply chain members ($A = 5, c = 0.1, \theta = 0.2$)



Change of the base wholesale price with respect to d

(a)

Change of the retail price with respect to d

(b)

Change of the quantity with respect to d

(c)

Figure 4.
Change of the base wholesale price, retail price and quantity with respect to d ($\theta = 0.2$, $c = 0.1$, $A = 5$)

(2) If $c < A < \widehat{A}$ and $0 < d \leq d_R$, $\Pi_i^{QD*} \leq \Pi_i^{MS*}$, where $\widehat{A} = c + \frac{2(\theta+2)^2}{(\theta+1)(2\theta+1)}$, $d_R = \frac{4(\theta+1)(\theta+2)^2 - 2(\theta+1)^2(2\theta+1)(A-c)}{(2\theta+1)^2(2\theta+3)}$.

Proposition 4 and Figure 5 show the relationships of the retailers' profits under two discount schemes. We can find that which discount scheme benefits retailers depends on the discount scale d and the product's market size A. Specifically, Proposition 4(i) reveals that when the product's market size is large, or the product's market size is small and the discount scale is high, two retailers tend to select QD. However, when the product's market size is small and the discount scale is low, MS becomes the more favorable choice of the two retailers (Proposition 4(ii)).

The explanations are as follows. If the product's market size is large, consumers are not sensitive to price and two retailers can set high selling price. Moreover, from Proposition 3, we obtain that the retailers' selling price in QD is higher than that in MS. Therefore, compared to low price in MS, two retailers can benefit from high price in QD.

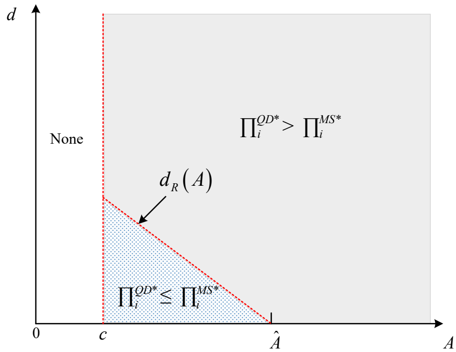


Figure 5. Relationships of retailers' profits between QD and MS

However, when the product's market size is small, which discount scheme is better to retailers depends on the discount scale. When the discount scale is high two retailers get higher profits in QD than that in MS. This is because, from the equilibrium solutions shown in Table 2, we can observe intuitively that the retailers' profits decrease in discount scale at a constant speed (i.e. 1) in QD while the decreasing rate of retailers' profits shows an accelerating tendency in MS with the increase of discount scale. If the product's market size is small and the discount scale is high, for two retailers, the benefits of low price cannot offset the loss caused by decreased profits in MS, and vice versa.

Proposition 5.

(1) If $A \geq \bar{A}$, or $c < A < \bar{A}$ and $d > d_M$, $\Pi_M^{QD*} < \Pi_M^{MS*}$;

(2) If $c < A < \bar{A}$ and $0 < d \leq d_M$, $\Pi_M^{QD*} \geq \Pi_M^{MS*}$, where $\bar{A} = c + \frac{2(\theta+2)}{(2\theta+1)}$, $d_M = \frac{4(\theta+1)(\theta+2) - 2(2\theta+1)(\theta+1)(A-c)}{(2\theta+1)^2}$.

Proposition 5 depicts relationships of the manufacturer's profit under two discount schemes in different parameter ranges. We find that if the product's market size is large, or the product's market size is small and the discount scale is high, the profit of the manufacturer in MS is higher than that in QD. Conversely, if the product's market size is small and the discount scale is low, the profit of the manufacturer in MS is lower than that in QD. The results in Proposition 5 can be pictorially shown in Figure 6.

