Mathematics and STEM teacher development for flipped education

Robert Weinhandl, Zsolt Lavicza and Tony Houghton
School of Education, STEM Didactics, Johannes Kepler University Linz, Linz, Austria

Received 28 January 2020
Revised 28 February 2020
Accepted 28 February 2020

Abstract

Purpose – Flipped classroom approaches (FCA) are an educational innovation that could increase students' learning outcomes in, and their enjoyment of, mathematics or STEM education. To integrate FCA into education sustainably, professional teacher development (PTD) is a promising tool. The research aim is to explore which aspects should be considered when developing and implementing professional mathematics or STEM teacher development for flipped approaches.

Design/methodology/approach – A total of 20 expert interviews were conducted and analysed according to a synthesis of grounded theory approaches and qualitative interview study principles.

Findings – Evaluating the interview data indicates that the characteristics of different teacher types in PTD, learning activities in PTD and the DSE model derived in this study could be vital elements in professional mathematics or STEM teacher development for flipped approaches.

Originality/value – Evaluating the interview data indicates that the characteristics of different teacher types in PTD, learning activities in PTD and the DSE model derived in this study could be vital elements in professional mathematics or STEM teacher development for flipped approaches.

Keywords Flipped classroom education, Professional teacher development, Mathematics education, STEM education, Technology-enhanced learning environments, Student-driven learning environments

Paper type Research paper

1. Introduction

Creativity, critical thinking, adaptability or other 21st-century skills should form one pillar of education according to experts in the field of education (Fadel, 2008; Saavedra and Opfer, 2012). Also, the European Commission (2019) stresses that students should achieve key competences, including mathematical, digital, personal or learning-to-learn competences, during their educational careers. To facilitate integrating 21st-century skills or key competences into education, it could be beneficial to use new educational technologies and approaches in teaching and learning. One educational approach, which is also based on using educational technologies, that could facilitate incorporating 21st-century skills or key competences into teaching and learning, is education following flipped classroom approaches (FCA).

Even though there is no uniform definition of a flipped classroom (Wolff and Chan, 2016) many experts (e.g. Enfield, 2016; Galway *et al.*, 2014; Wasserman *et al.*, 2015) emphasise that the core approach of flipped classroom education is that passive learning activities should take place outside a classroom and students construct their competencies themselves in class in an active and learner-centred manner. When teaching and learning according to modern FCA some experts add that students should acquire new competencies by solving real-world problems in groups (Choi, 2013) supported by new technologies in class (Galway *et al.*, 2014). Tackling real-world problems in education also implies that this process can usually not be

Journal of Research in Innovative Teaching & Learning
Vol. 13 No. 1, 2020
pp. 3-25
Emerald Publishing Limited
2397-7604
DOI 10.1108/JRIT-01-2020-0006

carried out by using concepts from one subject only. Many real-world problems associated with mathematics often require knowledge and competencies from the fields of science or technology. This diversity of knowledge and competencies in investigating real-world problems in FCA education means that FCA learning environments could also be suitable for fostering STEM education.

Recently, education following FCA has become increasingly popular (O'Flaherty and Phillips, 2015), especially in the field of natural sciences and mathematics (Muir and Geiger, 2016). Furthermore, Esperanza *et al.* (2016) were able to demonstrate that learning mathematics following FCA could have positive effects on both students' learning outcomes and enjoyment of mathematics. In order for scaling up (Cobb and Smith, 2008) promising approaches in mathematics education such as FCA, professional teacher development (PTD) could be one decisive factor (Selter *et al.*, 2015).

Therefore, our research aims to explore which aspects should be considered when developing and implementing professional mathematics or STEM teacher development for FCA, as an example of technology-enhanced and student-driven education.

2. Theoretical background

To identify essential aspects of professional mathematics or STEM teacher development for FCA, as one example for technology-enhanced and student-driven education, we firstly examine FCA with a focus on mathematics. Then, PTD and framework conditions for PTD are discussed whereby a special focus is put on conditions of professional mathematics teacher development in European countries. Investigating both FCA mathematics education and professional mathematics teacher development provides us with a framework to explore which aspects should be considered when developing and implementing professional mathematics or STEM teacher development for FCA.

2.1 Flipped classroom approaches and mathematics education

To explore which aspects should be considered when developing and implementing professional mathematics or STEM teacher development for FCA, a clear definition or description of flipped classroom education is crucial. Even though many researchers have been investigating FCA in recent years, there is no generally accepted definition of this educational approach (Wolff and Chan, 2016). However, the largest common divisor which holds many descriptions is that passive learning should take place outside or inside a classroom and in the classroom, learning should be active and learner-centred. As a result, lower levels of cognitive work (Krathwohl, 2002) are pursued in pre-class phases of education and then higher levels of cognitive work should be tackled in class (Enfield, 2016; Galway *et al.*, 2014; Wasserman *et al.*, 2015).

Both modern FCA (Galway *et al.*, 2014; Tsai *et al.*, 2016) and mathematics or STEM education (Chao *et al.*, 2016; Fogarty *et al.*, 2001) are often associated with using technologies, but flipping a class is much more than just using videos or technologies in teaching. In this paper, by modern technologies, we mean both mathematics-specific technologies, which can represent and solve mathematical problems (e.g. GeoGebra, MATLAB or wxMaxima), as well as learning management systems which should facilitate communication and collaboration (e.g. Mahara, Moodle or Blackboard). When learning mathematics or STEM according to FCA, these modern technologies could be used in pre-class phases for transmitting content, as a source of knowledge or for communication (Orlando and Attard, 2016), as well as drill programs through which students should acquire first knowledge on a topic (Samuelsson, 2006). In in-class phases of FCA mathematics or STEM education, technologies could facilitate students when conducting experimental activities or exploring real-world issues

(Samuelsson, 2006). Thus, combining FCA and mathematics or STEM education could facilitate developing technology-enhanced and student-driven learning environments. These technology-enhanced and student-driven learning environments should have a positive impact on students acquiring 21st-century skills or key competences and facilitate developing fruitful STEM learning environments.

Exploring real-world problems could be vital for FCA (Choi, 2013) and Gainsburg (2008) as well as Wilson *et al.* (2005) emphasise that high-quality mathematics education requires real-world content which is relevant for students. Working on such real-world issues often means that associated activities become more complex and learners have to form groups to deal with such demanding tasks (Choi, 2013; Herreid and Schiller, 2013; McNally *et al.*, 2016). Furthermore, tackling complex real-world problems also requires knowledge and competencies from different disciplines and, above all, a combination of knowledge and competencies from different disciplines. Combining knowledge and competencies from different disciplines is also a central element of STEM education.

The circumstance that learners (co-) determine real-world problems to be dealt with and tackle these problems as a group elucidates that FCA is learner-centred and students develop their competencies themselves. Students (co-) determining real world problems to be tackled in education means that students are involved in setting learning goals and can tailor the learning processes (Bergmann and Sams, 2014; Long *et al.*, 2016; Wasserman *et al.*, 2015). Also regarding mathematics education, discovering new content is described by Bell and Pape (2012), Elbers (2003) or Lee and Johnston-Wilder (2013) as a social process which should foster collaboration. However, Harkness and Stallworth (2013) stress that there are also students who prefer to discover mathematics alone and the needs of these students should also be considered. Tackling real-world problems and learning in different social forms (individual work, partner or group work) in education following FCA should also contribute to students developing personal or social competences which are vital according to the European Commission (2019).

Thus, mathematics or STEM teachers are confronted with a variety of tasks and challenges in FCA learning environments, which requires that teachers should progress from mere transmitters of knowledge to facilitators and guides (Klein, 2002; Wilson *et al.*, 2005). These diverse challenges in terms of content, pedagogies and technologies require a well-trained teacher so that 21st century-skills or key competences could actually be integrated into education and not degenerate into mere educational buzzwords. To reach a large number of well-trained teachers who can provide technology-enhanced and student-driven flipped classroom education, PTD could be a key.

2.2 Professional mathematics teacher development for FCA

When integrating new approaches or technologies into mathematics or STEM education to facilitate education including 21st-century-skills or key competences, PTD could be decisive (Selter *et al.*, 2015). To identify the framework conditions under which teachers should develop new skills in PTD, the conceptual framework of Maxwell (2010) will be adopted to illustrate the interactions or influences of PTD in the field of education. Here, a priority is set on both the specific context of our study and characteristics when mathematics or STEM education following FCA is the subject of PTD (see Figure 1).

