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The bundling sales pricing strategy of the closed-loop supply chain under patent protection

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Abstract

Purpose – The purpose of this article is based on the unit patent license fee model in the closed-loop supply chain.
Design/methodology/approach – This paper analyzes the impact of the bundling strategy of the retailer selling new products and remanufactured products on the closed-loop supply chain under the condition that the original manufacturer produces new products and the remanufacturer produces remanufacturing products.
Findings – The results show that alternative products can be bundled, and in many cases, the bundling of remanufactured products and new products is better than selling alone.
Originality/value – If the retailer chooses bundling, for the remanufacturer, when certain conditions are met, the benefits of bundling are greater than the separate sales at that time; for the original manufacturer, when the recycling price sensitivity coefficient is high, the bundling is better than separate sales.

Keywords Closed-loop supply chain, Bundling sales, Alternative product, Patent licensing, Stackelberg game
Paper type Research paper

1. Introduction

Product bundling is one of the simplest and most widely used business strategies to increase the profitability of a seller with little extra effort when the marginal cost of the product is low. In production, distribution and consumer markets, the value of a product is often created by various actors in the supply chain, whether independent companies or allied firms, who cooperate and simply play a zero-sum game, but are developing harmoniously and

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establishing stable bundling relationships in a balanced state. Common bundling practices exist in all aspects of the industry, such as home TV and pay-per-view, technology products and services, promotions and services on e-commerce platforms, multisource data platforms and patent pools, digital music and albums, etc. However, as early as 1976, Adams and Yellen (1976) established three main framework strategies for bundled sales, namely separate sales, pure bundled sales and mixed bundled sales, which provide the basis for the theoretical research of many subsequent generations of scholars. There are risks associated with product bundling, but it also increases customer willingness to purchase new products and add value, and it increases consumer convenience due to lower search and assembly costs (Eppen *et al.*, 1991; Harris, 2006; Kim *et al.*, 2008; Reinders *et al.*, 2010; Simonin and Ruth, 1995).

The economic structure of many markets is such that value is created jointly by multiple producers whose output is collected and sold by a consortium of producers or independent retailers as a common bundle of products. As a result, products that act as substitutes for each other are often bundled and sold as well. An inspiring example is video entertainment consumed on TVs and other personal devices, where bundled service providers (cable and satellite TV providers or streaming services) combine movies and TV shows from oligopolistic content providers such as movie studios and programming networks. The collection is offered as a bundle from which buyers can select and consume any item from multiple producers (e.g. documentaries, animated films, romance and action films). This bundling structure is favored when buyers want to diversify their available merchandise and have flexibility in choosing what and how much they consume at any given time, or when economic or technological factors make it difficult for producers to sell directly to consumers.

However, in many examples of bundled sales, the market structure is relatively stable, with many companies competing for business. In such cases, bundled sales are usually not intended to exclude competitors from the market, but are a business strategy to attract consumers to buy more products from the same company. For example, according to seller ratings, Consumer Packaged Goods Industry Organization (CPGIO) is the trusted top trusted seller of more than 70,000 Amazon.com customers in the United States, who are strong Amazon TOP 100 sellers, ranking 62nd overall. CPGIO is best at helping brand sellers to issue: creative product bundling strategies, packaging and logistics solutions, and using data-driven operations to expand the business scale. Among them, upselling and bundling are the most proud projects of CPGIO, and this method has created a negligible sales myth. There are several ways of CPGIO bundling: product mix and match package, personalized bundling, excess inventory bundling, new product package, etc. Creative strategies such as product bundling and value-added supporting logistics solutions can effectively increase the order scale and overall revenue, thus bringing a positive experience to customers.

To mitigate climate change in the context of increasing annual sales of electric vehicles, Alfons and Nina (2020) conduct a joint choice-based experiment with 393 respondents in Austria to investigate the impact of bundled sales of electric vehicles, solar panels with photovoltaics and battery storage products on purchase intentions. It is found that not only are most potential electric vehicle (EV) drivers more willing to buy bundles, but that the willingness to buy a bundle of EVs with photovoltaic vehicle and base stations is twice as high compared to stand-alone EVs. To investigate and determine the price sensitivity of consumers in three- and five-star hotels, a study is conducted to increase sales through bundling strategies. The findings reveal the optimal and acceptable price ranges for three- and five-star hotels and help hotel managers and administrators to manage prices without losing market share or revenue (Dominique-Ferreira and Antunes, 2019). In summary, retailers use bundling as an effective marketing tool to increase consumer demand at discounted prices (Chen *et al.*, 2019). Therefore, it is of great practical importance to consider bundling similar products from the same retailer in this paper.

In order to implement the “Circular Economy Promotion Law” and promote a resource-saving and environment-friendly society, as early as 2010, the National Development and

Reform Commission of China indicated that remanufacturing is an advanced form of "reuse" in the circular economy. However, researches on remanufacturing usually focus on product recycling channels and strategies (Pal and Sarkar, 2021; Pan and Lin, 2021), product pricing decision and coordination (Lang and Jia, 2021; Li *et al.*, 2021a, b), emission reduction and technologies (Chen *et al.*, 2022; Li *et al.*, 2021a, b; Yilmaz *et al.*, 2021), and so on, but relatively little on the sales of remanufactured products.

This paper focuses on the optimal pricing decision problem for channel members, including the original manufacturer with its own product, the remanufacturer who develops the remanufacturer's product and the retailer who distributes both products. The retailer can either sell the manufacturer's product alone or sell both products in a bundle. Note that bundling is an effective marketing tool for retailers, but it reduces consumers' individual valuation of the bundled products (Prasad *et al.*, 2015). Thus, a comprehensive analysis of bundling strategies is necessary, and this paper investigates the impact of bundle selling in retailer-driven distribution channels to achieve decision optimization for channel members and provides insights for practice and theory. Bundling usually takes two strategic forms: pure bundling and mixed bundling. Pure bundling refers to selling goods only in a bundle, whereas mixed bundling allows the components to be purchased separately, as well as in a bundle. Obviously, pure bundling is a special case of mixed bundling, so we focus on mixed bundling here.

In this paper, we identify some characteristics of mixed bundling and investigate the channel members' optimal pricing behaviors, attempting to answer the following research questions: (1) What are the optimal decisions of the retailer, the original manufacturer and the remanufacturer in the mixed bundling setting? (2) Does a mixed bundling strategy always perform better than a separate sales strategy for the retailer and two manufacturers when alternative products exist? (3) How should the retailer, the original manufacturer and the remanufacturer choose the form of bundling (i.e. mixed bundling or separate sales) given the impact of product substitution and the heterogeneity of consumer valuation of products? To address these research questions, we develop a functional optimization model based on product substitutability. Bundling sales can lower the price of a single product, consumers can start with a cheap price and businesses can take a "small profits" route to expand brand awareness, first let their own brand in the market to ensure a high share. This paper discusses the bundling pricing decision with the participation of original manufacturers and remanufacturers under patent protection, which has important practical significance.

The rest of this article is organized as follows. In Section 2, we briefly review the literature on the bundling sales of products. In Section 3, we introduce the description and assumptions of the models. Section 4 discusses the results of the model including insights into some propositional analysis and management. In Section 5, we perform a sensitivity analysis to further explore the impact of parameter changes on the supply chain. Finally, Section 6 concludes this paper and points out the limitations of this paper and the directions for further research.

2. Literature review

The purpose of this paper is to discuss the issue of bundled sales of new and remanufactured products with product substitution. The literature related to this study mainly includes the following three streams: research on the value of bundling, bundle model and strategy, and the sales of new and remanufactured products.

2.1 The value of bundling

Chao and Derdenger (2013) find that the sales price decreases with bundling in a bilateral market, but the participation in both platforms and social welfare both increase. Bhargava

(2021) performs bundling pricing for a uniformly distributed two product line. Performing bundle pricing reduces multiproduct pricing to a single variable nonlinear optimization problem in the bundle price. Banciu *et al.* (2010) consider bundling two products with limited quantities and conclude that the optimal strategy depends on the absolute and relative availability of the two resources and the degree of subadditivity of the product quality. Further, Abdallah (2018) suggests that a pure bundling mechanism is able to extract consumer surplus in the presence of non-negative marginal costs and associated distributions. For the impact of bundling strategies, Cao *et al.* (2019) find that bundling helps retailers mitigate the adverse consequences of demand uncertainty for primary products and consumer price discrimination for secondary products, but reduces retailers' wholesale price elasticity, while bundling benefits the manufacturer.

Honhon and Pan (2017) demonstrate that each bundling strategy chosen in a firm's vertically managed product sales has optimal expression. Chen *et al.* (2019) investigate how valuation discounts affect decision optimization in the distribution channel. Alfons and Nina (2020) segment four potential customer groups to study the market for bundled sales of electric vehicles and find that policy incentives are more effective when product bundles are tagged with price tags that have already been discounted by subsidies. Chen *et al.* (2023) investigate how valuation discounts influence the online retail platforms' decision optimization in bundling. Tang *et al.* (2023) find that whether initiated by suppliers or platforms, invariably benefits suppliers, both supplier- and platform-initiated bundling often increases the total profit. Li *et al.* (2023) proposes a dynamic bundle pricing strategy based on real-time consumer behaviors. Chirantan and Chandra (2024) find that the decision of which of two manufacturers will profit more depends on product complementarity and the residual values obtained from end-of-used products acquisition and their handling.

2.2 Bundle model and strategy

Chen and Zhang (2015) models interpersonal bundling for demand uncertainty, explaining that the profitability of interpersonal bundling depends on the nature of uncertainty and the parameters of the market environment when the group size is minimal or maximum. Giri *et al.* (2017) consider a dual oligopoly market and examine competition in a two-tier supply chain system. Dominique-Ferreira and Antunes (2019) survey and determine the price sensitivity of three and five-star hotel consumers and determine the impact of bundling strategies on consumer price sensitivity.

Gallego *et al.* (2020) design a dynamic nonlinear pricing model that allows the seller to dynamically select a price for each bundle size and restricts the seller to dynamically select a unit price for all bundle sizes. Li and Xing (2020) examine the regulation by a single principal of consumer bundling transactions between an intermediary and an agent with hidden characteristics. When it is up to the retailer to decide on a bundling strategy, Chen *et al.* (2020) study the impact of product relevance (substitutability and complementarity) on decision optimization.

2.3 Sales of new and remanufactured products

Ferrer and Swaminathan (2006) find that if remanufacturing is profitable, original equipment manufacturers may give up some of their profits to increase the number of core parts available for remanufacturing in the future at a reduced price. Atasu *et al.* (2008) investigate the case of remanufacturing as a marketing strategy. Kovach *et al.* (2018) analyze the impact of different incentive schemes on the sales of new and remanufactured products. Tang *et al.* (2019) investigate pricing and warranty decisions under two warranty models. Cheng *et al.* (2022) assume that all manufacturers are responsible for recycling and remanufacturing and assume that new and remanufactured products are homogeneous and have the same selling price in the demand market. Meng *et al.* (2020) study the impact of government consumer subsidies on the operation of closed-loop supply chains and find that the subsidies will decrease the demand for new products and increase the demand for remanufactured products. Shi *et al.* (2020) investigate the effect of a company's organizational structure on direct or indirect sales of these new and remanufactured products. Lou *et al.* (2020) are interested in multichannel cooperation between the original manufacturer and two competing retailers. When bundling products and warranties with alternative sales, it is observed that product substitutability and supply chain power structures do not influence product pricing and warranty decisions. The results show that offering warranties for remanufactured products affects new products pricing and remanufactured product pricing thereby increasing remanufacturing, individual and channel profitability and consumer surplus.

