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The optimal insurance demand under an ambiguity aversion

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Abstract

This study investigates insurance demand in a two-period model when a decision-maker (DM) is averse to the ambiguity of loss distributions. This study derives sufficient conditions such that the ambiguity-averse DM purchases more insurance than an ambiguity-neutral one when the DM maximises the expected utility. It also derives each sufficient condition to increase insurance demand as ambiguity aversion, ambiguity and downside ambiguity increase, respectively.

Keywords Ambiguity aversion, Ambiguity, Downside ambiguity, Insurance demand

Paper type Research paper

1. Introduction

The standard model for insurance demand developed by Mossin (1968) has been discussed in terms of a single period (or one period). The one-period model implies that purchasing insurance and a loss due to an accident occur simultaneously within a single period. In a one-period model, the level of optimal insurance demand is determined to hedge the risk of loss states. Thus, the utilities between loss and no loss states can be smoothing. However, as Seog and Hong (2022) highlight, in reality, there is a time difference between purchasing insurance and occurrence of loss. That is, the standard one-period model does not consider the interactions between different times with regard to insurance demand.

From a similar viewpoint, some economic studies on self-protection were developed based on a multi-period model. Menegatti (2009) argues that efforts towards prevention precede the effect. Hence, the two-period framework is more suitable for analysing prevention. He finds that a prudent individual invests in more self-protection than a risk-neutral individual. Eeckhoudt *et al.* (2012), Nocetti (2016) and Wang and Li (2015) also consider a two-period model for optimal self-protection when income in the second period is uncertain due to background risk. They demonstrate that a prudent individual invests more in self-protection effort when there is background risk. Although these studies have examined a two-state model with loss and no loss occurrence states, Lee (2019) considers multiple loss states to analyse self-protection in a two-period model. Lee also demonstrates that a prudent individual makes greater efforts for self-protection when an increase in effort induces a first-order stochastic dominance (FSD) improvement in the loss distribution. All of these results are in contrast to those of Eeckhoudt and Gollier (2005), who show that a prudent individual invests less in self-protection than a risk-neutral individual in a one-period model.

The key feature of a two-period model is the separation of the insurance cost from the insurance benefit. That is, the cost and benefits are not concurrent. The cost to mitigate future risk occurs in the first period, whereas the benefit occurs in the second period. In addition,

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insurance demand is generally explored under conventional expected utility framework, with the assumption that policyholders have information for the loss distribution. In this conventional insurance demand model, it is well known that choosing a high deductible to decrease one's insurance premium is optimal by utility smoothing across the states (Arrow, 1974). However, in reality, many policyholders choose a low deductible despite an expensive insurance premium. Sydnor (2006) finds policyholders' preference for a low deductible in housing insurance. To illustrate this preference, some studies introduce the ambiguity preference that policyholders dislike the uncertainty over what distribution loss follows. For example, the decision-maker (DM) does not know the exact loss distribution for the loss x, but knows that the loss follows the conditional loss distribution on the parameter of second-order belief θ which follows a distribution. In a two-period model, the DM faces not only a risk of future loss but also an ambiguity with regard to probability distributions of loss. That is, one's attitude towards the ambiguity of DM affects insurance demand as well as risk aversion.

Since Ellsberg (1961) highlighted the phenomenon of ambiguity aversion, whereby individuals are likely to avoid the uncertainty of probabilities, the concepts of ambiguity and ambiguity aversion have received significant attention in the literature. Fei (2009) and Faria and Correia-da-Silvia (2016) find that the demand of ambiguity-averse DM for risky assets decreases as ambiguity increases. Gollier (2011) and Huang and Tzeng (2018) derive some conditions to increase the demand for risky assets as ambiguity aversion increases. Snow (2011) and Alary *et al.* (2013) investigate the optimal level of self-protection and self-insurance with ambiguity aversion in a one-period model. With ambiguity aversion, Osaki and Schlesinger (2014) provide a condition to increase precautionary saving in a one-period model, whereas Berger (2014) derives such a condition in a two-period model. Peter (2019) also analyses a two-period model, but assumes that the utility function is not time-separable, whereas Berger (2014) assumes that it is time-separable.