Figure 1 illustrates that in the course of development of education teachers, courses and schools influence each other and are interdependent. The original figure is supplemented by a regulatory and social context in which PTD takes place because as Samuelsson (2006) already pointed out, each educational institution is also part of the respective society. Through the embedding of PTD courses into a wider context, it follows that changes that influence society also have an impact on educational institutions and therefore also on PTD courses. These changes also affect the content to be learned in PTD for flipped mathematics or STEM education. In our concrete case it is 21st-century skills or key competences that should be integrated into education as a consequence of the societal change towards education. As one way of integrating 21st-century skills or key competences into education, we investigate in our study PTD for flipped classroom education. In PTD for flipped mathematics or STEM education, it is often TPACK (Technological Pedagogical Content Knowledge (Mishra and Koehler, 2006) which is the centre of the courses. Considering the TPACK model in PTD for flipped mathematics or STEM education can be justified by the circumstance that flipped mathematics or STEM teachers require expertise in the three areas of educational technologies, pedagogy and content.

In teaching and learning following FCA, it is crucial that students can develop new concepts on their own and that students can utilise their new knowledge in many ways. Using educational technologies should facilitate students to discover new knowledge (pre-class phases) and then apply the new knowledge to real-world issues (in-class phases). Since students learn both independently and in groups, in the classroom and at home, and since real-world issues and content relevant for students should be at the centre of education, flipped classroom learning environments are usually diverse and multifaceted. To develop such diverse learning environments, teachers need rich pedagogical knowledge. Since students should explore real-world issues as well as issues that are meaningful to students in flipped classroom education, students (co-)determine the direction and focus of education. Co-determining and focussing of education by students lead to the circumstance that teachers in flipped learning environments need much more in-depth content knowledge than in teacher-driven approaches to education.

Since the current TPACK model (see Figure 2) has also been extended to include XK (conteXtual Knowledge) and both interactions and influences in PTD courses, and the TPACK model are essential when exploring which aspects should be considered when developing and implementing professional mathematics or STEM teacher development for FCA an attempt has been made to align these two frameworks. Furthermore, since the beginning of the 21st century, significant regulatory changes have been implemented top down into education in many European countries and these regulatory changes could also have an impact on PTD. Since the majority of the experts in our study have an Austrian educational background, we will briefly present regulatory changes and educational framework conditions in Austria with a particular focus on mathematics education. Since the academic year 2013/14, a central and standardised written school-leaving examination in

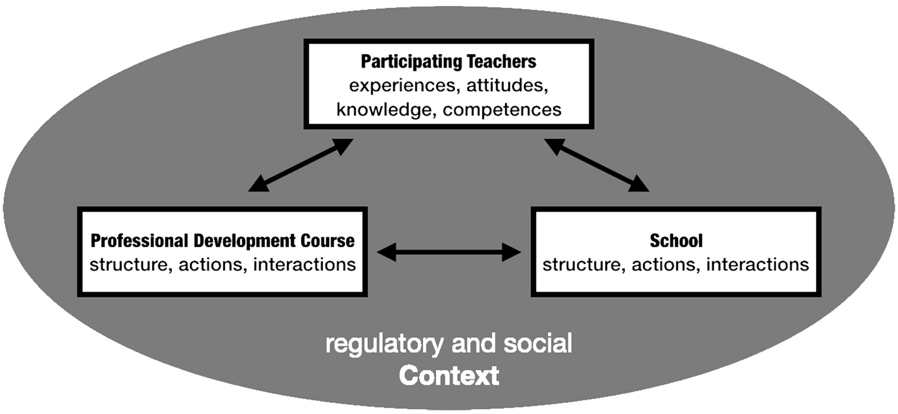

Figure 1.
Interactions and influences in PTD. Based on Maxwell's (2010) Conceptual framework for examining LSS trainee training

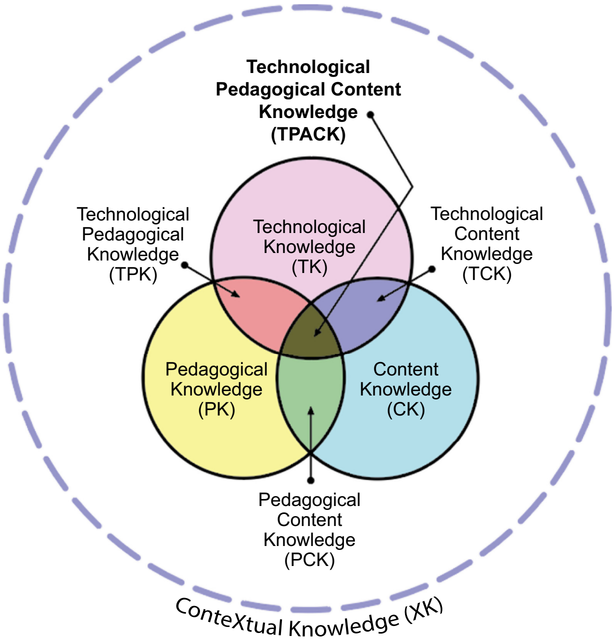

Figure 2.
TPACK diagram http://tpack.org (reproduced by permission of the publisher, © 2012 by tpack.org)

mathematics has been in place in Austria. A centralised and standardised written school-leaving examination in mathematics means that the tasks are developed centrally and that only predefined answer formats such as multiple-choice or assignment tasks are used in 50% of the school-leaving examination. A further change in the framework conditions for the school-leaving examination in mathematics was implemented in the academic year 2017/18. Since then, it has been mandatory to use higher-quality technologies such as GeoGebra or TI-Nspire for the written school-leaving examination in mathematics. A new regulatory requirement that affects all subjects in secondary education is that basic digital education (Digitale Grundbildung) is mandatory starting in the academic year 2018/19. In this context, basic digital education means that for example, digital communication, computational thinking or social media must be integrated into lessons and documented in teaching reports. These various changes in the framework conditions of education in Austria have led to a situation in which some teachers have become insecure about good teaching. However, individual changes in the framework conditions or elements of the PTD triangle (see Figure 1) and its context should not be interpreted as static but influence and change each other when PTD is carried out.

If, as it could be facilitated by FCA in mathematics or STEM education and is required under Austrian circumstances, new technologies and approaches should be implemented in teaching, the TPACK framework developed by Mishra and Koehler (2006) and further developed continuously (see Figure 2) could provide a theoretical basis for PTD. We consider the TPACK model as a promising framework for PTD for FCA, since mathematics or STEM education following FCA requires a synthesis of teacher knowledge from the fields of technologies (designing digital learning materials for the pre-class phase and utilising technologies to discover mathematics in the class), pedagogy (blended learning in the pre-class phases and student-centred constructivist learning in the class) and content (potentially an extension of the curriculum, as students co-determine learning goals and pathways).

Mishra and Koehler (2006) explain that teachers in technology-enhanced learning settings need knowledge in the individual bodies of knowledge (TK, PK and CK). Additionally, teachers should also possess new knowledge if two of these bodies of knowledge (TCK, TPK and PCK) or all three (TPACK) are combined. Based on this, it could be concluded that FCA teachers need knowledge in all of the seven areas of the TPACK model to provide helpful learning environments for mathematics education following FCA.

The theoretical framework concerning PTD for FCA indicates that flipped classroom education could be one way of integrating 21st-century skills and key competences into education. The potential of FCA education for integrating 21st-century skills and key competences into education can be justified by the circumstance that, according to this approach to education, students should learn both alone and in groups, students should tackle real-world problems, and students should use educational technologies in educational processes in a targeted manner. In this way, digital, social or learning-to-learn competencies as well as creativity and adaptivity should be strengthened in education.

In order to integrate FCA or other technology-enhanced and student-driven approaches to education into teaching and learning, it could be useful to consider the TPACK model of Mishra and Koehler (2006) as well as regulatory changes and framework conditions. When considering the TPACK model of Mishra and Koehler (2006) as well as regulatory changes and framework conditions, PTD courses should always offer a synthesis of theory and practice. The concrete research questions of our study on PTD for FCA as an example of an educational approach that should contribute to the achievement of 21st-century skills and key competences will be outlined in the next section.

2.3 Research questions investigated in our study

Considering the particularities of FCA mathematics education and professional mathematics teacher development, and according to our research aim to explore vital aspects when developing and implementing professional mathematics teacher development for FCA, the following research questions result:

(1) Which elements of professional teacher development could be crucial when designing courses for flipped classroom approaches in mathematics education?