In summary, we have reviewed the bundling strategy, product pricing, coordination mechanisms and the impact of bundled sales on the supply chain, as well as the sales of new and remanufactured products in a closed-loop supply chain. In this paper, we consider the bundling of new products and remanufactured products into the same sales mix and adopt a unified sales strategy to promote the sales of remanufactured products; in addition, it can also be seen that bundling mostly occurs between products with complementary characteristics. This paper studies the bundling and selling strategy under patent protection, in which the remanufacturer needs to be licensed by the original manufacturer to remanufacture the used products into remanufactured products. The original manufacturer and remanufacturer then sell new products and remanufactured products to the retailer, respectively, and the retailer will decide whether to bundle the products or sell them separately. We summarize the difference between our paper and related literature in Table 1. We contribute to the literature as follows:

(1) We analyze the closed-loop supply chain pricing decision problem under the bundling sales mode;

(2) We study the bundling sales pricing problem under the patent licensing mode.

Research paper	The value of bundling	Bundle model and strategy	New and remanufactured products	Patent licensing	Supply chain
Bhargava (2021)	√	√			
Abdallah (2018)	√	√			
Cao *et al.* (2019)	√	√			√
Alfons and Nina (2020)	√	√			
Gallego *et al.* (2020)	√	√			
Dominique-Ferreira and Antunes (2019)	√	√			
Li and Xing (2020)	√	√			
Chen *et al.* (2020)	√	√			√
Cheng *et al.* (2022)	√		√		√
Shi *et al.* (2020)			√		
Lou *et al.* (2020)	√	√	√		√
Tang *et al.* (2019)	√		√		√
This paper	√	√	√	√	√

Source(s): Table created by authors

Table 1. Comparisons between our study and the related literature

3. Model description and assumptions

As there are two manufacturers, Hershey's and Mars, who produce two different qualities of children's cartoon characters and sell them through the same retailer 7–11, with some substitutability between the two products. So in this paper, we examine the game between the original manufacturer, the remanufacturer and the retailer. In a single product monopoly market, we consider a two-tier supply chain system consisting of an original manufacturer who produces new products, a remanufacturer who produces remanufactured products and a retailer who sells both new and remanufactured products, where the remanufactured products are remanufactured through patent licensing deals with the original manufacturer, so there is no significant performance difference between the new and remanufactured products. The original manufacturer and the remanufacturer distribute their products to the same retailer at different wholesale prices, and the retailer can choose different marketing bundling strategies to sell the products, such as separate sales and bundling sales to maximize the profits. In this paper, the superscript separate retails (SR) is used to denote the case of separate sales model and the superscript bundled retails (BR) is used to denote the case of bundled sales model. The two sales models are shown in Figure 1. We summarize all the notations in Table 2.

Regardless of the marketing strategy adopted by the retailer, there will be two products or a combination of products in the market. Using the approach of describing multi-product demand as a linear relationship between demand and price and product substitutability (Chen *et al.*, 2008; Yan *et al.*, 2018), when the retailer adopts a bundling model to sell new and remanufactured products, the product demand function can be expressed as:

$$q_{12} = d - p_{12} * \beta + \theta\left(p_i + p_j\right), (i,j = 1,2;\ i \neq j) \tag{1}$$

$$q_i = d - p_i * \beta + \theta\left(p_{12} + p_j\right), (i,j = 1,2;\ i \neq j) \tag{2}$$

Without considering the increase in sales as product substitution, this function is able to describe the relationship between changes in demand, price sensitivity coefficients, prices, substitution coefficients, etc., without affecting the solution to the final price decision.

In the same way, when the retailer sells two products separately, the product demand function can be expressed as:

$$q_i = d - p_i * \beta + \theta * p_j, (i,j = 1,2;\ i \neq j) \tag{3}$$

The notations and definitions of this paper are shown in Table 2.

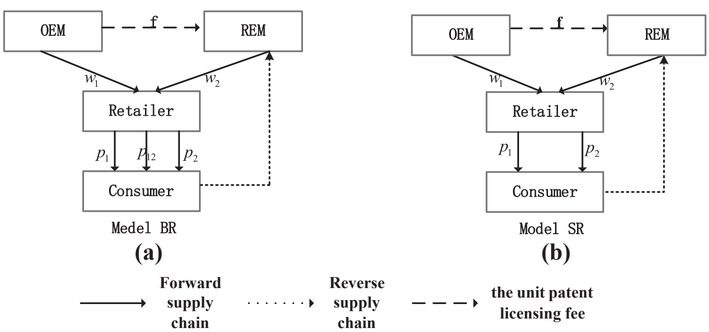

Figure 1. Remanufacturing closed-loop supply chain in bundling modes

Note(s): OEM denotes the original manufacturer and REM denotes the remanufacturer

Source(s): Figure created by authors

Symbols	Definitions
Decision variables	
p_i	Selling price of product i $(i = 1, 2)$
p_{12}	Selling price of bundled products
w_i	Wholesale price of the product i $(i = 1, 2)$
Decision objectives	
$\prod_i^{SR}, \prod_i^{BR}, \prod_{12}^{SR}, \prod_{12}^{BR}, \prod_R^{SR}, \prod_R^{BR}$	Optimal profit for the original manufacturer, remanufacturer and retailer in separate sales and bundle sales $(i = 1, 2)$
Model parameters	
I_1, I_2, I_{12}	Separate new products, remanufactured products and bundled products
q_i	Sales of products i ($i = 1, 2$, where 1 denotes new products, and 2 denotes remanufactured products)
q_{12}	the demand of a unit bundled product combined by one unit of original product and one unit of remanufactured product
p_r	Recycling price of used products
c_m, c_r	represents the production cost of new and remanufactured products, respectively, and $c_m > c_r$
d	maximum potential market demand
β	Price sensitivity coefficient of demand
θ	Product substitution factor
f	unit patent licensing fee
a	Base recycling price
φ	Ratio of unit patent licensing fee to wholesale price of remanufactured product
γ	Recycling price sensitivity factor

Source(s): Table created by authors

Table 2. Notations and definitions

We make the following assumptions.

Assumption 1. The retailer has complete freedom to choose the sales model for new and remanufactured products, i.e. separate sales and bundled sales. When selling separately, the new products and the remanufactured products will be in perfect competition and will exist independently in the market, whereas when selling as a bundle, the new product and the remanufactured product will not only be sold as one good in a 1:1 ratio, but will also be sold independently. For example, the bundling strategy of mobile phones, headphones, wireless chargers and other products, as well as the product and service bundling strategy of mobile phones and telecom services.

Assumption 2. The original manufacturer and the remanufacturer are the leaders of a closed-loop supply chain with equal power to each other and the retailer is a follower (Heese, 2012; He and Cao, 2016)

Assumption 3. The original manufacturer, the remanufacturer and the retailer are all rationalistic and have complete information, with profit maximization as the decision objective.

Assumption 4. New products, remanufactured products and bundled products share the same product market, i.e. the same maximum potential market demand, and are competing with each other as substitutes, excluding the retailer's cost of goods sold to simplify the calculation (Chen *et al.*, 2008; Giri *et al.*, 2017; Honhon and Pan, 2017).

Assumption 5. Without loss of generality, it is assumed that the demand for remanufactured products is met by recovered products and that the amount of new products recovered equals the amount of remanufactured demand (Gan *et al.*, 2017).

Assumption 6. Although new and remanufactured products are homogeneous (Cheng *et al.*, 2022), the price sensitivity coefficient is more sensitive to the impact on own demand than the impact of product substitutability on product demand due to, for example, differences in consumer perceptions of the value of the two products, i.e. $0 < \theta < \gamma < \beta < 1, 0 < \varphi < 1, 0 < 2\theta < \beta < 1$ (Prasad *et al.*, 2015).

Similar to Chen *et al.* (2017), this paper describes the unit patent licensing fee as a proportion of the wholesale price of the remanufactured product, $f = \varphi * w_2$ where φ is the said proportion; in addition, referring to the assumptions of Bakal and Akcali (2006) in the closed-loop supply chain, the supply of the remanufactured product is regarded as a function of the recycling price, i.e. $G(r) = a + b * r$, and this paper takes the inverse function of its recycling volume, then the recycling price can be expressed as $p_r = a + \gamma * q_2$, where γ is the recycling price sensitivity factor and a is the base recycling price.

4. Analysis

In the Stackelberg game of a supply chain dominated by two manufacturers, the two manufacturers make the first decision simultaneously as leaders, i.e. they give their respective wholesale prices, and the retailer make their decisions accordingly, determining the selling prices of new and remanufactured products, the selling prices of bundled products and their respective demand quantities. At the same time, in the closed-loop supply chain game, the retailer chooses whether to bundle the new and remanufactured products according to their own profit maximization objectives. Thus, the optimization problems of the retailer, the manufacturer and the remanufacturer can be expressed separately as follows.

4.1 Separate retails model (SR)

In this model, the retailer sells the new and remanufactured products separately by wholesaling the products from the original manufacturer and the remanufacturer. The decision sequence is as follows: the original manufacturer and remanufacturer first decide their own wholesale prices and the retailer make decisions accordingly to determine the sales prices of new and remanufactured products. The decision problems for the retailer, the original manufacturer and the remanufacturer are as follows.