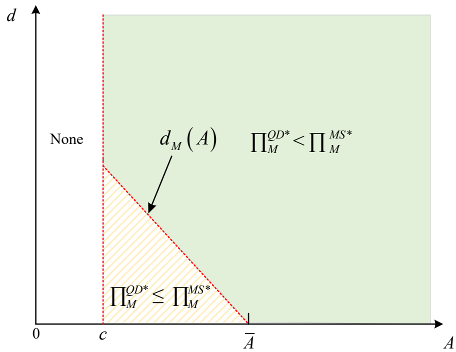


Figure 6. Relationships of the manufacturer's profit between QD and MS

The interpretation of the above results is as follows. From Proposition 3, we obtain that the manufacturer's optimal base wholesale price in QD is always higher than that in MS and the total order quantity in QD is also lower than that in MS. When the product's market size is large, the benefits of increased order quantities can offset the loss caused by double marginalization in MS. Thus, the manufacturer gains a higher profit in MS than in QD. But when the product's market size is small, whether or not the manufacturer's profit in QD is higher than that in MS depends on the discount scale. If the discount scale is high, the manufacturer's profit in MS is also higher than that in QD, and if the discount scale is low, the relations reverse. This is because, from the Proposition 2 and Figure 3, we can observe intuitively that the manufacturer's profits in two discount schemes increase in the discount scale, but the manufacturer's profit increases in the discount scale at a faster speed in MS compared with in QD.

From Propositions 4 and 5, we find that the retailers' and the manufacturer's choice of discount schemes maybe conflict with each other. Now we explore whether there exists the possibility of Pareto improvement between supply chain members.

Proposition 6.

(1) When $A \in \left[A', \widehat{A}\right]$, and $d \in [d_M, d_R]$, MS is the strategy of pareto improvement.

(2) When $A \in \left(c, A'\right]$, and $d \in [d_R, d_M]$, QD is the strategy of pareto improvement, where $A' = c + \frac{2(\theta+1)}{2\theta+1}$.

The results of Proposition 6 are shown in Figure 7. From the Proposition 6 and Figure 7, we find that there exist two different strategies of Pareto improvement: MS or QD. Specifically, if the product's market size is moderate ($A \in \left[A', \widehat{A}\right]$) and the discount scale is relatively low ($d \in [d_M, d_R]$), there exists a win-win situation in which the manufacturer and retailers all choose the MS because their profits in MS are higher than that in QD. However, if the product's market size is small ($A \in \left(c, A'\right]$) and the discount scale is relatively high ($d \in [d_R, d_M]$), the QD becomes the more favorable choices of the manufacturer and retailers. The reasons are the same as those in Propositions 4 and 5.

Proposition 7.

(1) If $c < A < c + \frac{(2\theta+1)d}{2}$, $\Pi_T^{QD*} > \Pi_T^{MS*}$;

(2) If $A \geq c + \frac{(2\theta+1)d}{2}$, $\Pi_T^{QD*} \leq \Pi_T^{MS*}$, where $\Pi_T^{j*} = \Pi_M^{j*} + \sum_{i=1}^{2} \Pi_i^{j*}$, $i = 1, 2$, and $j \in \{MS, QD\}$.

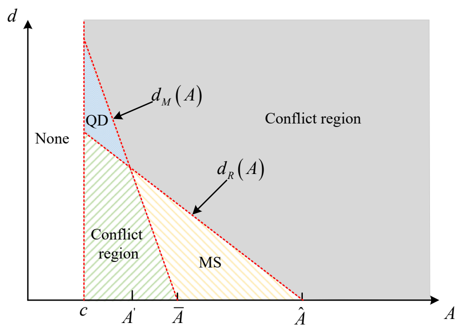


Figure 7.
The strategy of pareto improvement

Proposition 7 demonstrates the relationships of the optimal total profits of the supply chain in different discount schemes. From Proposition 7, we find that the total profit of the supply chain in QD is larger than that in MS if the product's market size is small. However, the supply chain is more likely to benefit from MS rather than QD if the product's market size is large. The reason is that when the product's market size is small, the profit increased by the manufacturer can offset the profits decreased by two competing retailers in QD. Then it is straightforward to see that the total profit of the supply chain is higher in QD than in MS. The reasons are the same when the product's market size is large. The results in Proposition 7 are shown in Figure 8.

6. Conclusions, managerial implications and future research

In this paper, we study the problem of the manufacturer's choice of discount schemes in a supply chain composed of a manufacturer and two competing retailers. We develop a quantity or market share discount framework and identify competition intensity, discount scale and the market size as the key performance drivers. The key findings with managerial implications are summarized below.

(1) **Finding 1:** Compared to the QD, the MS leads to lower selling prices and higher sale quantities.

Managerial implication 1: This finding implies that the MS intensifies the market competition between retailers. Moreover, the consumers can benefit from the MS compared with the QD.