(2) Which activities in professional mathematics teacher development could be promising when teachers are introduced to education according to flipped classroom approaches?

By exploring the two research questions above, our study should contribute to implementing technology-enhanced and student-driven approaches, as required for FCA education, into everyday mathematics education. Implementing technology-enhanced and student-driven approaches into mathematics education should also have positive impacts on achieving 21st-century skills or key competences.

3. Research design and methods

Our research aims to develop new theories and hypotheses on vital aspects when developing and implementing professional mathematics or STEM teacher development for FCA. By developing these vital aspects of PTD for FCA education, our study should also provide a means to discover how mathematics or STEM education could be developed towards acquiring 21st-century skills and key competences. In order to develop these new theories, we have conducted expert interviews and analysed the interview data according to grounded theory approaches (GTA) and qualitative interview study (QIS) principles. After analysing the interview data following inductive thematic principles (Braun and Clarke, 2006; Ezzy, 2002) and an initial and open approach (Charmaz, 2006; Strauss and Corbin, 1997), it became evident that the majority of experts considered teachers, as well as educational policy changes and their impacts on teachers, as central to PTD. By focusing on teachers during the initial and open coding process, we could deduce that teachers' needs and expectations, as well as educational policy changes and their impacts on teachers could be key components of crucial elements for PTD focussing on FCA. Therefore, we incorporated the different teachers' needs and characteristics as well as regulatory changes and their impacts on teachers and PTD courses into our research and adapted our interview guideline accordingly.

3.1 Participants and data collection

In order to obtain data to explore promising learning activities in PTD for FCA in mathematics or STEM education and to investigate the characteristics and needs of teachers attending PTD, expert interviews were conducted. Experts who provide information about PTD for FCA in mathematics or STEM education were instructors who offer PTD courses for secondary teachers on topics of FCA or using technologies when teaching and learning mathematics or STEM. Expert interviews are considered fruitful data to explore learning activities and characteristics of teachers in PTD because the selected experts have already been offering PTD from 3 to over 25 years, which should also enable a temporal comparison. In addition, the circumstance that assessments after PTD courses are rare could enable teacher trainers to adopt the positions of neutral observers. Since teacher trainers do not have to assess teachers' performances in most PTD, teacher trainers can focus on to be trained teachers' questions and problems which could have a positive effect on being neutral observers. By being able to adopt positions as neutral observers we anticipate that experts could provide perspectives from both experienced and new teachers.

Participants of our study were 17 experts, who agreed to be interviewed. Out of the 17 experts, 8 persons are FCA experts, 4 persons are experts for using technologies in mathematics or STEM education, and 5 persons are experts in both areas. Five of the experts of our study work at research universities as professors, six work at teacher training colleges as lecturers and researchers, and six are mathematics or STEM teachers at secondary schools as well as provide PTD courses. Experts in our study are aged from 34 to 58 and 14 of the 17 experts have their professional focus in urban regions. There are 13 experts employed at Austrian educational institutions and four experts work in a foreign European education sector. With regard to gender distribution, male dominance is evident: 12 experts are men and 5 experts are women.

The interviews were conducted over a period of 9 months, with a division into two parts regarding our guidelines for data collection. In principle, findings of data collection were constantly integrated into guidelines in order to obtain specific information. After eight interviews, it became evident that a fundamentally new guideline was needed in order to obtain further information and thus get closer to questions guiding the research. The first guideline focuses on (1) methods of motivation for attending PTD and motivating participants in courses, (2) current interests of teachers in general and in particular interests concerning applications of learner-centred teaching methods and technologies in mathematics teaching, and (3) temporal and spatial structures of PTD and, wishes and experiences of teacher trainers regarding structures. In the second guideline topics were (1) activities of teachers who attend a training course, (2) framework conditions for in-service PTD, (3) linking PTD and everyday school life as well as associated challenges when new knowledge and skills are applied in real settings, (4) mindsets of teachers and their role in

PTD, and (5) necessary training and characteristics of teacher educators. The interviews lasted from 14 to 53 min (average 29 min) and were conducted at experts' workplaces or online.

3.2 Data analysis

After conducting each interview, we have listened to the recordings repeatedly in order to derive first patterns and topics (Ritchie, 2012). Then all interviews were completely transcribed and analysed using a synthesis consisting of (1) open, axial and selective coding technique according to the GTA of Charmaz (2006) and Strauss and Corbin (1997), as well as (2) generating initial codes, searching for themes and reviewing themes following thematic analysis when conducting a QIS (Braun and Clarke, 2006). This synthesis of GTA and QIS principles was chosen in order to integrate both existing literature findings and own knowledge as teacher trainers for FCA in mathematics education into our research process and design. When analysing the data, we examined the experts' information in their entirety, separately according to experts' nationality (Austria and not Austria) and thematic specialisations of experts (FCA vs. technologies in mathematics or STEM education) to deduce findings about PTD courses for FCA.

In coding and analysing the interview data, we followed a threefold division of the research process: open coding – axial coding – selective coding. Throughout the entire coding and analysis process, we applied the principles of theoretical sampling. The coding and sampling processes used in our study are based on the models of Charmaz (2006) and Strauss and Corbin (1997).

Open coding aimed to break up the collected data and to derive first units of information. In order to derive the first units of information, we asked the questions what, how, who, and why to the collected data. This allowed us to develop codes of lower levels of abstraction (Table 1, first and second column). To increase the level of abstraction of the open codes, we grouped open codes with similar definitions and properties and added new tags (Table 1, third column). These new open codes of a higher level of abstraction were then used for axial coding. The goal of axial coding was to re-establish a synthesis of the open codes.

In order to achieve a synthesis of open codes, always one central open code of a higher level of abstraction was selected (= phenomenon) and then further investigated. In order to be able to further investigate central open codes of a higher level of abstraction, the other open codes of a higher level of abstraction were arranged around this phenomenon (see Figure 3). When arranging open codes of higher abstraction level around a phenomenon, the threefold division of causes, activities and consequences was used. For a further abstraction of our data, we focused on those codes which could be assigned to activities. By utilising axial coding, our data could be raised to an even higher level of abstraction (Table 1, fourth column).

In the next step, the codes of axial coding were used for selective coding. The goal of selective coding was to develop first theories and to close possible research gaps. In order to develop first theories and to discover and close research gaps, the codes of axial coding were connected to each other in selective coding. The aim of this connection was also to investigate the dependencies of the individual axial codes and to identify which axial codes form a centre in the connecting process.

The principles of theoretical sampling were applied throughout the entire coding process. Theoretical sampling means that new data, new survey instruments and new analysis methods always depend on the current state of research. In this way, it should be possible to keep the research process dynamic and to develop rich theories and hypotheses.

Since experts on FCA or technologies in mathematics or STEM education have given precise diagnoses of current situations in PTD, but have mentioned only superficial solutions,

Codes of lower level of abstraction First series of interviews	Codes of lower level of abstraction Second series of interviews	Grouped codes of open coding Higher level of abstraction	Categories based on axial coding
Applicability [16]	Sharing [64] > examples > feelings > success > fear	Routine-favoured teachers	Mindset
Demonstration [13]	Didactical knowledge [57] > materials > goals > teaching	Innovation-shy teachers	Examine / Assessment
Examples [21]	Feedback [13]	Error-shy teachers	Motivation
Obstacles [22]	Regulations and laws [16]	Educational discomfort	Learning activities
Hands-on learning [13]	Learning mode [26] > individual > group > peers	Guidelines for teaching	Support
Heterogeneity > technologies [18]	Heterogeneity [23]	Heterogenous teachers	Professional teacher trainer
Methodophobia [13]	Learning by doing [47]	Flexible PTD	……
Methods [18]	Learning hands-on [11]	Sharing experiences and examples	
Mindset [11]	Hands-on learning [18]>first application > time	Developing knowledge	
Motivation [28]	Mindset [31]	Success in/through PTD	
Professional Trainer [14]	Fear [23]	Confidence in PTD	
Workload [15]	Avoiding mistakes [29]	. . .	
. . .	Reflection [21]		
	Support [23]		
	Confidence [26]		
	Interlocking PTD and teaching [23]		
	Advantages [18]		
	Demonstrating [26]		
	. . .		