$$\max_{p_1,p_2} \Pi_R^{SR} = (p_1 - w_1)q_1 + (p_2 - w_2)q_2 \tag{4}$$

$$\max_{w_1} \Pi_1^{SR} = (w_1 - c_m)q_1 + \varphi * w_2 q_2 \tag{5}$$

$$\max_{w_2} \Pi_2^{SR} = (w_2 - \varphi * w_2 - c_r - p_r)q_2 \tag{6}$$

Lemma 1. In the SR model, the recycling price of used products is

$$p_r^{SR} = \frac{\begin{array}{c}-d\beta\gamma(2\beta+\theta)(-1+\varphi) + a\left(2\beta^3\gamma - 8\beta^2(-1+\varphi) - \beta\gamma\theta^2(1+\varphi) + 2\theta^2\left(\varphi^2-1\right)\right) + \\ \beta\gamma\left(-\beta\theta(\varphi-1)c_m + \left(-2\beta^2+\theta^2(1+\varphi)\right)c_r\right)\end{array}}{2L_1}, \tag{7}$$

and the sales quantities of new and remanufactured products are as follows, respectively:

$$q_1^{SR}=\frac{\begin{array}{c}-a\beta^2\theta(-1+\varphi)+d\left(\beta^3\gamma-\beta^2(2+\gamma\theta)(-1+\varphi)+\theta^2(-1+\varphi)\varphi+\beta\theta(1+\varphi(-2-\gamma\theta+\varphi))\right)\\+\beta\left(-\beta^3\gamma+\beta\gamma\theta^2+2\beta^2(-1+\varphi)-\theta^2(\varphi-1)\right)c_m-\beta^2\theta(-1+\varphi)c_r\end{array}}{2L_1}, \tag{8}$$

$$q_2^{SR}=\frac{\beta\left(-2a\beta^2-d(2\beta+\theta)(\varphi-1)+a\theta^2(1+\varphi)-\beta\theta(\varphi-1)c_m+\left(-2\beta^2+\theta^2(1+\varphi)\right)c_r\right)}{2L_1}, \tag{9}$$

$$w_1{}^{SR}=\frac{\begin{array}{c}a\beta\theta(1+\varphi)+\alpha\left(\theta+\beta(2+\gamma(\beta+\theta))+\beta(-2+\gamma\theta)\varphi-\theta\varphi^2\right)\\+\beta^2(2+\beta\gamma-2\varphi)c_m+\beta\theta(1+\varphi)c_r\end{array}}{2\beta^3\gamma-4\beta^2(-1+\varphi)-\beta\gamma\theta^2(1+\varphi)+\theta^2(-1+\varphi^2)}, \tag{10}$$

$$w_2{}^{SR}=\frac{2a\beta^2+\alpha(2\beta+\theta)(1+\beta\gamma-\varphi)+\beta\theta(1+\beta\gamma-\varphi)c_m+2\beta^2c_r}{2\beta^3\gamma-4\beta^2(-1+\varphi)-\beta\gamma\theta^2(1+\varphi)+\theta^2(-1+\varphi^2)}. \tag{11}$$

The solving process of equilibrium decisions in different models is shown in Appendix 1.

4.2 Bundled sales model (BR)

Under this model, the retailer wholesales products from the original manufacturer and the remanufacturer, mixes the new product and the remanufactured product in the same proportion of packaging as goods for sale, and sells the single new and remanufactured products independently. The decision sequence is as follows: the original manufacturer and remanufacturer first decide their own wholesale prices and the retailer make decisions accordingly to determine the sales prices of new and remanufactured products and the sales prices of bundled products. The decision problems for the retailer, the original manufacturer and the remanufacturer are as follows, respectively:

$$\max_{p_1,p_2,p_{12}}\Pi_R^{BR}=(p_{12}-w_1-w_2)q_{12}+(p_2-w_2)q_2+(p_1-w_1)q_1 \tag{12}$$

$$\max_{w_1}\Pi_1^{BR}=(w_1-c_m)(q_1+q_{12})+\varphi*w_2(q_2+q_{12}) \tag{13}$$

$$\max_{w_2}\Pi_2^{BR}=(w_2-\varphi*w_2-c_r-p_r)(q_2+q_{12}) \tag{14}$$

Lemma 2. In the BR model, the sales quantity of each product and recycling price of used products are as follows, respectively:

$$q_{12}^{BR}=\frac{\begin{array}{c}-2a(\beta-\theta)^2(\theta+\beta(\varphi-3)-3\theta\varphi)+d\\\left(-2\beta^3\gamma(\varphi-1)+2\beta\theta(-5+\gamma\theta-\varphi)(-1+\varphi)-\theta^2\left(3(-1+\varphi)^2+2\gamma(\theta+3\theta\varphi)\right)+\beta^2((-1+\varphi)(3+\varphi)+2\gamma(\theta+3\theta\varphi))\right)\\+2(\beta-\theta)^2\left(\left(2\beta^2\gamma-3\beta(\varphi-1)+\theta(\varphi-1-2\gamma\theta)\right)c_m-(\theta+\beta(\varphi-3)-3\theta\varphi)c_r\right)\end{array}}{L_4} \tag{15}$$

$$q_1^{BR}=\frac{\begin{array}{c}-2\alpha/(\beta-\theta)^2(\beta+5\theta+3\varphi-3\theta\varphi)+d\\(-2\beta^3\gamma(5+\varphi)+2\beta\theta(-(-1+\varphi)(3+\varphi)+\gamma\theta(5+\varphi))+\beta^2(6\gamma\theta(1+\varphi)+(\varphi-1)(9+\varphi))-\theta^2(6\gamma\theta(1+\varphi)+(-1+\varphi)(-1+3\varphi)))\\+4(\beta-\theta)\left(2\beta^3\gamma-2\beta^2(2\gamma\theta+\varphi-1)+\theta^2(4\gamma\theta+\varphi-1)+\beta\theta(-2\gamma\theta+3\varphi-3)\right)c_m-2(\beta-\theta)^2(\beta+5\theta+\beta\varphi-3\theta\varphi)c_r\end{array}}{L_4} \tag{16}$$

$$q_2^{BR}=\frac{\begin{array}{c}4a(\beta-\theta)\left(2\beta^2+\beta\theta(\varphi-3)-\theta^2(1+3\varphi)\right)+d\left(2\beta^2\gamma(-1+\varphi)+\theta^2(\varphi-1+2\gamma\theta)\right.\\\left.(1+3\varphi)-2\beta\theta(3+\gamma\theta-\varphi)(-1+\varphi)-\beta^2\left(9+2\gamma\theta-10\varphi+6\gamma\theta\varphi+\varphi^2\right)\right)-2(\beta-\theta)^2\\\left(\beta+2\beta^2\gamma+\theta(5-2\gamma\theta-5\varphi)-\beta\varphi\right)c_m+4(\beta-\theta)\left(2\beta^2+\beta\theta(\varphi-3)-\theta^2(3\varphi+1)\right)c_r\end{array}}{L_4} \tag{17}$$

$$p_r^{BR}=\frac{\begin{array}{c}2d\gamma\left(3\beta^2-4\beta\theta+\theta^2\right)(\varphi-1)+\alpha\left(\frac{1}{2}L_5+L_7-L_6-L_8\right)\\+\gamma(\beta-\theta)\left(-2(\beta-3\theta)(\beta-\theta)(\varphi-1)c_m-\left(-7\beta^2+10\beta\theta+\theta^2+(\beta-3\theta)^2\varphi\right)c_r\right)\end{array}}{L_5+L_7-L_6-L_8} \tag{18}$$

$$w_1{}^{BR}=\frac{\frac{4\alpha}{-\beta+\theta}-4c_m+\frac{(\beta-3\theta)(1+\varphi)(a(\beta-\theta)+\alpha(1+2\beta\gamma-2\gamma\theta-\varphi)+(\beta-\theta)c_r)}{(\beta-\theta)^2(1+\beta\gamma-\gamma\theta-\varphi)}}{8(1-\frac{(\beta-3\theta)^2(1+2\beta\gamma-2\gamma\theta-\varphi)(1+\varphi)}{16(\beta-\theta)^2(1+\beta\gamma-\gamma\theta-\varphi)}} \tag{19}$$

$$w_2{}^{BR}=\frac{\begin{array}{c}-8a(\beta-\theta)^2-2\alpha(3\beta-\theta)(1+2\beta\gamma-2\gamma\theta-\varphi)+\\2(\beta-3\theta)(\beta-\theta)(1+2\beta\gamma-2\gamma\theta-\varphi)c_m-8(\beta-\theta)^2c_r\end{array}}{\begin{array}{c}2\beta^3\gamma(-7+\varphi)+2\beta\theta((-1+\varphi)(-13+3\varphi)+3\gamma\theta(-3+5\varphi))-\beta^2\left((-15+\varphi)(-1+\varphi)\right.\\\left.+2\gamma\theta(-17+7\varphi)\right)-\theta^2((-1+\varphi)(-7+9\varphi)+2\gamma(\theta+9\theta\varphi))\end{array}} \tag{20}$$

P1. In the sales market for new and remanufactured products (SR), where new and remanufactured products are sold separately, we have $w_1^{SR} > w_2^{SR}$, $p_1^{SR} > p_2^{SR}$ and $q_1^{SR} > q_2^{SR}$.

Proposition 1 suggests that in a separate sales market, the wholesale price, sales price and even sales volume of new products are greater than that of remanufactured products, and remanufactured products are clearly at a competitive disadvantage; although the remanufacturer is required to pay patent protection fees to the original manufacturer and to pay consumers for recovered products, it is clear that the reduction in manufacturing costs brought about by the acquisition of core technology is greater than the first two expenses.

Proof of propositions under different models are shown in Appendix 2.

P2. In markets where bundled products exist (BR), there are $p_{12}^{BR} < p_2^{BR} + p_1^{BR}$, $p_{12}^{BR} > p_i^{BR}$ and $2q_{12}^{BR} < q_2^{BR} + q_1^{BR}$, $q_{12}^{BR} < q_i^{BR}$.

Proposition 2 states that the price of the bundled product is less than the sum of the prices of the individual products, which is apparently also in line with consumer psychological expectations and market mechanisms, because the bundled product has a scale advantage over the individual products and can increase revenue by promoting consumption, but in the bundled market, the sum of the number of products sold individually is greater than the sum of products sold by bundling, mainly because the bundled product consists of products that are substitutes for each other, and there is no complementary effect to drive linked sales. This is mainly because bundles consist of products that are substitutes for each other and do not have a complementary effect to drive linked sales. Therefore, in terms of the number of products, when consumer demand is low, more products are sold individually than in bundles. However, the three products brought by such bundles can provide more choices for different types of consumers and meet different consumer needs, so it is crucial for the retailer to adopt bundles to increase their revenue.

P3. When $2L_1^{(2)}L_{10} > L_{11}L_9^{(2)}$, we get $\Pi_1^{BR} > \Pi_1^{SR}$; on the contrary, when $2L_1^{(2)}L_{10} < L_{11}L_9^{(2)}$, we can obtain $\Pi_1^{BR} < \Pi_1^{SR}$.

P4. When $4L_{12}L_1^{(2)} > L_9^{(2)}L_{13}$, we get $\Pi_2^{BR} > \Pi_2^{SR}$; on the contrary, when $4L_{12}L_1^{(2)} < L_9^{(2)}L_{13}$, we can obtain $\Pi_2^{BR} < \Pi_2^{SR}$.

Propositions 3 and 4 illustrate that there is no single sales model that is always dominant, whether sold separately or bundled. For both the original manufacturer and the remanufacturer, although it is the retailer's choice whether to bundle or not, it also influences the original manufacturer upstream in the supply chain. When $2L_1^{(2)}L_{10} > L_{11}L_9^{(2)}$, the revenue generated by bundling with the remanufactured product is greater for the original manufacturer than by selling separately, as $p_{12}^{BR} < p_2^{BR} + p_1^{BR}$ and $2q_{12}^{BR} < q_2^{BR} + q_1^{BR}$ can be seen from Proposition 2, so when the bundled price advantage of the bundled product is greater than the volume advantage of selling separately, the bundled sale at that point generates a greater profit for the original manufacturer is greater. When $4L_{12}L_1^{(2)} < L_9^{(2)}L_{13}$, for the remanufacturer, the revenues generated through separate sales are greater than those from bundled sales, the presence of the bundle is clearly an infringement of the remanufacturer's profits and not as profitable as when sold separately, because the bundle appears in the market as a third product, competing for market share with the single product of the remanufactured product. This is because the bundle appears in the market as a third product, competing for market share with the single product of the remanufactured product and thus potentially reducing the remanufacturer's profits.