(2) **Finding 2:** Intuitively, retailers can benefit from a high discount scale, so they are willing to accept the price discount offered by the manufacturer. However, we find that the two retailers' profits decrease in the discount scale while the manufacturer's profit increases in the discount scale.

Managerial implication 2: This finding is a warning to retailers that indiscriminately accepting a high price discount may result in a loss of profit. Moreover, for the manufacturer, if its wholesale price is nonlinear decline in the retailer's order quantity or market share, it can increase profit by improving the discount scale.

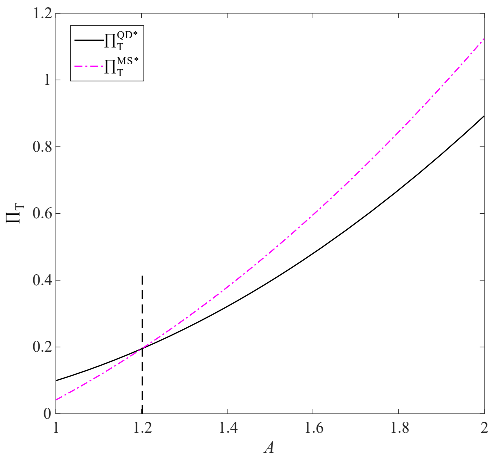


Figure 8. Relationships of the whole supply chain's profit between QD and MS ($\theta = 0.2$, $c = 0.5$, $d = 1$)

(3) **Finding 3:** We find that which discount scheme is a better choice for the manufacturer depends on the discount scale and the product's market size. If the product's market size is large, or the product's market size is small and the discount scale is high, the profit of the manufacturer in the MS is higher than that in the QD, while the results are reversed for retailers.

Managerial implication 3: This finding suggests that when the manufacturer plans to introduce a discount scheme to its downstream retailers, it should consider the discount scale and the product's market size. If the product's market size is large, or the product's market size is small and the discount scale is high, the MS is a better choice. Otherwise, the QD makes the manufacturer better off.

(4) **Finding 4:** We find that under certain conditions, the manufacturer and two retailers can achieve a win-win situation by using the same discount scheme. Specifically, if the product's market size is moderate (small) and the discount scale is relatively low (high), the strategy of Pareto improvement is the market share (quantity) discount scheme.

Managerial implication 4: This finding tells the manufacturer and retailers that they can achieve a win-win situation by using the same discount scheme under some conditions.

This paper has a few limitations. On the one hand, we assume that the two retailers are symmetrical in market size and operation efficiency. It would be interesting to study the effect of different discount schemes on retailers when the retailers have different market sizes or operating efficiency. On the other hand, we study the manufacturer's choice of discount schemes in a supply chain with one common manufacturer and two competing retailers. However, in practice, there exist other supply chain structures, such as the supply chain with two competing manufacturers selling their products through their own retailers. Future research can examine the problem of choices of discount schemes in other different supply chain structures.

References

Abhishek, V., Jerath, K. and Zhang, Z.J. (2016), "Agency selling or reselling? Channel structures in electronic retailing", *Management Science*, Vol. 62 No. 8, pp. 2259-2280.

Ahmadi, A., Pishvaee, M.S. and Heydari, M. (2018), "How group purchasing organisations influence healthcare-product supply chains? An analytical approach", *Journal of the Operational Research Society*, Vol. 70 No. 2, pp. 280-293, doi: 10.1080/01605682.2018.1434403.

Auto, S. (2018), "BMW announced a large cash subsidy to dealers", available at: http://www.sohu.com/a/257870579 100143567

Buchanan, J.M. (1952), "The theory of monopolistic quantity discounts", *The Review of Economic Studies*, Vol. 20 No. 3, p. 199, doi: 10.2307/2295890.

Cachon, G.P. and Kök, A.G. (2010), "Competing manufacturers in a retail supply chain: on contractual form and coordination", *Management Science*, Vol. 56 No. 3, pp. 571-589, doi: 10.1287/mnsc.1090.1122.

Calzolari, G. and Denicolò, V. (2020), "Loyalty discounts and price-cost tests", *International Journal of Industrial Organization*, Vol. 73, 102589, doi: 10.1016/j.ijindorg.2020.102589.

Chen, R.R. and Roma, P. (2010), "Group buying of competing retailers", *Production and Operations Management*, Vol. 20 No. 2, pp. 181-197, doi: 10.1111/j.1937-5956.2010.01173.x.