Table 1. Prototypical codes of different levels of abstraction

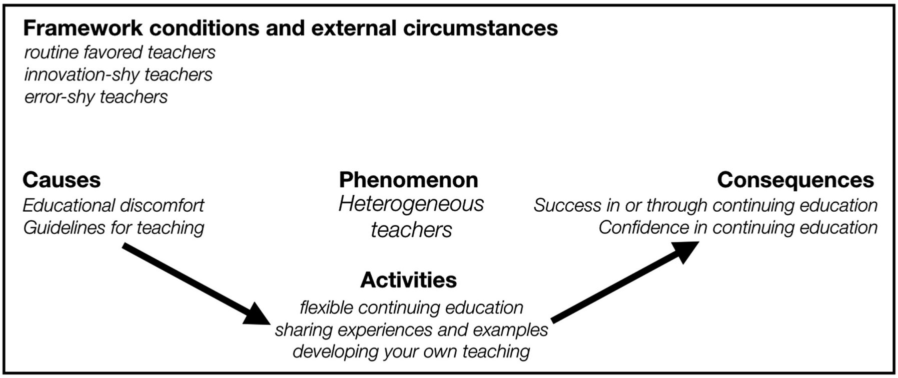

Figure 3.
Prototype procedure for axial coding

further sources have been integrated into our research following theoretical sampling principles (Charmaz, 2006; Strauss and Corbin, 1997). These further sources were expert opinions from educational scientists with a focus on scientists who conduct research in the field of teacher and teaching development. Concerning this field of educational science three experts were willing to be interviewed. These interviews lasted from 21 to 44 min and focused on areas of motivations and fears of teachers in mathematics or STEM PTD for new technologies and approaches to reduce research gaps from expert interviews described above. The generation and comparison of data-based concepts and categories led to three types of teachers who distinguish themselves in terms of motivation for PTD and participation in courses as well as three types of learning activities in PTD, which could be decisive for success of PTD courses for technology-enhanced and student-driven approaches of mathematics or STEM education such as flipped classroom education.

4. Results

The results section of this paper is divided into two sub-areas, whereby the first section investigates in detail the needs, characteristics and attitudes of teachers attending PTD for FCA in mathematics education, and how the given frameworks affect teachers' needs, characteristics and attitudes. In the second section, potential learning activities in PTD are examined, which could be helpful when PTD focuses on technology-enhanced and student-driven methods of mathematics education.

All quotes from the interviews, which were mainly conducted in German, were translated into English by the authors. The pseudonyms of the experts in our study were also provided with information regarding gender (f . . . female or m . . . male), field of expertise (fca . . . FCA or t . . . technologies in mathematics or STEM education) and field of work (u . . . university, tt . . . teacher training college or s . . . secondary school).

4.1 Types of teachers in PTD for FCA in mathematics education

According to interview data, we deduced that participants in PTD could be grouped into types of teachers according to the core categories of goals, activities and participation in training, as well as attitudes towards PTD. Analysing the data also indicates that motivational triggers for participating in PTD influence these core categories. Corresponding to these motivational triggers as well as how teachers participate in PTD for FCA or technologies in mathematics or STEM education PTD courses participants could be grouped into three types of teachers – namely: the delegates, the extrinsically motivated and the intrinsically motivated. These three groups of teacher types are intended to reflect the requests, needs and characteristics of teachers participating in PTD courses, but cannot be distinguished sharply from each other.

4.1.1 The delegates. Teachers, which can be counted as delegates, take part in PTD as required by their schools or a school authority, and content of PTD as well as place and time of training are often determined by others. Most teachers of this type want to fulfil their duty by participating in PTD and show a phlegmatic attitude as well as a low interest in training content and management of PTD. In PTD courses, the delegates usually behave passively and demonstrate little or no interest in cooperation with other participants.

Expert E [m,t,s]: certain further training courses have to be attended – they are prescribed by the regional school board, as a result there is a lack of capacity for voluntary further training [. . .] you notice this [that compulsory PTD courses prevent voluntary PTD courses from being attended] in [low] motivation during the course.

4.1.2 The extrinsically motivated. The extrinsically motivated teachers take part in PTD because regulatory changes have occurred which affect the teaching of these teachers or the subjects they teach. In order to receive support regarding regulatory changes and its consequences, teachers can determine place or time of PTD themselves, so respective PTD institutes have appropriate offers. The aims of the extrinsically motivated teachers are to obtain quick and easy answers or recipes concerning their lessons. Since in Austria in general and in mathematics education in particular there have been many regulatory changes concerning teaching in recent years, many teachers of this type are annoyed by attending PTD courses and are neutral to negative towards content of PTD. In PTD courses many extrinsically motivated teachers follow a minimal principle which means that they want to acquire knowledge and competencies with the least possible effort, so that teaching can be offered in accordance with new frameworks.

> Expert A [m,t,tt]: A large group only wants to hear what they need next in class: e.g. in the AHS [grammar schools], then when it comes to the SRDP [school leaving examination] or NOST [new organisation of the upper secondary] – then they want to have very concrete specifications, example types, tasks – everything very concretely

4.1.3 The intrinsically motivated. If a potential for improvement of their own teaching is recognised and teachers want to use this potential or if teachers have a fundamental interest in content of PTD and this triggers teachers to participate in a course, these teachers can be counted as intrinsically motivated. It is an expansion of a repository of knowledge and skills or a development of their own teaching that intrinsically motivated teachers pursue in PTD courses. These teachers show a positive to euphoric attitude towards content of courses both before and during PTD. The intrinsically motivated teachers want to be active in PTD and participate in the construction of knowledge.

> Expert E [m,t,s]: [. . .] today more and more rarely, in former times it was more often that a teacher says "I want to change something in my lessons" – then, the motivation of a teacher came more from the inside –teachers said "I want to improve something" – that is rare today

4.1.4 Location of individual teachers on the teacher type segment. The individual groups of teacher types cannot be clearly distinguished from each other. Rather, there is a smooth transition between the individual groups of teacher types who attend PTD (see Figure 4). Likewise, teachers who participate in a course cannot be assigned to a point on the segment which should represent the groups of teacher types in a course. It is often the case that teachers can be assigned to several groups from which it follows that individual teachers in this model should be represented as pair of two points on the teachers' types segment.

The distance of this pair of two points representing a teacher is determined by the external trigger of a PTD courses (left end), and the motivation and participation in PTD (right end) (see Figure 4). With regard to qualities and consequences, the two points clearly distinguish themselves. The left point is usually determined by others and the right point is usually determined by the teachers themselves. Additionally, the left point determines whether a teacher participates in PTD and the right point is most decisive for how active a teacher is in PTD. Furthermore, if a teacher wants to attend several PTD courses and this leads to time overlaps or other conflicts, the left points are usually more relevant than the right points.

In this context it should be mentioned that different teachers have different levels of PTD capacities and obligations in Austria. These various capacities and obligations regarding PTD result in individual PTD opportunities which depend both on a school authority, and on the teacher as a person. In Austria, it is the school authorities that can require teachers to participate in PTD and these authorities also have to approve PTD requests of teachers. In addition to institutional requirements, personal circumstances of teachers also influence whether certain PTD courses are attended. When teachers decide for or against a course, important factors could be the duration and timing of PTD courses, costs of a training or the location of a training.

> Expert L [f,fca,tt]: often there are organisational or scheduling matters as well [that decide whether a course will be attended] – if it is just before the Matura or just before Christmas holidays it becomes more difficult [that teachers can be motivated to participate in the course]

If teacher capacities, which depend on both vocational and private commitments, are combined with partially existing obligations to attend PTD courses and the existing regulatory framework as well as its changes, this results in the demand for PTD per teacher. However, here a minimum of demand for PTD per teacher is determined by others and corresponds to the demand of the type "the delegates".

4.2 Promising activities in PTD courses

There are three types of learning activities which could be of great importance in PTD courses on FCA mathematics education according to interview evaluations: (1) learning by doing, (2) enhancing prior knowledge and (3) sharing experiences and examples.

4.2.1 Learning by doing. Experts declare in the interviews that it is important that teachers who participate in a course are active. Being active could be crucial for a learning progress both in PTD for new methods and technologies. For this reason, it is recommended that teachers who participate in PTD courses do something themselves during most of a training period.