5. Numerical analysis

To verify the relevant proposition, on the basis of $d = 1$, $c_m = 4$, $c_r = 2$, $a = 0.5$, we discuss performance of each wholesale price, selling price and profit of each entity under the bundled versus separate sales model when $(\beta, \theta, \varphi, \gamma)$ in different situations, respectively. We let $(\beta, \theta, \varphi, \gamma) = (0.8, 0.35, 8\%, 0.6)$, $(\beta, \theta, \varphi, \gamma) = (1, 0.45, 10\%, 0.8)$, $(\beta, \theta, \varphi, \gamma) = (1.8, 0.85, 18\%, 1.2)$. The specific results are shown in Tables 3 and 4.

β	θ	φ	γ	w_1	w_2	p_1	p_2	p_{12}	π_1	π_2	π_r
0.8	0.35	8	0.6	3.91	3.86	6.96	6.93	8.89	0.45	0.10	4.64
1	0.45	10	0.8	3.75	3.54	6.88	6.77	8.65	0.41	0.14	5.20
1.8	0.85	18	1.2	3.68	3.25	6.84	6.63	8.47	1.44	0.71	7.54

Source(s): Table created by authors

Table 3. Prices and profits under the bundling model

β	θ	φ	γ	w_1	w_2	p_1	p_2	π_1	π_2	π_r
0.8	0.35	8	0.6	2.88	2.67	2.15	2.05	0.02	0.36	0.64
1	0.45	10	0.8	2.67	1.92	1.92	1.55	0.85	0.17	0.61
1.8	0.85	18	1.2	3.12	2.45	2.09	1.75	0.69	0.26	1.57

Source(s): Table created by authors

Table 4. Prices and profits in individual sales

As shown in Table 2, when the retailer bundles new and remanufactured products and $(\beta, \theta, \varphi, \gamma) = (0.8, 0.35, 8\%, 0.6)$ turns into $(\beta, \theta, \varphi, \gamma) = (1.8, 0.85, 18\%, 1.2)$, the wholesale price and sales price of the new and remanufactured products gradually decrease as the magnitude of each parameter significantly, while the profits of the original manufacturer and retailer increase, and the profits of the remanufacturer decrease and then increase. It is very clear that despite the reduction in selling price, this leads to an increase in sales volume and therefore profits for the original manufacturer and retailer, where the sales volume advantage from the reduction in price is much greater than the price disadvantage of selling at a lower price. In addition, for remanufacturer' profits, it is clear that $0.45 > 0.41, 0.41 < 0.44$, when $(\beta, \theta, \varphi, \gamma)$ rises gradually, so that remanufacturer profits will be lowest when $(\beta, \theta, \varphi, \gamma)$ is in the middle. For the remanufacturer, the higher margins require reasonable avoidance of $(\beta, \theta, \varphi, \gamma)$ and strict attention to their size trends. For the retailer, the bundling of similar products can lead to increased profits, proving that it is profitable for the retailer to bundle substitutes.

As can be seen through Table 3, when selling products separately, when $(\beta, \theta, \varphi, \gamma) = (0.8, 0.35, 8\%, 0.6)$ turns into $(\beta, \theta, \varphi, \gamma) = (1.8, 0.85, 18\%, 1.2)$, with the increase of various parameters, the wholesale price and sales price of new products and remanufactured products first decrease and then increase, while the profit of the original manufacturer increases first and then decreases, then the manufacturer's profit first decreases and then increases, and the retailer's profit decreases, indicating that the separate sales model has damage to the profit of retailers.

Combining Tables 2 and 3, it can be seen that whatever $(\beta, \theta, \varphi, \gamma)$ is at, there is a case that $w_1 > w_2$; $p_1 > p_2$; $p_1 + p_2 > p_{12}$ in Tables 2 and 3, validating part of the previous proposition. Furthermore, with a significant increase in $(\beta, \theta, \varphi, \gamma)$, for the retailer, there exists a situation where the retailer makes more profit through bundled sales than through separate sales under the two different sales models. It is highly likely that the retailer will choose to bundle the remanufactured product with the new products to achieve higher profits.

To further validate the proposition and to supplement the simulation, the following section explores the impact of changes in relevant parameters on the profitability of the original manufacturer, the remanufacturer and the retailer, such as the demand price sensitivity factor, the product substitution factor, the ratio of the unit patent licensing fee to the wholesale price of the remanufactured product, and the price demand sensitivity factor of the used product, etc. Based on the relevant literature and assumptions, the relevant parameters are set as follows: $d = 1$, $c_m = 400$, $c_r = 200$, $a = 300$.

5.1 Sensitivity analysis of β

This subsection first examines the impact of the price sensitivity coefficient on the profitability of the original manufacturer, the remanufacturer and the retailer, based on a comparison of the separate and bundled sales models, with the following parameters: $\theta = 0.5$, $\varphi = 5\%$, $\gamma = 0.5$, and when $\beta \in [0, 1]$, we can obtain Figures 2–4.

As can be seen from Figures 2–4, in the bundling and separate sales models, when $0.15 < \beta < 0.25$ the impact of the demand price sensitivity coefficient on the profit of the original manufacturer, remanufacturer and retailer is relatively obvious. When $0.15 < \beta < 0.20$, the profit of the original manufacturer and remanufacturer decrease with the increase of the demand price sensitivity coefficient, and the retailer increases; meanwhile, when $0.20 < \beta < 0.25$, the profit of the original manufacturer and remanufacturer increases, while the retailer decreases.

When the demand price sensitivity coefficient is small $0.15 < \beta < 0.25$, that is, the consumer's sensitivity coefficient to demand price is weak, for the two manufacturers,

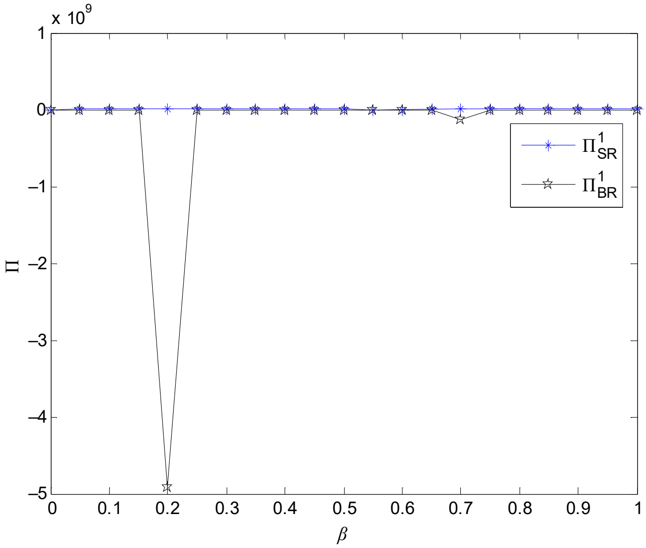

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Figure 2.
Impact of β on the profit of OEM

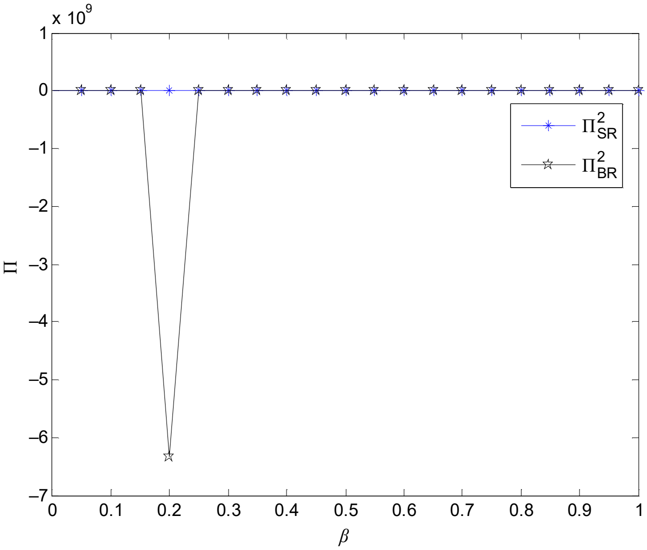

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Figure 3.
Impact of β on the profit of REM

the revenue from separate sales is much greater than bundled sales, while for the retailer, the revenue from bundling sales is much greater than separate sales. At this time, the product has just entered the market, the price sensitivity coefficient of the demand is small, and the retailer is less sensitive to the price. Even if the original manufacturer and remanufacturer significantly reduce the wholesale price, the retailer will not increase the purchase volume

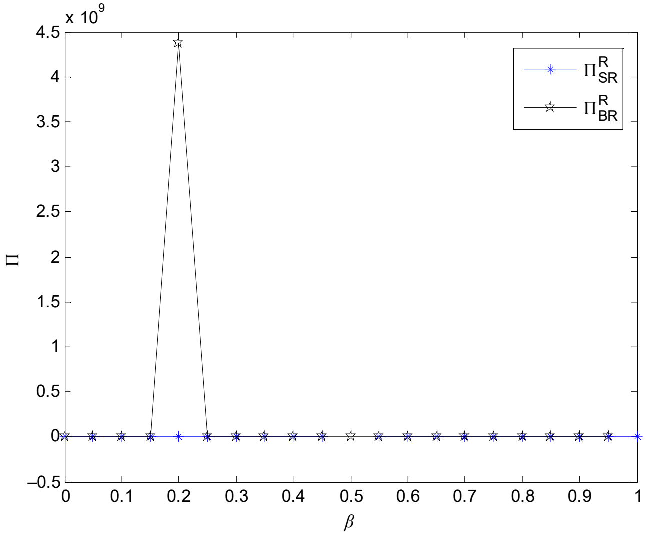

Figure 4.
Impact of β on the profit of the retailer

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and bundle the original product and remanufactured product for sales. Due to the substantial price reduction of the original manufacturer and remanufacturer, coupled with the small demand, both lost a large part of their profits, so the profit of the original manufacturer and remanufacturer under the bundling strategy are negative. The retailer has a certain safety inventory, their sales activities are not affected, they still bundle the original product and remanufactured product for sales, showing a positive profit.

While $0 < \beta < 0.15$ and $0.25 < \beta < 1$, there is no significant impact on the profit's of the original manufacturer, remanufacturer and retailer, no matter how β changes, there is no difference between separate sales or bundled sales. Within the scope of $0.15 < \beta < 0.25$ the demand price sensitive coefficient is low, the impact on demand compared to the substitution coefficient is weak, so for the retailer, bundling is better than separate sales, but the bundling strategy is not adopted by the original manufacturer and remanufacturer, for both profits have a lot of damage. When choosing separate sales or bundled sales, sometimes it is impossible to take into account the interests of the original manufacturer, remanufacturer and retailer at the same time, so it is necessary to avoid the price sensitivity coefficient of demand in $0.15 < \beta < 0.25$, so that achieve a win-win situation.