Chen, Z. and Shaffer, G. (2014), "Naked exclusion with minimum-share requirements", *The RAND Journal of Economics*, Vol. 45 No. 1, pp. 64-91, doi: 10.1111/1756-2171.12042.

Chen, Z. and Shaffer, G. (2019), "Market share contracts, exclusive dealing, and the integer problem", *American Economic Journal: Microeconomics*, Vol. 11 No. 1, pp. 208-242, doi: 10.1257/mic.20160350.

Deng, S., Fu, K., Xu, J. and Zhu, K. (2021), "The supply chain effects of trade credit under uncertain demands", *Omega*, Vol. 98, 102113, doi: 10.1016/j.omega.2019.102113.

Dolan, R.J. (1987), "Quantity discounts: managerial issues and research opportunities", *Marketing Science*, Vol. 6 No. 1, pp. 1-22, doi: 10.1287/mksc.6.1.1.

Gabor, A. (1955), "A note on block tariffs", *The Review of Economic Studies*, Vol. 23 No. 1, p. 32, doi: 10.2307/2296149.

Heydari, J. and Momeni, B. (2021), "Retailers' coalition and quantity discounts under demand uncertainty", *Journal of Retailing and Consumer Services*, Vol. 61, 102557, doi: 10.1016/j.jretconser.2021.102557.

Inderst, R. and Shaffer, G. (2010), "Market-share contracts as facilitating practices", *The RAND Journal of Economics*, Vol. 41 No. 4, pp. 709-729.

Ingene, C.A. and Parry, M.E. (1995), "Channel coordination when retailers compete", *Marketing Science*, Vol. 14 No. 4, pp. 360-377.

Jackson, J.E. and Munson, C.L. (2016), "Shared resource capacity expansion decisions for multiple products with quantity discounts", *European Journal of Operational Research*, Vol. 253 No. 3, pp. 602-613, doi: 10.1016/j.ejor.2016.03.024.

Jadidi, O., Jaber, M.Y., Zolfaghri, S., Pinto, R. and Firouzi, F. (2021), "Dynamic pricing and lot sizing for a newsvendor problem with supplier selection, quantity discounts, and limited supply capacity", *Computers and Industrial Engineering*, Vol. 154, 107113, doi: 10.1016/j.cie.2021.107113.

Jeuland, A.P. and Shugan, S.M. (1983), "Managing channel profits", *Marketing Science*, Vol. 2 No. 3, pp. 239-272, doi: 10.1287/mksc.2.3.239.

Kwon, Y.W., Sheu, J.B. and Yoo, S.H. (2022), "Quadratic quantity discount contract under price-dependent demand and consumer returns", *Transportation Research Part E: Logistics and Transportation Review*, Vol. 164, 102756, doi: 10.1016/j.tre.2022.102756.

Li, G., Tian, L. and Zheng, H. (2021), "Information sharing in an online marketplace with co-opetitive sellers", *Production and Operations Management*, Vol. 30 No. 10, pp. 3713-3734, doi: 10.1111/poms.13460.

Liang, J. and Gu, C. (2021), "The value of target sales rebate contracts in a supply chain with downstream competition", *International Journal of Production Economics*, p. 242, 108290, doi: 10.1016/j.ijpe.2021.108290.

Mansini, R., Savelsbergh, M.W. and Tocchella, B. (2012), "The supplier selection problem with quantity discounts and truckload shipping", *Omega*, Vol. 40 No. 4, pp. 445-455, doi: 10.1016/j.omega.2011.09.001.

Mantena, R. and Saha, R.L. (2021), "Market share contracts in b2b procurement settings with heterogeneous user preferences", *Production and Operations Management*. doi: 10.1111/poms.13611.

Mills, D.E. (2010), "Inducing downstream selling effort with market share discounts", *International Journal of the Economics of Business*, Vol. 17 No. 2, pp. 129-146, doi: 10.1080/13571516.2010.483082.

Raju, J. and Zhang, Z.J. (2005), "Channel coordination in the presence of a dominant retailer", *Marketing Science*, Vol. 24 No. 2, pp. 254-262, doi: 10.1287/mksc.1040.0081.

Sawik, T. (2010), "Single vs. multiple objective supplier selection in a make to order environment", *Omega*, Vol. 38 Nos 3-4, pp. 203-212, doi: 10.1016/j.omega.2009.09.003.