> Expert D [m,t,u]: [. . .] work is done immediately – you should not deal with terms and concepts too long, but do something right away

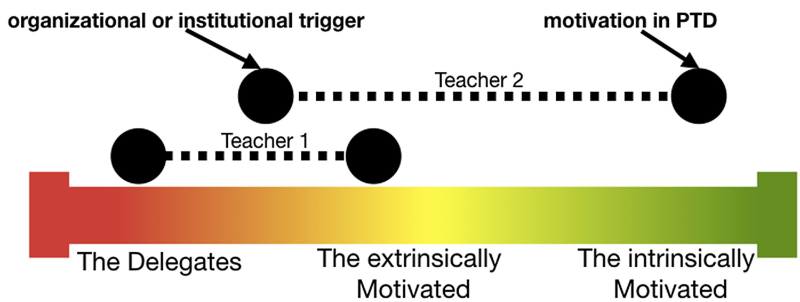

Figure 4.
Participants' segments in a PTD course

So that participating teachers can act in in-course phases it would be important that knowledge is acquired in a pre-course phase of PTD or by examples. When using examples in PTD courses, it could be helpful that theory and practice are dovetailed. Interview data indicate that it could be fruitful if activities in PTD are triggered by real problems and real frameworks should not be lost sight of when trying to solve real problems.

> Expert F [m,fca,u]: I meet with them at the schools – I ask what you want to prepare – ok let us plan something, I help them to plan and create something.

Using and developing examples concerning real frameworks should also make it easier to individualise PTD. Hence, different competencies of participating teachers could be taken into account and resulting learning artefacts should be feasible in respective teachers' lessons. Real problems as well as frameworks mean that learning activities carried out could be complex which indicates that in many cases a joint processing of tasks is more expedient than individual work.

> Expert J [f,fca,tt]: [. . .] this results in challenging tasks, which they [participating teachers] should work on in groups of 2, 3 or 4. So, even if someone has not done the tasks of the pre-phase, this should not be immediately noticeable.

In spite of complex learning actions, interview data make it evident that it is recommended to start with simple aspects of technologies or method applications. This beginning focussing on simple elements should help to minimise fear of teachers who participate in a course of new teaching materials or methods.

> Expert U [m,t,s]: I always start with simple elements. When I then realise that participants can do that then content and tasks become more complex.

If it is possible, learning actions in PTD courses should lead to a concrete learning product. Through such self-made artefacts the acquired knowledge can be put into tangible materials and the motivation of participating teachers could be increased.

> Expert A [m,t,tt]: [. . .] the app is presented, then people should immediately make trials [. . .] the teachers are active here themselves and should then have something of their own in their hands

Then, learning artefacts created by teachers who participate in a PTD courses should also be used in schools under real conditions. The expert recommendations indicate that teachers should be active both in PTD and in school, and that learning should therefore take place in both places.

> Expert C [f,fca,s]: it is always great when there is a phase in between [two or more training units], so participants can also try it [new methods or technologies] out at school

When using learning artefacts from a PTD courses at schools, experts emphasise that first self-developed learning products should be as good as possible. Applying high-quality teaching and learning materials should help teachers to trust in a new method or technologies, and facilitate the transition of a learning content from PTD to real school settings. It makes sense that both in PTD and in schools, teachers who participate in a course are supported, because learning takes place in both places and therefore questions or problems could arise in both places.

> Expert S [f,fca,tt]: The first application should be planned as precisely as possible, because when you are teaching, you have little time to create new materials [. . .] this should also lead to a successful experience with first applications.

If teachers who participate in PTD are active and supported by leaders of PTD courses, this should also have an impact on teachers' mindsets and how they interpret the role of a teacher.

According to the experts in our study, this form of experiential learning should have a stronger impact on teachers' mindset and the role of teachers than direct instruction could. Therefore, many experts recommend that learning settings and learning actions in PTD are designed in a way that participating teachers are supposed to teach afterwards.

> Expert F [m,fca,u]: They [participating teachers] should learn [in PTD courses] in the same way they should teach afterwards

By first learning according to new methods or using technologies, teachers should also become aware of any weaknesses in these forms of education and should then be able to react better to potential difficulties in their own lessons. However, interview data show that it could be problematic to learn according to FCA in PTD because one should be prepared to learn according to FCA. This means that sufficient time should be scheduled so that on the one hand learners take pre-class phases of FCA seriously and learners actually become active. On the other hand, a certain familiarisation phase to learning following an FCA is also needed so that time and possibilities of in-course phases are used fruitfully.

> Expert J [f,fca,tt]: Risk here [PTD following FCA] is, that only few do that, only few read texts or watch videos [before the course] – you [teacher trainer] have to be patient and you have to stay tuned, you should not think just because I do it once, that it will work immediately

4.2.2 Enhancing prior knowledge. According to the interview data, it is often the case that changes in regulation or changes in learners' needs lead to discontent among teachers in class. To ease this discontent, interviewed experts state that it is recommended that PTD as a whole, as well as methods, actions and technologies of PTD, should not lead to a sharp contrast to previous teaching. It would be better if in PTD an attempt is made to regard previous training and teaching of participating teachers as a basis and to connect at this point in PTD.

> Expert C [f,fca,s]: then we collect ideas and also experiences or knowledge – which methods are known by which teachers, i.e. you discover which methods teachers of a training course know and therefore what can be simply used in first flipped classroom applications

Also when introducing technologies in PTD courses in general and concerning mathematics-specific technologies in particular it could be helpful to connect with teachers' experiences and prior knowledge. The experts recommend that simple technologies or those familiar to teachers who participate in a course should be applied at the beginning of PTD.

> Expert K [m,t,u]: It is important that you start with a very low-threshold, it is important that simple elements of technologies also appear, that you start with technologies that teachers already know from their everyday life [. . .] through this one gets quickly into doing, into acting.

In particular in PTD for FCA in mathematics education, linking PTD content to teachers' experiences could be appropriate in terms of teaching methods and using technologies. Teaching methods or student arrangements (e.g. partner work or group work) already used by teachers in their lessons could be door openers for student-driven classes such as those according to FCA. One objective of PTD for FCA should be for teacher trainers and teachers who participate in a course to work on how FCA and using technologies can be integrated into their regular teaching. Often teachers are already using technologies in their teaching and PTD courses should be oriented towards these technologies and develop on a basis of these. If new methods and technologies are introduced in this way in PTD, this smooth transition from familiar teaching to new approaches and technologies could reduce a rejection of the new by teachers.

> Expert C [f,fca,s]: Often the participants already have knowledge or experience with technologies. Teachers should consider in groups how they can use existing knowledge or material for FC lessons.

4.2.3 Sharing. Sharing is described as one of the central learning activities in PTD by both experts on FCA and experts on using technologies in mathematics education. Many experts place gaining knowledge through sharing above theoretical knowledge acquisition such as offered by a lecture. When extending knowledge and competencies through sharing two central aspects can be identified: Examples could be shared, and experiences could be shared. In many cases, a PTD instructor initiates a process of sharing. In the later course of PTD, however, teachers who participate in PTD courses should also spread their expertise and experiences.

> Expert B [m,fca,tt]: Then there are examples from practice –I [teacher trainer] present examples from my own lessons [. . .] this is important, teachers need this, concrete examples from practice.

4.2.3.1 Sharing examples. The evaluation of interviews indicates that experts recommend a tripartite division when sharing examples in PTD courses: first a varied repository of examples should be created by a PTD management, then how examples could be applied should be demonstrated and finally teachers who participate in a course should work with these examples and develop them further or create their own examples to extend the repository. The experts stated that a repository of examples should be as comprehensive as possible. A comprehensive repository means that different examples should include various student arrangements, teaching methods, and technology applications. It is also recommended that several experts create an initial repository because examples developed by different experts are at least slightly heterogeneous. Additionally, different experts could offer different perspectives on FCA and how technologies could be applied in mathematics education.

> Expert C [f,fca,s]: [. . .] the two-part training courses are offered with another colleague so that they [participating teachers] see several perspectives and also so that they see how differently we do this, so that they get to know different examples of flipped classroom applications right at the beginning.

When demonstrating how examples from a repository could be used, experts recommend being as close as possible to everyday school life and considering constraints and limitations of everyday lessons. If examples are selected, it is equally advantageous if the examples are simple and concrete and if teachers who participate in a course can also adapt them to their own everyday school life. Adapting examples means that teachers can change, delete or add individual aspects of the teaching materials provided. If individual examples are worked on, experts advocate that advantages of this teaching artefacts and related methods or technologies over conventional mathematics education be highlighted.

> Expert K [m,t,u]: also to present good practice examples is always well-appreciated – examples where something has been successful and then continue to work on it in small groups, adapting it for one's own lessons.