5.2 Sensitivity analysis of θ

This sub-section examines the impact of the degree of substitution between products on the profit levels of the original manufacturer, the remanufacturer and the retailer, comparing the trends based on separate and bundled sales models. The following assumptions are made for the relevant parameters: $\beta = 1.2, \varphi = 5\%, \gamma = 0.5, \theta \in [0, 1]$, and when substituting the data, we can get the following graphs.

As can be seen from Figures 5–7, in the bundling and separate sales models, when $0.70 < \theta < 1$ the impact of the degree of substitution between products on the profit of the original manufacturer, remanufacturer and retailer is relatively obvious. When $0.70 < \theta < 0.85$, the profit of the original manufacturer and remanufacturer increase with the increase of the degree of substitution between products, and the retailer decreases;

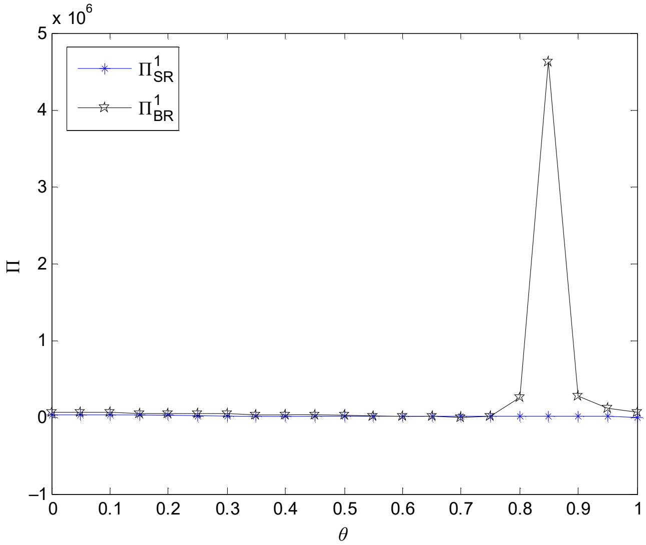

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Figure 5.
Impact of θ on the profit of OEM

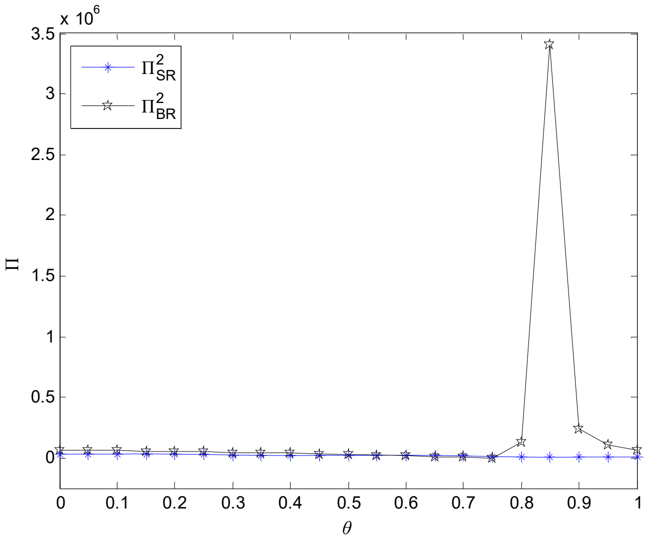

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Figure 6.
Impact of θ on the profit of REM

meanwhile, when $0.85 < \theta < 1$, the profit of the original manufacturer and remanufacturer decrease, while the retailer increases, which is consistent with the reality. Due to the increase of product substitution degree, product homogenization is serious, and the difference between products becomes smaller, leading to the fluctuation of demand, so that the profit fluctuation is large. The same is, for the two manufacturers, when $0.70 < \theta < 0.85$, with the

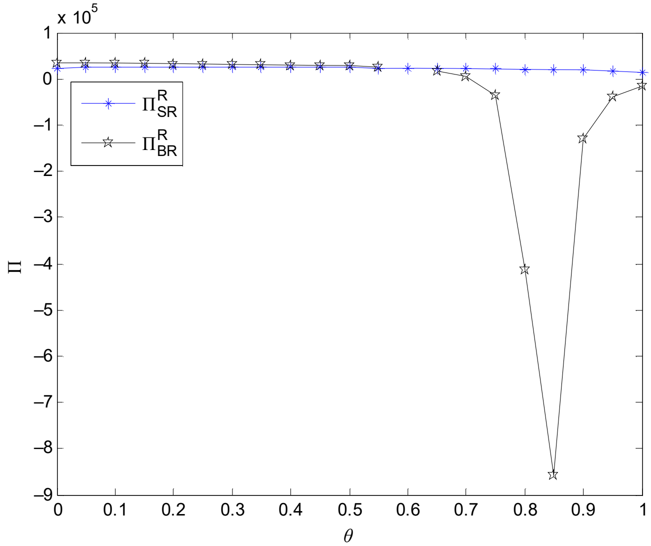

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Figure 7.
Impact of θ on the profit of the retailer

increase of θ, the bundling sales is always yield more than separate sales, the original manufacturer's and manufacturer's advantage strategy are bundling sales, within this scope by the bundling sales revenue is always greater than separate sales, compared to separate sales, the change of product substitution coefficient is more sensitive to bundling, so in the bundling, will further consider the influence of product substitution coefficient. For the retailer, its advantage strategy is separate sales, this is because in separate sales, although there is alternative between I_1 and I_2, but each other is different product, can be decided by different consumers to choose and buy rather than bundling, with the increase of the product substitution coefficient, the retailer's bundling profit is always lower than separate sales, the alternative lead to reducing the demand. When choosing separate sales or bundled sales, sometimes the interests of the original manufacturer, remanufacturer and retailer cannot be considered at the same time, so it is necessary to avoid the product substitution coefficient within the range $0.70 < \theta < 1$, so that can achieve a win-win situation.

5.3 Sensitivity analysis of φ

This section examines the impact of changes in the ratio of the unit patent licensing fee to the wholesale price of the remanufactured product on the profit levels of the original manufacturer, remanufacturer and retailer, based on a comparison of separate sales and bundled sales models. The following assumptions are made about the relevant parameters: $\beta = 1.2$, $\theta = 0.5$, $\gamma = 0.5$, and when $\varphi \in [0, 1]$, we can get the following graphs.

As can be seen in Figures 8 and 9, the higher the ratio of the unit patent license fee to the wholesale price of the remanufactured product, the lower the profit of the original manufacturer, and the higher the profit of the manufacturer and retailer. When $0 < \varphi < 0.5$, the bundling sales is better for the retailer, otherwise, when $0.5 < \varphi < 1$, the separate sales would bring more profits for the retailer. Compared with bundling sales, with the change of φ, the change of profits of the original manufacturer, remanufacturer and retailer are more sensitive, but for the original manufacturer and remanufacturer, no matter how the proportion of unit patent licensing fee and the wholesale price change, the bundling strategy's

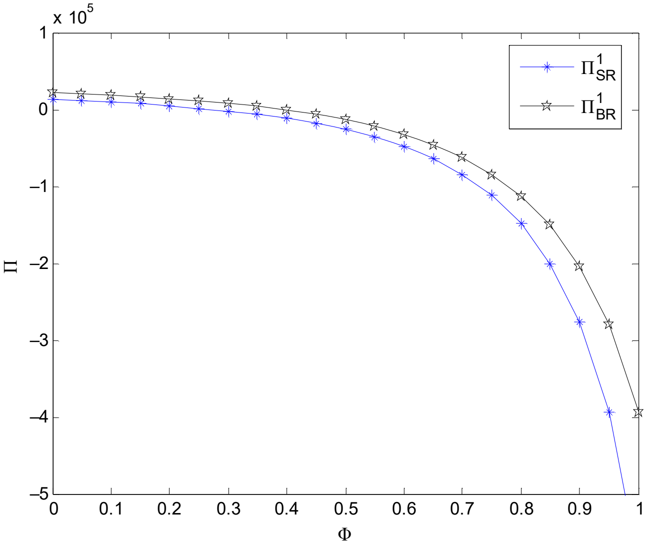

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Figure 8.
Impact of φ on the profit of OEM

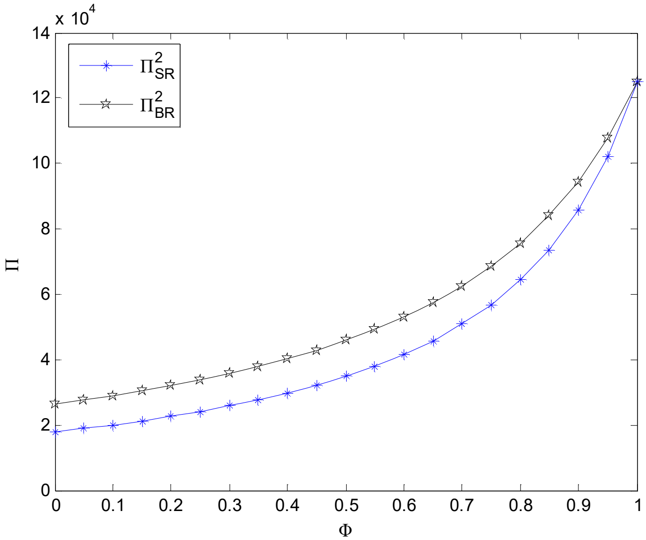

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Figure 9.
Impact of φ on the profit of REM

revenue is always greater than the separate sales. From Figure 10, compared with the original manufacturer and remanufacturer on the bundling sales, the retailer needs to according to the unit patent license fee and the proportion of remanufacturing product wholesale price to choose separate or bundling sales to get greater advantage, therefore, the size of the unit patent license fee and wholesale price is not only bargaining between two manufacturers, in

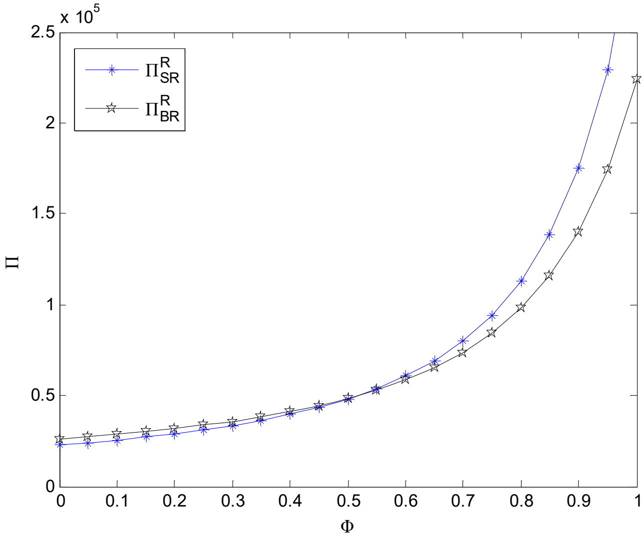

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Figure 10. Impact of φ on the profit of the retailer

the closed loop supply chain also about the retailer's profit. Therefore, when adopting the bundling strategy, when the decision-maker decides the unit patent license fee and the wholesale price, should consider the interests of the retailer and control the proportion as far as possible, so that the original manufacturer, remanufacturer and retailer can be profitable under the bundling.