Spector, David. (2005), "Loyalty rebates: an assessment of competition concerns and a proposed rule of reason", *Competition Policy International*, Vol. 1 No. 1, pp. 89-114.

Tian, L., Vakharia, A.J., Tan, Y. and Xu, Y. (2018), “Marketplace, reseller, or hybrid: strategic analysis of an emerging e-commerce model”, *Production and Operations Management*, Vol. 27 No. 8, pp. 1595-1610, doi: 10.1111/poms.12885.

Tom, Willard K., Balto, David A. and Averitt, Neil W. (1999), “Anticompetitive aspects of market-share discounts and other incentives to exclusive dealing”, *Antitrust Law Journal*, Vol. 67, p. 615.

Vassallo, A. (2012), “Market share discounts”, Tech. rep., Citeseer.

Wu, S. and Li, Q. (2021), “Emergency quantity discount contract with suppliers risk aversion under stochastic price”, *Mathematics*, Vol. 9 No. 15, p. 1791, doi: 10.3390/math9151791.

Xiao, L., Xu, M., Zheng, J.J. and Huang, S. (2020), “Inducing manufacturer’s quality enhancement via retailer’s acquisition strategy”, *Omega*, Vol. 93, 102032, doi: 10.1016/j.omega.2019.02.001.

Yan, Y., Zhao, R. and Lan, Y. (2017), “Asymmetric retailers with different moving sequences: group buying vs. individual purchasing”, *European Journal of Operational Research*, Vol. 261 No. 3, pp. 903-917, doi: 10.1016/j.ejor.2017.02.025.

Appendix

Proof of Theorem 1:

The derivative of Π_1 with respect to p_1 gives

$$\frac{\partial \Pi_1}{\partial p_1} = A - 2(1+\theta)p_1 + \theta p_2 + (c + w_q)(1+\theta) \tag{A1}$$

And the derivative of Π_2 with respect to p_2 gives

$$\frac{\partial \Pi_2}{\partial p_2} = A - 2(1+\theta)p_2 + \theta p_1 + (c + w_q)(1+\theta) \tag{A2}$$

Thus, applying the first-order condition, we have

$$p_1^{QD*}(p_2) = \frac{A + \theta p_2 + (w_q + c)(1+\theta)}{\theta + 1}, p_2^{QD*}(p_1) = \frac{A + \theta p_1 + (w_q + c)(1+\theta)}{\theta + 1}.$$

Then we obtain the equilibrium by solving these two equations simultaneously.

Proof of Theorem 2:

Taking the first derivative of Eq. (6) with respect to w_q gives

$$\frac{\partial \Pi_M}{\partial w_q} = \frac{2(\theta+1)(A - c - 2w_q)}{\theta + 2} \tag{A3}$$

Thus, applying the first-order condition, we have $w_q^{QD*} = \frac{A-c}{2}$.

By plugging w_q^{QD*} into p_1^{QD*} and p_2^{QD*} in Theorem 1, we can get the retailers’ optimal retail prices are

$$p_1^{QD*} = p_2^{QD*} = \frac{(3+\theta)A + (1+\theta)c}{2(\theta+2)} \tag{A4}$$

Then, putting w_q^{QD*}, p_1^{QD*} and p_2^{QD*} into Equations (1)-(5), we get the equilibriums of retailer *i*’s selling quantity, the manufacturer’s final wholesale price and profits of retailers, the manufacturer and the whole supply chain in QD, which are shown in Table 2.

$$q_i^{QD*} = \frac{(A-c)(1+\theta)}{2(2+\theta)},$$

$$w_i^{QD*} = \frac{A-c}{2} + \frac{2(\theta+2)d}{(1+\theta)(A-c)},$$

$$\Pi_i^{QD*} = \frac{(\theta+1)(A-c)^2}{4(\theta+2)^2} - d, \quad \text{(A5)}$$

$$\Pi_M^{QD*} = \frac{(\theta+1)(A-c)^2}{2(2+\theta)} + 2d,$$

$$\Pi_T^{QD*} = \frac{(\theta+3)(\theta+1)(A-c)^2}{2(\theta+2)^2}.$$

Proof of Theorem 3:

The derivative of Π_1 with respect to p_1 gives

$$\frac{\partial \Pi_1}{\partial p_1} = A - 2(1+\theta)p_1 + \theta p_2 + (c+w_r)(1+\theta) + d \quad \text{(A6)}$$