Once teachers have gained initial experiences with examples of a new method or technologies, teachers should also apply such examples themselves and then develop them. For an initial application of teaching examples, teachers of a PTD courses should start with best practice examples, which have already been tested and developed under real circumstances. Using tested and further developed examples should increase the probability that teachers who participate in a course will be successful first applying a new method or technologies. After a first utilisation of supplied teaching materials, applied examples should also be improved by teachers or they should develop new examples themselves.

> Expert S [f,fca,tt]: At teacher meetings we try to integrate it [sharing experiences and examples]. That teachers talk about their lessons, what was going well – where were problems. Now we also organise mini-workshops where we share our experiences and examples so that others can use them as well.

These new or improved examples of applying new methods or technologies should also be integrated into a repository. Through this integration of examples on the one hand a collection of teaching materials should grow and on the other hand it should be made easier for teachers who participate in a course to become a part of a community of practitioners or experts.

> Expert C [f,fca,s]: would be beneficial that they [participating teachers] can submit their documents and examples on a platform, so sharing even after the course becomes possible

With all these learning actions, experts advise that a management of PTD should remain modest and meet the learning teachers at eye level. If the power and knowledge asymmetry is exemplified clearly, there is a high probability that teachers who participate in a course could lose confidence in PTD and that content of PTD would rarely or never be applied.

> Expert H [m,fca,tt]: You [teacher trainer] have to be willing to share your own experiences and examples. Without running down the others [participating teachers] – you have to be very respectful with the others.

4.2.3.2 Sharing experiences. If experiences with modern technologies or methods are shared in PTD, a leader of PTD should begin with this process. It is recommended that when leaders initiate a sharing process that they start with their first experiences of FCA or using modern technologies. Then leaders of PTD courses should explain step by step further developments of their education where new technologies or methods have been used. This sharing of experience with first applications of new technologies or methods should make it easier for teachers who participate in a course to recognise the genesis of an educational approach.

> Expert S [f,fca,tt]: You have to talk to them [participating teachers] a lot, you have to share your experiences, you have to be open to your colleagues – how you felt when you started teaching according to the flipped classroom principle.

From this relevance of sharing experiences, it follows that it would be advantageous if leaders of a PTD have ample experiences of applying PTD content. If experiences are shared, then it could be helpful if leaders of a PTD dedicate themselves particularly to problems experienced thereby, with it connected experienced feelings, and then solution attempts. These discussions of problems and related attempts to solve these issues should not only increase the knowledge of teachers but also increase trust within a learning group. If teachers have also gained their first experiences with new methods or technologies, experiences gained should also be shared with the entire learning group. According to expert opinions, this process of sharing initial experiences needs to be institutionalised. Institutionalisation of exchanging experiences means that leaders of a PTD course should offer and moderate (online) meetings for sharing experiences and knowledge gained in these gatherings should also be recorded. Among other things, this exchange of experiences should lead to teachers losing uncertainty about new learning environments and simplifying a formation of a community.

> Expert I [f,fca,tt]: We strongly encourage people to build a learning network. They also need information, people need to know how to develop, how to develop their [flipped classroom] teaching.

In order for sharing processes described above to be realised, it is important that there is trust among a community of teachers in a course. Trust in this context means both that teachers of a PTD courses trust each other and that teachers of a PTD have confidence in the content of a PTD.

Expert H [m,fca,tt]: listen to what they [participating teachers] say and think [. . .] you [teacher trainer] have to build trust. They must have confidence that they will be helped when they get into problems.

In order to make it easier for teachers who participate in a course to trust new teaching methods or technologies, it is important that a PTD leader is convinced of the content being presented. Experts emphasise that if a leader of a PTD is persuaded of the content presented and can convey this, teachers who participate in a course would trust in new methods or technologies.

> Expert H [m,fca,tt]: It is also important that a teacher trainer strongly believes in the method and that one has a lot of experience with this method [. . .] that one shares experiences with learners and that one is enthusiastic about content.

In order to establish trust between the participants of PTD, the experts recommend that all concerns and problems of teachers who participate in PTD be taken seriously and treated with respect. If teachers' problems are dealt with openly in PTD, this could be key to trust. It should be possible to address all concerns in PTD and avoid making participating teachers feel that they are asking stupid questions and being treated from above.

> Expert O [m,fca,s]: learners should not have inhibitions to ask also "stupid questions" and a teacher trainer should not look down on learners [. . .] learners should really have fun in the course and learners should also be able to get rid of their "concerns".

If confidence is established in a PTD course, the probability increases that teachers will contact other learners or the leader of a PTD after the course with problems. This staying in touch could lead to circumstances in which content of PTD are increasingly applied in everyday school life.

> Expert U [m,t,s]: The real problems usually arise weeks after the course, then they [participating teachers] contact you [teacher trainer] and hope that you can help them.

4.2.4 The DSE model of learning activities in PTD. Doing, enhancing and sharing (DSE) can each be a central activity if PTD for technology-enhanced and student-driven approaches such as FCA is to be offered for mathematics teachers. But, experts in didactics as well as experts in educational science explain that it could be fruitful to combine two or even all three of these activities in PTD for technology-enhanced and student-driven approaches. In the following diagram (see Figure 5), central activities in PTD are visualised and how these

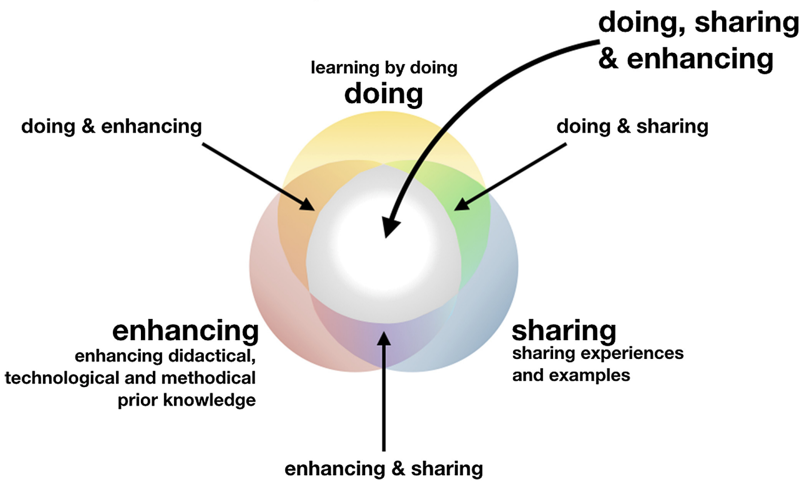

Figure 5.
DSE model of learning activities in PTD

actions can overlap. Here, the central activities in PTD, namely DES, as well as their mutual influence and interdependence form the basic elements of the DSE model of learning activities in PTD.

Interviewed experts explain that it could be helpful when sharing and doing are linked. For instance, it is described that examples are provided to teachers and then these examples are actively developed by teachers themselves.

> Expert K [m,t,u]: an experienced teacher assists another teacher as an "eBuddy" [. . .] they should plan and implement lessons together [. . .] thereby new didactic impulses are to be set, and they should support themselves mutually

But also enhancing and doing are connected by some experts. When enhancing and doing are combined, experts recommend this approach with technologies or teaching methods, which are already known to participating teachers. Teachers who participate in PTD should then work with or on what is familiar which should lead to existing competencies being expanded and, ideally, new teaching products being developed.

> Expert C [f,fca,s]: They should think about how they can use existing material for FC lessons. Here, learning by doing is always very important.

The colour gradient of the individual actions in the DSE model of learning activities in PTD (see Figure 5) is intended to reflect the fact that individual activities can only rarely be found in their pure form in PTD. This is also exemplified by the interview data by the fact that in many cases, a central activity can be identified in the expert statements, but this activity is often linked to a second activity. Since many of these activities could be assigned multiple times, it can be recognised that individual activities are fluently connected with each other. To illustrate this fluent transition of activities, the graphic representation does not include circular lines. Similarly, a visualisation of the three central activities in the DSE model (see Figure 5) differs from other circular models (e.g. TPACK Model (Mishra and Koehler, 2006) or Model of a family business (Zimmermann, 2012) insofar as circles overlap considerably more. This larger overlapping of the circles of the individual activities is intended to make it apparent that a majority of activities in PTD is formed by combining two or three activities.

5. Discussion

Our research aims to elucidate how to develop and implement PTD for FCA in mathematics or STEM education. Since experts of our study described teachers' characteristics, needs and wishes as vital for PTD for FCA, we have incorporated them as well into our research.