5.4 Sensitivity analysis of γ

This section examines the impact of changes in the recycling price sensitivity coefficient on the profit levels of the original manufacturer, the remanufacturer and the retailer, comparing the trends based on separate sales and bundled sales models. The following assumptions are made about the relevant parameters: $\beta = 1.2$, $\theta = 0.5$, $\varphi = 5\%$, $\gamma \in [0, 1]$.

Figures 11–13 show that whether sold separately or bundling, as the recycling price sensitivity coefficient increases, both the remanufacturer's and retailer's profit are all decline, while the original manufacturer shows an increasing trend. This is because the greater the recycling price sensitive coefficient, the more sensitive the consumer to the recycling price, the higher the recycling price, the more the consumer willing to recycle waste products for second-hand processing. Compared with higher recycling prices, the remanufacturer' profits are reduced, while the original manufacturer' profits increase. In addition, the increase of the recycling price sensitivity coefficient will also lead to the remanufacturer to raise the wholesale prices, the retailer's profit will be relatively reduced. Similarly, for both manufacturers and retailer, with the increase of the recycling price sensitivity coefficient, for the original manufacturer, remanufacturer and retailer, bundling sales is always better than the separate sales, and the optimal strategy for all the three is bundling. In addition, for the remanufacturer and retailer, under the two different sales strategies, the profit difference between the two will decrease with the increase of the recycling price sensitivity coefficient, indicating that bundled sales are more sensitive than separate sales. Therefore, when the manufacturer, remanufacturer and retailer choosing bundled sales, the original manufacturer

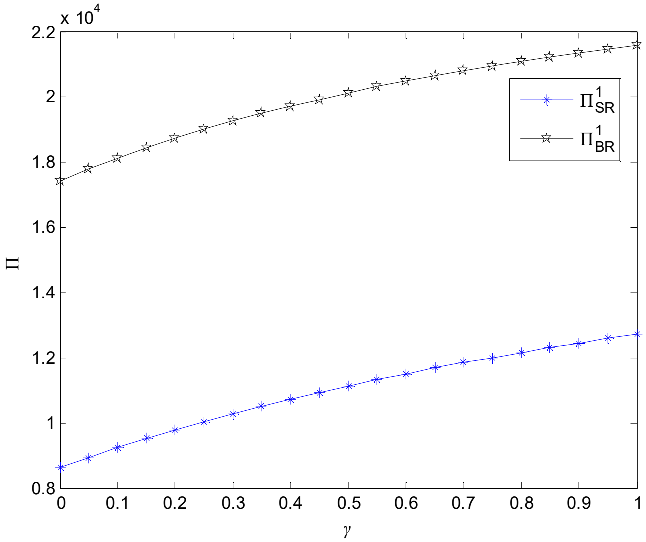

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Figure 11.
Impact of γ on the profit of OEM

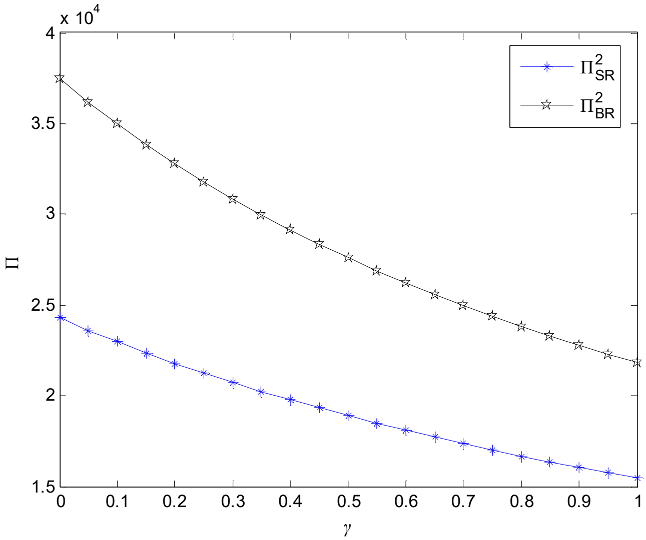

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Figure 12.
Impact of γ on the profit of REM

should take into account the behavior of recycling waste products, consider the impact of the recycling price of waste products on consumers' demand, and make comprehensive decisions to make all three benefits.

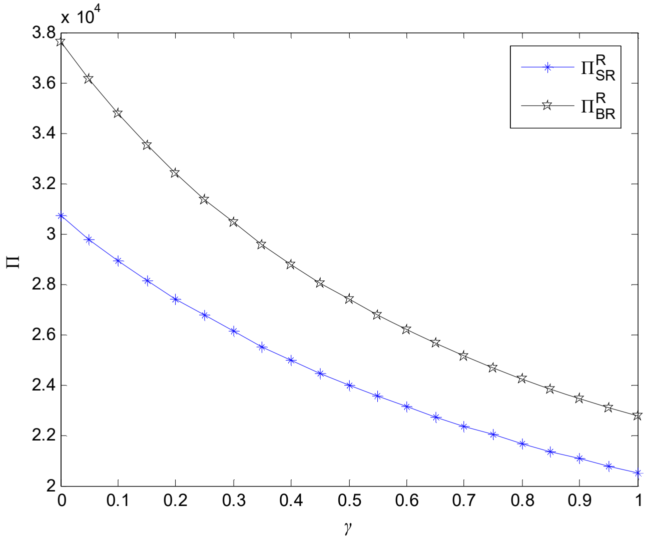

Figure 13.
Impact of γ on the profit of the retailer

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6. Concluding remarks

Based on the existence of mutual substitution of similar products, this paper investigates a closed-loop supply chain system consisting of an original manufacturer, a remanufacturer, and a retailer, in which two bundling models are formed by these members. We discuss the recycling and sales of new and remanufactured products in the closed-loop supply chain, providing pricing decisions and theoretical guidance for the different members. More importantly, we provide a comparative analysis of equilibrium decisions and profit distribution in separate sales and mixed bundles. Therefore, we summarize the results, and this study draws the following conclusions based on the two bundling models.

(1) In the separate sales market, the sales price, wholesale price and sales volume of new products are always ahead of the remanufacturing products. For the remanufacturer, it is at a competitive disadvantage compared to the original manufacturer, but it is still the primary means of competition to increase sales volume and open the market by obtaining technological advantages and saving manufacturing costs through patent licensing.

(2) In the bundled sales market, although the number of bundled products sold is smaller than the sum of individual products, for the remanufacturer, the sales of bundled products are smaller than the sales of remanufactured products through the bundled sales strategy, and for the retailer, diversified marketing methods are the key to increase revenue.

(3) For the retailer, when the demand price sensitivity coefficient and the proportion of unit patent licensing fee and remanufacturing products' wholesale price are in the reasonable range, taking bundling sales is always better than separate sales, therefore, the retailer needs to strictly examine the upstream supplier interaction behavior, not just focus on the wholesale price. The benefits to the remanufacturer of bundling sales are greater when compared to separate sales when $4L_{12}L_1^{(2)} > L_9^{(2)}L_{13}$.

(4) For the original manufacturer, bundling sales is superior to separate sales only when the product substitution coefficient is relatively high. In most cases, the retailer's bundling behavior can greatly improve the original manufacturer and remanufacturer's earnings, the occurrence of bundling behavior depends on long-term cooperation with suppliers, and the behavior is also the retailer's spontaneous behavior according to the market demand, and for the remanufacturer, improve the quality of products and service is always the main way to ensure its core revenue.

Bundling affects the wholesale pricing decisions of the original manufacturer and remanufacturer in an unexpected way. When the retailer implements bundling, 2 manufacturers may reduce wholesale prices. Our analysis shows that when considering retail order uncertainty, while 2 manufacturers always benefit from expected retail bundling, bundling may be worse depending on the realization of uncertain market sizes. This explains why retail bundling has been a concern for some major manufacturers who do not prohibit early bundling. When demand exceeds supply, the retailer resorts to bundling to deal with backlogs. Our analysis shows that downstream retailers' bundling choice has a number of implications in the supply chain. First, bundling benefits the retailer by mitigating the adverse consequences of demand uncertainty as a means of selling season. In particular, when demand is greater than supply, bundling allows the retailer to extract additional surplus from the bundle mix compared to no bundling. When demand is less than supply, bundling allows the retailer to clear some excess inventory of new and remanufactured products. Second, compared to an unbundled benchmark, bundling can change ordering decisions by (1) the retailer should order more products, and (2) manufacturers benefit from downstream bundling by raising wholesale prices or increasing order quantities (or both). The retailer benefits from bundling choice by lowering wholesale prices or reducing the adverse effects of uncertainty, but the retailer may be harmed if the benefits of bundling to the retailer do not compensate for the losses retailer incur as a result of manufacturer wholesale price exploitation. Therefore, the retailer should think carefully before deciding to bundle products when faced with a mismatch between supply and demand, as bundling may reduce profits.

Although this paper explores the different impacts of the retailer' bundling behavior on each member of the closed-loop supply chain under patent protection, there are still some limitations, which lead to future research directions. In the closed-loop supply chain, there are powerful retailers, manufacturers, etc. In this paper, we consider that the original manufacturer and the remanufacturer have equal power together as the leader of the game, and there are inevitably limitations. Therefore, we can consider different members as future supply chain leaders and then compare them with each other to provide reference for related companies. In addition, the study is based on two types of bundled sales behavior of alternatives. There are bundles involving other products in reality, such as pure bundles, bundles of three and more products, etc (Chen *et al.*, 2020; Giri *et al.*, 2017). Although the remanufacturing license model is widely available in the industry, this paper adopts the fixed license fee model for enterprises, so the article can add the yield-centered unit license fee model for comparison, and further improve the application of remanufacturing license for supply chain enterprises through the comparison of fixed fee and unit fee (Kovach *et al.*, 2018; Cheng *et al.*, 2022).

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Appendix 1

The solving process of equilibrium decisions in different models

In order to facilitate the calculation and simplify the results, we make some parameters replacement in the calculation process, and let

$$L_1 = 2\beta^3\gamma - 4\beta^2(-1+\varphi) - \beta\gamma\theta^2(1+\varphi) + \theta^2\left(\varphi^2 - 1\right),$$

$$L_2 = 1 - \beta\gamma - \gamma\theta - \varphi,$$

$$L_3 = 1 + 2\beta\gamma - 2\gamma\theta - \varphi,$$

$$\begin{aligned} L_4 = {} & 4\beta^3\gamma(-7+\varphi) + 4\beta\theta((\varphi-1)(3\varphi-13) + 3\gamma\theta(5\varphi-3)) \\ & -2\beta^2((\varphi-15)(\varphi-1) + 2\gamma\theta(-17+7\varphi)) \\ & -2\theta^2((\varphi-1)(9\varphi-7) + 2\gamma(\theta+9\theta\varphi)) \end{aligned},$$

$$L_5 = -2\beta^3\gamma(-7+\varphi),$$

$$L_6 = \beta^2\left((\varphi-1)(\varphi-15) + \gamma\theta(-17+7\varphi)\right),$$

$$L_7 = 2\beta\theta((\varphi-1)(3\varphi-13) + 3\gamma\theta(5\varphi-3)),$$

$$L_8 = \theta^2((\varphi-1)(9\varphi-7) + \gamma(\theta+9\theta\varphi)),$$

$$L_4 = 2(L_5 - L_6 + L_7 - L_8).$$

The solving process of Model SR:

It is easy to obtain the Hessian matrix of the retailer's profit function: $H_R^{SR} = \begin{bmatrix} -2\beta & 2\theta \\ 2\theta & -2\beta \end{bmatrix}$

For $|H_R^1| = -2\beta < 0$, if $|H_R^2| = 4(\beta^2 - \theta^2) > 0$, that is to say $\beta > \theta$ is satisfied. So the Hessian matrix is negatively definite, and taking the first-order derivative of Π_R^{SR} with respect to p_1^{SR} and p_2^{SR}, we can obtain the unique optimal solution. Then we can derive the following equilibrium solution by using the reverse recursion method.

$$\begin{cases} p_1^{SR} = \dfrac{1}{2}\left(\dfrac{d}{\beta-\theta} + w_1^{SR}\right) \\ p_2^{SR} = \dfrac{1}{2}\left(\dfrac{d}{\beta-\theta} + w_2^{SR}\right) \end{cases} \tag{A1}$$

Substituting (A1) into (5) and (6), and taking the first-order derivative of Π_1^{SR} and Π_2^{SR} with respect to w_1^{SR}, w_2^{SR}, we can obtain the unique optimal solution of Π_1^{SR} and Π_2^{SR} with respect to w_1^{SR} and w_2^{SR}.