And the derivative of Π_2 with respect to p_2 gives

$$\frac{\partial \Pi_2}{\partial p_2} = A - 2(1+\theta)p_2 + \theta p_1 + (c+w_r)(1+\theta) + d \quad \text{(A7)}$$

Thus, applying the first-order condition, we have

$$p_1^{MS*}(p_2) = \frac{A+b+\theta p_2 + (w_r+c)(1+\theta)}{2(\theta+1)}, p_2^{MS*}(p_1) = \frac{A+b+\theta p_1 + (w_r+c)(1+\theta)}{2(\theta+1).}.$$

Then we obtain the equilibrium by solving these two equations simultaneously.

Proof of Theorem 4:

Taking the first derivative of Equation (9) with respect to w_r gives

$$\frac{\partial \Pi_M}{\partial w_r} = \frac{2(\theta+1)(A-c-2w_r) - 2(2\theta+3)b}{\theta+2} \quad \text{(A8)}$$

Thus, applying the first-order condition, we have $w_r^{MS*} = \frac{(1+\theta)(A-c) - (2\theta+3)d}{2(1+\theta)}$.

By plugging w_r^{MS*} into p_1^{MS*} and p_2^{MS*} in Theorem 1, we can get the retailers' optimal retail prices are

$$p_1^{MS*} = p_2^{MS*} = \frac{(\theta+3)A + (\theta+1)c - (2\theta+1)d}{2(\theta+2)} \quad \text{(A9)}$$

Then, Putting w_r^{MS*}, p_1^{MS*} and p_2^{MS*} into Equations (1), (3), (7) and (8), we get the equilibriums of retailer i's selling quantity, the manufacturer's final wholesale price and profits of retailers, the manufacturer and the whole supply chain in MS, which are shown in Table 2.

$$q_i^{MS*} = \frac{(1+\theta)(A-c)+(2\theta+1)d}{2(2+\theta)},$$

$$w_i^{MS*} = \frac{(\theta+1)(A-c)+(2\theta+1)d}{2(\theta+1)},$$

$$\Pi_i^{MS*} = \frac{((\theta+1)(A-c)+(2\theta+1)d)((\theta+1)(A-c)-(2\theta+1)(2\theta+3)d)}{4(\theta+2)^2(\theta+1)}, \quad \text{(A10)}$$

$$\Pi_M^{MS*} = \frac{((1+\theta)(A-c)+(2\theta+1)d)^2}{2(\theta+1)(2+\theta)},$$

$$\Pi_T^{MS*} = \frac{(\theta+1)(\theta+3)(A-c)^2+2(2\theta+1)(A-c)d-(2\theta+1)^2d^2}{2(\theta+2)^2}.$$

Proof of Proposition 1:

According to Equations (A5) and (A10), we have

$$\frac{\partial \Pi_i^{QD*}}{\partial \theta} = -\frac{(A-c)^2\theta}{4(\theta+2)^3} < 0;$$

$$\frac{\partial \Pi_M^{QD*}}{\partial \theta} = \frac{(A-c)^2}{2(\theta+2)^2} > 0;$$

$$\frac{\partial \Pi_i^{MS*}}{\partial \theta} = \frac{-(\theta+1)^2\theta(A-c)^2-2(\theta+1)^2(5\theta+4)(A-c)b-(2\theta+1)\left(10\theta^2+25\theta+16\right)b^2}{4(\theta+1)^2(\theta+2)^2} < 0;$$

$$\frac{\partial \Pi_M^{MS*}}{\partial \theta} = \frac{((\theta+1)(A-c)+(4\theta+5)b)((\theta+1)(A-c)+(2\theta+1)b)}{2(\theta+1)^2(\theta+2)^2} > 0.$$

Proof of Proposition 2:

According to Equations (A5) and (A10), we have

$$\frac{\partial \Pi_i^{QD*}}{\partial d} = -1 < 0;$$

$$\frac{d\Pi_M^{QD*}}{dd} = 2 > 0;$$

$$\frac{\partial \Pi_i^{MS*}}{\partial d} = -\frac{(2\theta+1)\left((\theta+1)^2(A-c)+(2\theta+1)(2\theta+3)d\right)}{(\theta+1)(\theta+2)^2} < 0;$$

$$\frac{\partial \Pi_M^{MS*}}{\partial d} = \frac{(2\theta+1)((\theta+1)(A-c)+(2\theta+1)d)}{(\theta+1)(\theta+2)} > 0.$$