The results indicate that a central aspect when developing and implementing PTD are current regulatory changes affecting mathematics or STEM education as well as heterogeneity within teachers. The importance of a context of PTD is already emphasised in the theoretical background of our paper based on Maxwell (2010), Samuelsson (2006) and Mishra and Koehler (2006) and our study was able to extend context by educational policy changes.

Heterogeneity of teachers and PTD participants has already been extensively researched (e.g. Kraft and Papay, 2014) and our research extends these insights into different levels of heterogeneity. Our data illustrate that different knowledge and competencies of teachers exist in the subject area which consists of seven sub-areas according to the TPACK model (Mishra and Koehler, 2006) and in each sub-area heterogeneity could occur. In addition, there are also clear heterogeneities within participants of PTD courses in terms of motivation, attitudes, beliefs and obligations. If technologies are integrated into PTD, this often increases the heterogeneity among teachers in respective areas. This elucidates that heterogeneity among teachers in PTD should not be considered as an entity, but as a multivariate phenomenon.

A combination of high impact potential of regulatory changes and a heterogeneity within teachers regarding knowledge and competencies related to educational technologies and learning approaches could make teacher trainers' knowledge about teachers participating in PTD decisive. The relevance of educators' knowledge about their learners was already illustrated by Wilson *et al.* (2005) concerning school mathematics education. Some aspects such as "a professional relationship with your kids and at the same time understand the difficulties and the problems they are having" (Wilson *et al.*, 2005, p. 92) what the teachers in the study of Wilson *et al.* (2005) indicated as vital for good school mathematics could also apply to professional mathematics teacher development.

In addition to who attends PTD courses for FCA in mathematics education, our research also focuses on how competencies should be acquired in PTD when teachers are familiarised with new approaches or technologies. Here it becomes apparent that three core activities are central for experts: learning by doing, enhancing prior knowledge and sharing experiences and examples. It is these three activities which form our DSE model.

Primus inter pares of these core activities should be learning by doing because doing is key to many further activities and thus central to enabling teachers to learn in PTD. Nevertheless, PTD should try to combine and offer as many activities as possible – that is moving to the centre of Figure 5. In the scientific frame of our research expert conclusions (Breckwoldt *et al.*, 2014; Chapman and An, 2017; Dekeyser *et al.*, 2005) have been integrated which highlight that it is important that content is applied and practical relevance is established. Applying and practical relevance are close to doing, but our data suggests a more general view on doing. A general view on doing means that already when knowledge is acquired, this should take place through teacher-active activities – that is, doing of our research is interpreted more holistically than that of the literature which forms a basis of the theoretical background.

6. Conclusions and implications

Since FCA are not precisely defined, we could not discover key aspects for developing and implementing professional mathematics teacher development for FCA through our study. Rather, we explored important aspects for PTD which focus on technology-enhanced, student-led and constructivist approaches in mathematics education. These approaches do not apply to FCA exclusively but could be important when developing and implementing PTD for FCA.

The grouping of teachers illustrates that heterogeneity could be found in PTD on many levels and heterogeneity depends on many aspects, especially concerning educational technologies and regulatory changes and its impacts on teachers. Consequently, heterogeneity should be thought of much more broadly than just in terms of competencies of participants in PTD.

A broad understanding of heterogeneity could result in an instructor's knowledge of learners' characteristics being essential in both a school and PTD context. Challenges that could be associated with heterogeneity in professional mathematics teacher development consists of PTD's temporal structure (usually one-off events) as well as changing compositions of participating teachers in PTD. Since PTD is usually too short to capture the heterogeneity of participating teachers as well as their needs and characteristics, utilising tools that could facilitate teacher trainers or PTD institutions to understand participating teachers' needs and characteristics before PTD courses could be promising. Such tools should be able to capture both hard facts (e.g. triggers of training attendance or competencies and experiences with educational technologies) and soft facts (e.g. attitudes and beliefs concerning educational innovations) from participating teachers. This insight could provide teacher trainers or PTD institutions with information that could facilitate tailoring PTD courses to participating teachers which could have positive effects on school education.

Doing, enhancing and sharing were identified by expert of our study as the most promising activities in PTD. On a larger scale, it can be concluded that it is these activities that put participants at the heart of PTD courses. It could also be deduced that it is not individual activities that should dominate PTD, but that a combination of different activities could lead to sustainable learning in PTD which is summarised in our DSE model.

In our DSE model, doing and therefore constructing or utilising knowledge and competencies can be characterised as the key activity. Since doing is often interlinked with other activities, it might be helpful to take into account as many aspects of our DSE model as possible when developing and evaluating activities in PTD. Taking into account as many aspects of the DSE model as possible means that activities are as close as possible to the centre of our model – combining doing, sharing and enhancing.

Since our DSE model focusses on how information should be acquired and the TPACK elucidates what should be taught in PTD for FCA a combination of these two models (see Table 2) could provide guidance in developing and evaluating PTD for technology-enhanced and student-driven approaches such as FCA in mathematics education.

A simple example of combining the TPACK model and the DSE model in professional mathematics teacher training for FCA education could be producing learning videos:

In PTD for FCA teachers should acquire competencies about how to present mathematical content through a learning video and these competencies should be acquired by learning activities through which teachers actively connect to prior knowledge and experience and share applied and newly acquired knowledge with colleagues.

Taking into account the TPACK model and the DSE model, this description of a learning process could be broken down and analysed as follows:

A distinct examination of learning processes in professional mathematics teacher development with regard to the above models could be helpful in planning and evaluating PTD courses to examine whether the learning content (TPACK) and promising learning activities (DSE) necessary for achieving the learning objective have been taken into account. Supplementing the TPACK model with the DSE model will not be a panacea in professional mathematics teacher development but could be an element to improve PTD for mathematics education.

TPACK model (see Figure 2) attribution	
How to present	(PK)
Mathematical content (CK)	(CK)
Through a learning video	(TK)
How to present mathematical content	(PCK)
How to present . . . through a learning video	(PTK)
How to present mathematical content through a learning video	(TPACK)
DSE Model (see Figure 5) attribution	
Teachers actively	(D)
Connect to prior knowledge and experience	(E)
Share applied and newly acquired knowledge with colleagues	(S)
Teachers actively connect to prior knowledge and experience	(DE)
Connect to prior knowledge/experience and share applied/newly acquired knowledge with colleges	(ES)
Teachers actively connect to prior knowledge and experience and share applied and newly acquired knowledge with colleagues	(DSE)

Table 2.
Combining TPACK and DSE model

References

Bell, C.V. and Pape, S.J. (2012), "Scaffolding students' opportunities to learn mathematics through social interactions", *Mathematics Education Research Journal*, Vol. 24, pp. 423-445, doi: 10.1007/s13394-012-0048-1.

Bergmann, J. and Sams, A. (2014), "Flipped learning: gateway to student engagement", *International Society for Technology in Education*.

Braun, V. and Clarke, V. (2006), "Using thematic analysis in psychology", *Qualitative Research in Psychology*, Vol. 3, pp. 77-101, doi: 10.1191/1478088706qp063oa.

Breckwoldt, J., Svensson, J., Lingemann, C. and Gruber, H. (2014), "Does clinical teacher training always improve teaching effectiveness as opposed to no teacher training? A randomized controlled study", *BMC Medical Education*, Vol. 14 No 6, doi: 10.1186/1472-6920-14-6.

Chao, T., Chen, J., Star, J.R. and Dede, C. (2016), "Using digital resources for motivation and engagement in learning mathematics: reflections from teachers and students", *Digital Experiences in Mathematics Education*, Vol. 2, pp. 253-277, doi: 10.1007/s40751-016-0024-6.

Chapman, O. and An, S. (2017), "A survey of university-based programs that support in-service and pre-service mathematics teachers' change", *ZDM Mathematics Education*, Vol. 49, pp. 171-185, doi: 10.1007/s11858-017-0852-x.

Charmaz, K. (2006), *Constructing Grounded Theory: A Practical Guide through Qualitative Analysis*, 1st ed., SAGE, London, Thousand Oaks, CA.

Choi, E.M. (2013), "Applying inverted classroom to software engineering education", *International Journal of e-Education, e-Business, e-Management and e-Learning*, Vol. 3 No. 121.

Cobb, P. and Smith, T. (2008), "The challenge of scale: designing schools and districts as learning organizations for instructional improvement in mathematics", *International Handbook of Mathematics Teacher Education*, Vol. 3, pp. 231-254.