Then, we can get the following equilibrium solution by using the reverse recursion method.

$$w_1^{SR} = \frac{\left(\alpha\beta\theta(1+\varphi) + d\left(\theta + \beta(2+\gamma(\beta+\theta)\right) + \beta(-2+\gamma\theta)\varphi - \theta\varphi^2\right) + \beta^2(2+\beta\gamma-2\varphi)c_m + \beta\theta(1+\varphi)c_r}{L_1} \tag{A2}$$

$$w_2^{SR} = \frac{2a\beta^2 + d(2\beta+\theta)(1+\beta\gamma-\varphi) + \beta\theta(1+\beta\gamma-\varphi)c_m + 2\beta^2 c_r}{L_1} \tag{A3}$$

$$p_1^{SR} = \frac{1}{2}\left(\frac{d}{\beta-\theta} + \frac{\left(\alpha\beta\theta(1+\varphi) + d\left(\theta+\beta(2+\gamma(\beta+\theta)\right) + \beta(-2+\gamma\theta)\varphi - \theta\varphi^2\right) + \beta^2(2+\beta\gamma-2\varphi)c_m + \beta\theta(1+\varphi)c_r}{L_1}\right) \tag{A4}$$

$$p_2^{SR} = \frac{1}{2}\left(\frac{d}{\beta-\theta} + \frac{2a\beta^2 + d(2\beta+\theta)(1+\beta\gamma-\varphi) + \beta\theta(1+\beta\gamma-\varphi)c_m + 2\beta^2 c_r}{L_1}\right) \tag{A5}$$

Therefore, the optimal profits of the alliance and the retailer are further obtained as follows:

$$\Pi_R^{SR} = \left(p_1^{SR} - w_1^{SR}\right)\left(d - \beta p_1^{SR} + \theta p_2^{SR}\right) + \left(p_2^{SR} - w_2^{SR}\right)\left(d - \beta p_2^{SR} + \theta p_1^{SR}\right) \tag{A6}$$

$$\Pi_1^{SR} = \left(w_1^{SR} - c_m\right)\left(d - \beta p_1^{SR} + \theta p_2^{SR}\right) + \varphi w_2^{SR}\left(d - \beta p_2^{SR} + \theta p_1^{SR}\right) \tag{A7}$$

$$\Pi_2^{SR} = \left(w_2^{SR} - \varphi w_2^{SR} - c_r - a - \gamma\left(d - \beta p_2^{SR} + \theta p_1^{SR}\right)\right)\left(d - \beta p_2^{SR} + \theta p_1^{SR}\right) \tag{A8}$$

Proof of Lemma 1:

From the above analysis it follows that $w_1^{SR}, w_2^{SR}, p_1^{SR}, p_2^{SR}$, $p_r = a - \gamma * q_2$, $q_1 = d - p_1\beta + \theta p_2$, $q_2 = d - p_2\beta + \theta p_1$.Therefore $p_r^{SR}, q_1^{SR}, q_2^{SR}$ are the above solutions.

The solving process of Model BR:

It is easy to obtain the Hessian matrix of the retailer's profit function:

$$H_R^{BR} = \begin{bmatrix} -2\beta & 2\theta & 2\theta \\ 2\theta & -2\beta & 2\theta \\ 2\theta & 2\theta & -2\beta \end{bmatrix}$$

For $|H_R^1| = -2\beta < 0$, if $|H_R^2| = 4(\beta^2 - \theta^2) > 0$, that is to say $\beta > \theta$ is satisfied. So the Hessian matrix is negatively definite, and taking the first-order derivative of Π_R^{BR} with respect to p_1^{BR}, p_2^{BR} and p_{12}^{BR}, we can obtain the unique optimal solution. So we joint system of equations:

$$\begin{cases} \dfrac{\partial \Pi_R^{BR}}{\partial p_1^{BR}} = 0 \\ \dfrac{\partial \Pi_R^{BR}}{\partial p_2^{BR}} = 0 \\ \dfrac{\partial \Pi_R^{BR}}{\partial p_{12}^{BR}} = 0 \end{cases} \tag{A9}$$

Then we can derive the following equilibrium solution by using the reverse recursion method.

$$\begin{cases} p_1^{BR} = \dfrac{1}{2}\left(\dfrac{d}{\beta - 2\theta} + w_1^{BR}\right) \\ p_2^{BR} = \dfrac{1}{2}\left(\dfrac{d}{\beta - 2\theta} + w_2^{BR}\right) \\ p_{12}^{BR} = \dfrac{1}{2}\left(\dfrac{d}{\beta - 2\theta} + w_1^{BR} + w_2^{BR}\right) \end{cases} \tag{A10}$$

Substituting (A10) into(11) and (12), and taking the first-order derivative of Π_1^{BR} and Π_2^{BR} with respect to w_1^{BR}, w_2^{BR}, we can obtain the unique optimal solution of Π_1^{BR} and Π_2^{BR} with respect to w_1^{SR} and w_2^{SR}.

Then, we can get the following equilibrium solution by using the reverse recursion method.

$$w_1^{BR} = \frac{\frac{4d}{\beta-\theta} + 4c_m - \frac{(\beta-3\theta)(1+\varphi)(a(\beta-\theta)+dL_3+(\beta-\theta)c_r)}{(\beta-\theta)^2 L_2}}{8\left(1 - \frac{(\beta-3\theta)^2 L_3(1+\varphi)}{16(\beta-\theta)^2 L_2}\right)} \tag{A11}$$

$$w_2^{BR} = \frac{-8a(\beta-\theta)^2 - 2d(3\beta-\theta)L_3 + 2(\beta-3\theta)(\beta-\theta)L_3 c_m - 8(\beta-\theta)^2 c_r}{\begin{array}{c}2\beta^3\gamma(-7+\varphi) + 2\beta\theta((-1+\varphi)(-13+3\varphi) + 3\gamma\theta(-3+5\varphi)) - \beta^2\big((-15+\varphi)(-1+\varphi)+ \\ 2\gamma\theta(-17+7\varphi)\big) - \theta^2((\varphi-1)(9\varphi-7) + 2\gamma(\theta+9\theta\varphi))\end{array}} \tag{A12}$$

$$p_1^{BR} = \frac{1}{2}\left(\frac{d}{\beta-2\theta} + \frac{\frac{4d}{\beta-\theta} + 4c_m - \frac{(\beta-3\theta)(1+\varphi)(a(\beta-\theta)+dL_3+(\beta-\theta)c_r)}{(\beta-\theta)^2 L_2}}{8\left(1 - \frac{(\beta-3\theta)^2 L_3(1+\varphi)}{16(\beta-\theta)^2 L_2}\right)}\right) \tag{A13}$$

$$p_2^{BR} = \frac{1}{2}\left(\frac{d}{\beta-2\theta} + \frac{-8a(\beta-\theta)^2 - 2d(3\beta-\theta)L_3 + 2(\beta-3\theta)(\beta-\theta)L_3 c_m - 8(\beta-\theta)^2 c_r}{\begin{array}{c}2\beta^3\gamma(-7+\varphi) + 2\beta\theta((-1+\varphi)(-13+3\varphi) + 3\gamma\theta(-3+5\varphi)) - \beta^2\big((\varphi-1) \\ (\varphi-15) + 2\gamma\theta(-17+7\varphi)\big) - \theta^2((\varphi-1)(9\varphi-7) + 2\gamma(\theta+9\theta\varphi)))\end{array}}\right) \tag{A14}$$

$$p_{12}^{BR}=\frac{\begin{array}{c}2a(\beta-2\theta)(\beta-\theta)(\theta+\beta(\varphi-3)-3\theta\varphi+d(\beta^2(2\gamma\theta(41-19\varphi)-3(\varphi-9)(\varphi-1)+\\ 6\beta^3\gamma(-5+\varphi)+2\beta\theta((-1+\varphi)(-27+8\varphi)+\gamma\theta(-25+37\varphi))-\theta^2(3(\varphi-1)(7\varphi-5)+\\ 2\gamma(\theta+21\theta\varphi)))+2(\beta-2\theta)(\beta-\theta)((-2\beta^2\gamma+\theta(1+2\gamma\theta-\varphi)+3\beta(-1+\varphi))c_m+\\ (\theta+\beta(\varphi-3)-3\theta\varphi)c_r)\end{array}}{\begin{array}{c}2(\beta-2\theta)(2\beta^3\gamma(-7+\varphi)+2\beta\theta((-1+\varphi)(-13+3\varphi)+3\gamma\theta(-3+5\varphi))-\\ \beta^2((-15+\varphi)(-1+\varphi)+2\gamma\theta(-17+7\varphi))-\theta^2((\varphi-1)(9\varphi-7)+2\gamma(\theta+9\theta\varphi)))\end{array}} \tag{A15}$$

Therefore, the optimal profits of the alliance and the retailer are further obtained as follows:

$$\begin{aligned}\Pi_R^{BR} &= (p_{12}^{BR}-w_1^{BR}-w_2^{BR})(d-\beta p_{12}^{BR}+\theta(p_2^{BR}+p_1^{BR}))+(p_2^{BR}-w_2^{BR})(d-\beta p_2^{SR}+\theta p_1^{SR})\\ &\quad+(p_1^{BR}-w_1^{BR})(d-\beta p_1^{SR}+\theta p_2^{SR})\end{aligned} \tag{A16}$$

$$\begin{aligned}\Pi_1^{BR} &= (w_1^{BR}-c_m)((d-\beta p_1^{BR}+\theta p_2^{BR})+(d-\beta p_{12}^{BR}+\theta(p_2^{BR}+p_1^{BR})))\\ &\quad+\varphi w_2^{BR}((d-\beta p_2^{BR}+\theta p_1^{BR})+(d-\beta p_{12}^{BR}+\theta(p_2^{BR}+p_1^{BR})))\end{aligned} \tag{A17}$$

$$\begin{aligned}\Pi_2^{BR} &= (w_2^{BR}-\varphi w_2^{BR}-c_r-(a+\gamma(d-\beta p_2^{BR}+\theta p_1^{BR})))((d-\beta p_2^{BR}+\theta p_1^{BR})\\ &\quad+(d-\beta p_{12}^{BR}+\theta(p_2^{BR}+p_1^{BR})))\end{aligned} \tag{A18}$$

Proof of Lemma 2:

From the above analysis we can get $w_1^{BR}, w_2^{BR}, p_1^{BR}, p_2^{BR}, p_{12}^{BR}$, since $p_r = a-\gamma * q_2, q_1 = d-p_1\beta+\theta p_2$, $q_2 = d-p_2\beta+\theta p_1$, and $q_{12} = d-p_{12}\beta+\theta(p_1+p_2)$, therefore $p_r^{BR}, q_1^{BR}, q_2^{BR}, q_{12}^{BR}$ are the above solutions.