Proof of Proposition 3:

Comparing w_q^{QD*} and w_r^{MS*}, we can derive $w_q^{QD*} - w_r^{MS*} = (2\theta+3)d/2(\theta+1) > 0$. Similarly, $p_i^{QD*} - p_i^{MS*} = (2\theta+1)d/2(\theta+2) > 0$; and $q_i^{QD*} - q_i^{MS*} = -(2\theta+1)d/2(\theta+2) < 0$.

Proof of Proposition 4:

Comparing Π_{Ri}^{QD*} and Π_{Ri}^{MS*}, we have

$$\Pi_{Ri}^{QD*} - \Pi_{Ri}^{MS*} = \frac{\left((2\theta+1)^2(2\theta+3)d - 4(\theta+1)(\theta+2)^2 + 2(\theta+1)^2(2\theta+1)(A-c)\right)d}{4(\theta+1)(\theta+2)^2},$$

whose sign depends on d:

(1) If $d > \frac{4(\theta+1)(\theta+2)^2 - 2(\theta+1)^2(2\theta+1)(A-c)}{(2\theta+1)^2(2\theta+3)}$, $\Pi_{Ri}^{QD*} > \Pi_{Ri}^{MS*}$;

(2) If $d \le \frac{4(\theta+1)(\theta+2)^2 - 2(\theta+1)^2(2\theta+1)(A-c)}{(2\theta+1)^2(2\theta+3)}$, $\Pi_{Ri}^{QD*} \le \Pi_{Ri}^{MS*}$.

Furthermore, note that $d > 0$ in our paper. Let $d_R = \frac{4(\theta+1)(\theta+2)^2 - 2(\theta+1)^2(2\theta+1)(A-c)}{(2\theta+1)^2(2\theta+3)}$. Then when $d_R > 0$, i.e. $A < c + \frac{2(\theta+2)^2}{(\theta+1)(2\theta+1)}$, there exists a d satisfied $d > d_R > 0$, making $\Pi_{Ri}^{QD*} > \Pi_{Ri}^{MS*}$. When $d_R \le 0$, i.e. $A \ge c + \frac{2(\theta+2)^2}{(\theta+1)(2\theta+1)}$, it is easy to find that $d > d_R$ always holds in this condition, then $\Pi_{Ri}^{QD*} > \Pi_{Ri}^{MS*}$ is always true for all $d > 0$.

To sum up, if $A \ge c + \frac{2(\theta+2)^2}{(\theta+1)(2\theta+1)}$, or $c < A < c + \frac{2(\theta+2)^2}{(\theta+1)(2\theta+1)}$, and $d > \frac{4(\theta+1)(\theta+2)^2 - 2(\theta+1)^2(2\theta+1)(A-c)}{(2\theta+1)^2(2\theta+3)}$, $\Pi_{Ri}^{QD*} > \Pi_{Ri}^{MS*}$, otherwise, $\Pi_{Ri}^{QD*} \le \Pi_{Ri}^{MS*}$.

Proof of Proposition 5:

According to Equations (A5) and (A10), we have $\Pi_M^{QD*} = \frac{(\theta+1)(A-c)^2}{2(2+\theta)} + 2d$, and $\Pi_M^{MS*} = \frac{((1+\theta)(A-c)+(2\theta+1)d)^2}{2(\theta+1)(2+\theta)}$. Comparing Π_M^{QD*} and Π_M^{MS*}, we get

$$\Pi_M^{QD*} - \Pi_M^{MS*} = \frac{\left(-(2\theta+1)^2 d - 2(2\theta+1)(\theta+1)(A-c) + 4(\theta+1)(\theta+2)\right)d}{2(\theta+1)(\theta+2)},$$

whose sign depends on d:

(1) If $d \le \frac{4(\theta+1)(\theta+2) - 2(2\theta+1)(\theta+1)(A-c)}{(2\theta+1)^2}$, $\Pi_M^{QD*} - \Pi_M^{MS*} \ge 0$;

(2) If $d > \frac{4(\theta+1)(\theta+2) - 2(2\theta+1)(\theta+1)(A-c)}{(2\theta+1)^2}$, $\Pi_M^{QD*} - \Pi_M^{MS*} < 0$.