Dekeyser, H.M., Van Rijn, F.H.M. and Jansen, D. (2005), "Teacher training on the job", in van Weert, T. and Tatnall, A. (Eds), *Information and Communication Technologies and Real-Life Learning: New Education for the Knowledge Society*, Springer, Boston, MA, pp. 179-188, doi: 10.1007/0-387-25997-X_20.

Elbers, E. (2003), "Classroom interaction as reflection: learning and teaching mathematics in a community of inquiry", *Educational Studies in Mathematics*, Vol. 54, p. 77, doi: 10.1023/B:EDUC.0000005211.95182.90.

Enfield, J. (2016), "The value of using an E-text in a flipped course", *TechTrends*, Vol. 60, pp. 449-455, doi: 10.1007/s11528-016-0100-1.

Esperanza, P., Fabian, K. and Toto, C. (2016), "Flipped classroom model: effects on performance, attitudes and perceptions in high school algebra", in Verbert, K., Sharples, M. and Klobučar, T. (Eds), *Adaptive and Adaptable Learning: 11th European Conference on Technology Enhanced Learning, EC-TEL 2016, Lyon, France, September 13-16, 2016, Proceedings*, Springer International Publishing, Cham, pp. 85-97.

European Commission (2019), *Key Competences for Lifelong Learning*, Publications Office of the European Union, Luxembourg, available at: https://op.europa.eu/en/publication-detail/-/publication/297a33c8-a1f3-11e9-9d01-01aa75ed71a1/language-en (accessed 21 December 2019).

Ezzy, D. (2002), *Qualitative Analysis: Practice and Innovation*, Allen and Unwin, Crows Nest, N.S.W.

Fadel, C. (2008), *21st Century Skills: How Can You Prepare Students For The New Global Economy*, Cisco Systems and OECD, Paris.

Fogarty, G., Cretchley, P., Harman, C., Ellerton, N. and Konki, N. (2001), "Validation of a questionnaire to measure mathematics confidence, computer confidence, and attitudes towards the use of technology for learning mathematics", *Mathematics Education Research Journal*, Vol. 13, pp. 154-160, doi: 10.1007/BF03217104.

Gainsburg, J. (2008), "Real-world connections in secondary mathematics teaching", *Journal of Mathematics Teacher Education*, Vol. 11, pp. 19-219, doi: 10.1007/s10857-007-9070-8.

Galway, L.P., Corbett, K.K., Takaro, T.K., Tairyan, K. and Frank, E. (2014), "A novel integration of online and flipped classroom instructional models in public health higher education", *BMC Medical Education*, Vol. 14, p. 181, doi: 10.1186/1472-6920-14-181.

Harkness, S.S. and Stallworth, J. (2013), "Photovoice: understanding high school females' conceptions of mathematics and learning mathematics", *Educational Studies in Mathematics*, Vol. 84, pp. 329-347, doi: 10.1007/s10649-013-9485-3.

Herreid, C.F. and Schiller, N.A. (2013), "Case studies and the flipped classroom", *Journal of College Science Teaching*, Vol. 42, pp. 62-66.

Klein, M. (2002), "Teaching mathematics in/for new times: a poststructuralist analysis of the productive quality of the pedagogic process", *Educational Studies in Mathematics*, Vol. 50, pp. 63-78, doi: 10.1023/A:1020566020275.

Kraft, M.A. and Papay, J.P. (2014), "Can professional environments in schools promote teacher development? Explaining heterogeneity in returns to teaching experience", *Educational Evaluation and Policy Analysis*, Vol. 36, pp. 476-500.

Krathwohl, D.R. (2002), "A revision of Bloom's taxonomy: an overview", *Theory into practice*, Vol. 41, pp. 212-218.

Lee, C. and Johnston-Wilder, S. (2013), "Learning mathematics—letting the pupils have their say", *Educational Studies in Mathematics*, Vol. 83, pp. 163-180, doi: 10.1007/s10649-012-9445-3.

Long, T., Cummins, J. and Waugh, M. (2016), "Use of the flipped classroom instructional model in higher education: instructors' perspectives", *Journal of Computing in Higher Education*, pp. 1-22, doi: 10.1007/s12528-016-9119-8.

Maxwell, B. (2010), "In-service initial teacher education in the learning and skills sector in England: integrating course and workplace learning", *Vocations and Learning*, Vol. 3, pp. 185-202, doi: 10.1007/s12186-010-9045-2.

McNally, B., Chipperfield, J., Dorsett, P., Del Fabbro, L., Frommolt, V., Goetz, S., Lewohl, J., Molineux, M., Pearson, A., Reddan, G., Roiko, A. and Rung, A. (2016), "Flipped classroom experiences: student preferences and flip strategy in a higher education context", *Higher Education*, pp. 1-18, doi: 10.1007/s10734-016-0014-z.

Mishra, P. and Koehler, M.J. (2006), "Technological pedagogical content knowledge: a framework for teacher knowledge", *Teachers College Record*, Vol. 108, p. 1017.

Muir, T. and Geiger, V. (2016), "The affordances of using a flipped classroom approach in the teaching of mathematics: a case study of a grade 10 mathematics class", *Mathematics Education Research Journal*, Vol. 28, pp. 149-171, doi: 10.1007/s13394-015-0165-8.

Orlando, J. and Attard, C. (2016), "Digital natives come of age: the reality of today's early career teachers using mobile devices to teach mathematics", *Mathematics Education Research Journal*, Vol. 28, pp. 107-121, doi: 10.1007/s13394-015-0159-6.

O'Flaherty, J. and Phillips, C. (2015), "The use of flipped classrooms in higher education: a scoping review", *The Internet and Higher Education*, Vol. 25, pp. 85-95.

Ritchie, S. (2012), "Incubating and sustaining: how teacher networks enable and support social justice education", *Journal of Teacher Education*, Vol. 63, pp. 120-131, doi: 10.1177/0022487111428327.

Saavedra, A.R. and Opfer, V.D. (2012), "Learning 21st-century skills requires 21st-century teaching", *Phi Delta Kappan*, Vol. 94, pp. 8-13, doi: 10.1177/003172171209400203.

Samuelsson, J. (2006), "ICT as a change agent of mathematics teaching in Swedish secondary school", *Education and Information Technologies*, Vol. 11, pp. 71-81, doi: 10.1007/s10639-005-5713-5.

Selter, C., Gräsel, C., Reinold, M. and Trempler, K. (2015), "Variations of in-service training for primary mathematics teachers: an empirical study", *ZDM*, Vol. 47, pp. 65-77, doi: 10.1007/s11858-014-0639-2.

Strauss, A. and Corbin, J.M. (1997), *Grounded Theory in Practice*, Sage.

Tsai, C.W., Shen, P.D., Chiang, Y.C. and Lin, C.H., (2016), "How to solve students' problems in a flipped classroom: a quasi-experimental approach", *Universal Access in the Information Society*, pp. 1-9, doi: 10.1007/s10209-016-0453-4.

Wasserman, N.H., Quint, C., Norris, S.A. and Carr, T. (2015), "Exploring flipped classroom instruction in calculus III", *International Journal of Science and Mathematics Education*, pp. 1-24, doi: 10.1007/s10763-015-9704-8.

Wilson, P.S., Cooney, T.J. and Stinson, D.W. (2005), "What constitutes good mathematics teaching and how it develops: nine high school teachers' perspectives", *Journal of Mathematics Teacher Education*, Vol. 8, pp. 83-111, doi: 10.1007/s10857-005-4796-7.

Wolff, L.C. and Chan, J. (2016), "Defining flipped classrooms", in Wolff, L.C. and Chan, J. (Eds), *Flipped Classrooms for Legal Education*, Springer Singapore, Singapore, pp. 9-13, doi: 10.1007/978-981-10-0479-7_2.

Zimmermann, W. (2012), "Familienunternehmen und Unternehmerfamilien: zwischen Gefühl und Geschäft", In *Unternehmer Sind Verrückte*, Gabler Verlag, pp. 205-232.

Further reading

Bishop, J.L. and Verleger, M.A. (2013), "The flipped classroom: a survey of the research - flipped-classroom-artikel.pdf", available at: http://www.studiesuccesho.nl/wp-content/uploads/2014/04/flipped-classroom-artikel.pdf (accessed 27 October 2016).

Corresponding author

Robert Weinhandl can be contacted at: robert.weinhandl@gmail.com