Appendix 2

Proof of propositions under different models

Proof of Proposition 1:

Since

$$w_1^{SR}-w_2^{SR}=\frac{\begin{array}{c}(a+c_r)\beta(-2\beta+\theta+\theta\varphi)+d(-\beta^2\gamma+\beta\gamma\theta\varphi-\theta(-1+\varphi)\varphi)+\beta(\beta^2\gamma+\theta(-1+\varphi)-\\ \beta(-2+\gamma\theta+2\varphi))c_m\end{array}}{L_1},$$

$1>\beta>\gamma>\theta>0$ and $1>\varphi>0$, therefore, this paper gets $L_1<0, w_1^{SR}-w_2^{SR}>0, w_1^{SR}>w_2^{SR}$. Furthermore, it is clear that $p_1^{SR}-p_2^{SR}=\frac{1}{2}(w_1^{SR}-w_2^{SR})$, so $p_1^{SR}>p_2^{SR}$; in addition, because of $q_i = d-p_i*\beta+\theta*p_j, q_1^{SR}-q_2^{SR}=(\beta+\theta)(p_2^{SR}-p_1^{SR})$, this paper obtains $q_1^{SR}>q_2^{SR}$, and proposition 1 is proved.

Proof of Proposition 2:

From the above model solution it follows that $p_{12}^{BR}=\frac{1}{2}\left(\frac{d}{\beta-2\theta}+w_1^{BR}+w_2^{BR}\right)p_1^{BR}=\frac{1}{2}\left(\frac{d}{\beta-2\theta}+w_1^{BR}\right)$ and $p_2^{BR}=\frac{1}{2}\left(\frac{d}{\beta-2\theta}+w_2^{BR}\right)$. Since $\beta-2\theta>0, p_{12}^{BR}-p_2^{BR}-p_1^{BR}=-\frac{d}{2(\beta-2\theta)}<0, p_{12}^{BR}-p_1^{BR}=\frac{1}{2}w_2^{BR}>0$ and $p_{12}^{BR}-p_2^{BR}=\frac{1}{2}w_1^{BR}>0$, then we can get $p_{12}^{BR}<p_2^{BR}+p_1^{BR}$, $p_{12}^{BR}>p_i^{BR}$, $q_{12}=d-p_{12}\beta+\theta(p_1+p_2)$, $q_i=d-p_i\beta+\theta(p_{12}+p_j)$, $q_{12}^{BR}-q_1^{BR}=(p_1^{BR}-p_{12}^{BR})(\beta+\theta)<0$, $q_{12}^{BR}-q_2^{BR}=(p_2^{BR}-p_{12}^{BR})(\beta+\theta)<0$, $2q_{12}^{BR}-q_2^{BR}-q_1^{BR}=(\beta+\theta)(p_1^{BR}+p_2^{BR}-2p_{12}^{BR})<0$. Therefore $q_{12}^{BR}<q_i^{BR}$ and $2q_{12}^{BR}<q_2^{BR}+q_1^{BR}$ are proved, and Proposition 2 is proved.

Proof of Proposition 3:

In model BR and model SR, it is calculated that $\Pi_1^{BR}-\Pi_1^{SR}=\frac{L_{10}}{L_9^{(2)}}-\frac{L_{11}}{2L_1^{(2)}}$ where we let

$$\begin{aligned}L_9 &= -2\beta^3\gamma(-7+\varphi)-2\beta\theta((\varphi-1)(3\varphi-13)+3\gamma\theta(5\varphi-3))+\beta^2((\varphi-15)(\varphi-1)\\ &\quad+2\gamma\theta(-17+7\varphi))+\theta^2((\varphi-1)(9\varphi-7)+2\gamma(\theta+9\theta\varphi))\end{aligned}$$

$$L_{10}=2(\beta-\theta)\varphi\Big(4a(\beta-\theta)^2+d(3\beta-\theta)(1+2\beta\gamma-2\gamma\theta-\varphi)-(\beta-3\theta)(\beta-\theta)(1+2\beta\gamma$$
$$-2\gamma\theta-\varphi)c_m+4(\beta-\theta)^2c_r\Big)+\Big(-2d(3\beta-\theta)(\varphi-1)+a\Big(-7\beta^2+10\beta\theta+\theta^2$$
$$+(\beta-3\theta)^2\varphi\Big)+2(\beta-3\theta)(\beta-\theta)(\varphi-1)c_m+\Big(-7\beta^2+10\beta\theta+\theta^2+(\beta-3\theta)^2\varphi\Big)c_r\Big)$$
$$+\Big(2a(\beta-3\theta)(\beta-\theta)^2(\varphi-1)+d\big(2\beta^3\gamma(2+\varphi)+\theta^2\big(2+4\gamma\theta-5\varphi+6\gamma\theta\varphi+3\varphi^2\big)$$
$$+2\beta\theta(-\gamma\theta(2+\varphi)+(\varphi-1)(4+\varphi))-\beta^2((\varphi-1)(6+\varphi)+2\gamma\theta(2+3\varphi))\big)$$
$$+(\beta-\theta)\big(\big(-6\beta^3\gamma+2\beta\theta(5+3\gamma\theta-5\varphi)-\theta^2(-1+10\gamma\theta+\varphi)+\beta^2(-7+10\gamma\theta+7\varphi)$$
$$\times\big)c_m+2(\beta-3\theta)(\beta-\theta)(\varphi-1)c_r\big)\Big)\big(\big(2\beta^3\gamma(-3+\varphi)-\beta^2\big(7-8\varphi+\varphi^2$$
$$+2\gamma\theta(-5+7\varphi)\big)+\theta^2(1+(8-9\varphi)\varphi-2\gamma\theta(5+9\varphi))+2\beta\theta\big(5-8\varphi+3\big(\varphi^2$$
$$+\gamma(\theta+5\theta\varphi)\big)\big)\big)c_m-2\big(a(\beta-3\theta)(\beta-\theta)(1+\varphi)+d\big(\theta+2\beta^2\gamma(\varphi-1)-\beta(3$$
$$+\varphi(-4+8\gamma\theta+\varphi))+\theta(\varphi(-4+3\varphi)+2\gamma(\theta+3\theta\varphi))\big)+(\beta-3\theta)(\beta-\theta)(1+\varphi)c_r\big)\big)$$

$$L_{11}=\big(-a\beta^2\theta(\varphi-1)+d\big(\beta^3\gamma-\beta^2(2+\gamma\theta)(\varphi-1)+\theta^2(\varphi-1)\varphi+\beta\theta(1+\varphi(-2-\gamma\theta+\varphi))\big)$$
$$+\beta\big(-\beta^3\gamma+\beta\gamma\theta^2+2\beta^2(\varphi-1)-\theta^2(\varphi-1)\big)c_m-\beta^2\theta(\varphi-1)c_r\big)\big(a\beta\theta(1+\varphi)+d\big(\theta$$
$$+\beta(2+\gamma(\beta+\theta))+\beta(-2+\gamma\theta)\varphi-\theta\varphi^2\big)+\big(-\beta^3\gamma+2\beta^2(\varphi-1)+\beta\gamma\theta^2(1+\varphi)$$
$$-\theta^2\big(\varphi^2-1\big)\big)c_m+\beta\theta(1+\varphi)c_r\big)-\beta\varphi\big(2a\beta^2+d(2\beta+\theta)(1+\beta\gamma-\varphi)$$
$$+\beta\theta(1+\beta\gamma-\varphi)c_m+2\beta^2c_r\big)\big(2a\beta^2+d(2\beta+\theta)(-1+\varphi)-a\theta^2(1+\varphi)+\beta\theta(\varphi-1)c_m$$
$$+\big(2\beta^2-\theta^2(1+\varphi)\big)c_r\big),$$

so we can obtain $\Pi_1^{BR}-\Pi_1^{SR}=\frac{L_{10}}{L_9^2}-\frac{L_{11}}{2L_1^{(2)}}=\frac{2L_1^{(2)}L_{10}-L_{11}L_9^{(2)}}{2L_1^{(2)}L_9^{(2)}}$. It is clear that when $2L_1^{(2)}L_{10}>L_{11}L_9^{(2)}$, we can get $\Pi_1^{BR}>\Pi_1^{SR}$. On the contrary, when $2L_1^{(2)}L_{10}<L_{11}L_9^{(2)}$, we can obtain $\Pi_1^{BR}<\Pi_1^{SR}$. So Proposition 3 is proved.

Proof of Proposition 4:

In Model BR and Model SR, it is calculated that $\Pi_2^{BR}-\Pi_2^{SR}=\frac{L_{12}}{L_9^{(2)}}-\frac{L_{13}}{4L_1^{(2)}}$ where we let

$$L_{12}=(\beta-\theta)L_2\Big(-2d(3\beta-\theta)(\varphi-1)+a\Big(-7\beta^2+10\beta\theta+\theta^2+(\beta-3\theta)^2\varphi\Big)+2(\beta-3\theta)(\beta-\theta)(\varphi-1)c_m$$
$$+\Big(-7\beta^2+10\beta\theta+\theta^2+(\beta-3\theta)^2\varphi\Big)c_r\Big)^2L_{13}$$
$$=\beta(2+\beta\gamma-2\phi)\big(2a\beta^2+d(2\beta+\theta)(\phi-1)-a\theta^2(1+\phi)+\beta\theta(\phi-1)c_m+\big(2\beta^2-\theta^2(1+\phi)\big)c_r\big)^2$$

so we can obtain $\Pi_2^{BR}-\Pi_2^{SR}=\frac{L_{12}}{L_9^{(2)}}-\frac{L_{13}}{4L_1^{(2)}}=\frac{4L_{12}L_1^{(2)}-L_9^{(2)}L_{13}}{4L_1^{(2)}L_9^{(2)}}$. It is clearly that when $4L_{12}L_1^{(2)}>L_9^{(2)}L_{13}$, we get $\Pi_2^{BR}>\Pi_2^{SR}$. On the contrary, when $4L_{12}L_1^{(2)}<L_9^{(2)}L_{13}$, we can obtain $\Pi_2^{BR}<\Pi_2^{SR}$. So Proposition 4 is proved.

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