Let $d_M = \frac{4(\theta+1)(\theta+2) - 2(2\theta+1)(\theta+1)(A-c)}{(2\theta+1)^2}$. Solving $d_M > 0$, we get $A < c + \frac{2(\theta+2)}{(2\theta+1)}$. In this case, there exists a d satisfied $0 < d \le d_M$ making $\Pi_M^{QD*} \ge \Pi_M^{MS*}$. And if $d_M \le 0$, i.e., $A \ge c + \frac{2(\theta+2)}{(2\theta+1)}$, $d > 0 > d_M$ always holds, implying $\Pi_M^{QD*} < \Pi_M^{MS*}$ always holds for all $d > 0$.

To sum up, when $A \ge c + \frac{2(\theta+2)}{(2\theta+1)}$, or $c < A < c + \frac{2(\theta+2)}{(2\theta+1)}$ and $d > \frac{4(\theta+1)(\theta+2) - 2(2\theta+1)(\theta+1)(A-c)}{(2\theta+1)^2}$, $\Pi_M^{QD*} < \Pi_M^{MS*}$, otherwise, $\Pi_M^{QD*} \ge \Pi_M^{MS*}$.

Proof of Proposition 6:

Denote $\widehat{A} = c + \frac{2(\theta+2)^2}{(\theta+1)(2\theta+1)}$, and $A = c + \frac{2(\theta+2)}{(2\theta+1)}$, or $c < A < c + \frac{2(\theta+2)}{(2\theta+1)}$. According to the proofs of Propositions 4 and 5, we have

(1) If $A \ge \widehat{A}$, or $c < A < \widehat{A}$, and $d > d_R$, $\Pi_i^{QD*} > \Pi_i^{MS*}$, otherwise, $\Pi_i^{QD*} \le \Pi_i^{MS*}$;

(2) If $A \ge A$, or $c < A < A$ and $d > d_M$, $\Pi_M^{QD*} < \Pi_M^{MS*}$, otherwise, $\Pi_M^{QD*} \ge \Pi_M^{MS*}$.

Comparing $\widehat{A}$ and A, we can derive $\widehat{A} - A = \frac{2(\theta+2)}{(\theta+1)(2\theta+1)} > 0$, then $\widehat{A} > A$. Comparing d_R and d_M, we can derive $d_R - d_M = \frac{2(\theta+1)(\theta+2)((2\theta+1)(A-c) - 2(\theta+1))}{(2\theta+1)^2(2\theta+3)}$, then we get if $A = A' = c + \frac{2(\theta+1)}{2\theta+2}$, $d_R = d_M$; if $A > A'$,

$d_R > d_M$; if $A < A'$, $d_R < d_M$. Furthermore, solving $\dot{A} - A'$, we get $\dot{A} - A' = \frac{6(\theta+1)}{(2\theta+2)(2\theta+1)} > 0$; solving $\widehat{A} - A'$, we get $\widehat{A} - A' = \frac{(\theta+3)}{(2\theta+2)(2\theta+1)} > 0$.

Thus, based on the above results, we have if $A' \leq A \leq \widehat{A}$, and $d_M < d < d_R$, $\Pi_M^{QD*} < \Pi_M^{MS*}$ and $\Pi_i^{QD*} < \Pi_i^{MS*}$, then MS is the strategy of pareto improvement in this condition. And if $c \leq A \leq A'$, and $d_R < d < d_M$, $\Pi_M^{QD*} \geq \Pi_M^{MS*}$ and $\Pi_i^{QD*} \geq \Pi_i^{MS*}$, then QD is the strategy of pareto improvement in this condition.

Proof of Proposition 7:

Comparing Π_T^{QD*} and Π_T^{MS*}, we can derive

$$\Pi_T^{QD*} - \Pi_T^{MS*} = -\frac{(2\theta+1)(2(A-c)-(2\theta+1)d)d}{2(\theta+2)^2},$$

whose sign depends on A, if $c < A < c + \frac{(2\theta+1)d}{2}$, $\Pi_T^{QD*} > \Pi_T^{MS*}$; and if $A \geq c + \frac{(2\theta+1)d}{2}$, $\Pi_T^{QD*} \leq \Pi_T^{MS*}$.

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