In line with these studies, this study aims to identify the conditions under which the demand for insurance increases as ambiguity aversion and ambiguity increase, respectively. We also set a two-period model to examine insurance demand in an intertemporal context. The DM is assumed to face not only a risk of future loss but also an ambiguity with regard to probability distributions of loss. However, this study differs from previous studies on ambiguity in the following points. First, we focus on the effect of ambiguity and ambiguity aversion on the demand of insurance rather than that of risky asset. Most studies have examined the saving and portfolio choice problem (Gollier, 2011; Osaki and Schlesinger, 2014; Berger, 2014; Huang and Tzeng, 2018; Peter, 2019). Second, we consider an intertemporal decision in a two-period model with multiple loss states, whereas Alary *et al.* (2013) investigate insurance demand in a one-period and binary state of loss model. Third, we investigate a change in insurance demand in a two-period model as ambiguity aversion and ambiguity change, respectively. Although Huang and Tzeng (2018) examine the effect of change in ambiguity on the demand for risky assets in a one-period model, to the best of our knowledge, insurance demand in a two-period model as ambiguity aversion and ambiguity change has yet to be explored.

Our findings are summarised as follows. We adopt Klibanoff *et al.*'s (2005, 2009) smooth ambiguity aversion approach to consider the intertemporal effect of ambiguity aversion and ambiguity on insurance demand. We first find that an ambiguity-averse DM purchases more insurance than an ambiguity-neutral DM when the expected utility and the expected marginal utility of the second period are anti-comonotone in terms of the ambiguity parameter, and the ambiguity preference exhibits non-decreasing absolute ambiguity aversion. Second, insurance demand increases as ambiguity aversion increases when the expected utility and the expected marginal utility of the second period are anti-comonotone with regard to the ambiguity parameter, the ambiguity preference presents constant absolute ambiguity aversion, and the second-order belief has deteriorated following the maximum likelihood ratio (MLR) order. Third, we investigate the effect of an increase in ambiguity and downside ambiguity on insurance demand. Note that an increase in ambiguity and an increase in downside ambiguity imply second- and third-degree increases in ambiguity, respectively. We find that ambiguity prudence and ambiguity temperance affect the increase in insurance demand as ambiguity and downside ambiguity increase, respectively. In particular, if the elasticity of the expected utility of the second period is less than that of the expected marginal utility, insurance demand increases as ambiguity increases when ambiguity prudence is greater than 2. Further, insurance demand increases as downside ambiguity increases when ambiguity temperance is greater than 3 under a relative risk aversion of less than 1. In Huang and Tzeng (2018) and Peter (2019), the elasticity is always less than 1, whereas in the current study, the elasticity can be greater or less than 1.

In recent years, natural disasters such as floods and droughts by climate change are causing massive losses in property and human life. Insurance is the most representative measure to cope with these disasters. However, in the case of the catastrophe risk, it is difficult to accurately predict the loss distribution, and insurance demand may vary depending on the individual's belief in the subjective probability distribution. That is, the ambiguity of the loss distribution affects the individual's insurance demand. In addition, the shorter cycle of disaster due to climate change can affect the ambiguity of the loss distribution and lead to a change in ambiguity aversion. In this respect, this study contributes to the literature in that this study examines the insurance demand under ambiguity aversion and analyses comparative static analysis following the change in ambiguity and ambiguity aversion. This study is also different from previous studies that it studies the long-term decision-making of an individual by adopting a two-period model rather than a one-period model.

The rest of this paper is organised as follows. In the next section, we introduce the general model assumptions. In Section 3, we compare insurance demand between an ambiguity-averse and an ambiguity-neutral DM. We also investigate the effect of an increase in ambiguity aversion, ambiguity and downside ambiguity on insurance demand. Finally, concluding remarks are provided in Section 4.

2. Model description

Following Seog and Hong (2022), we also consider a two-period model. The utility function, with respect to income, is denoted as $u(W)$, where W_t denotes income at time t, $t = 1{,}2$. This utility function is a strictly increasing and concave function in income. That is, $u'(W_t) > 0$, $u''(W_t) < 0$, respectively. Unlike Peter (2019), in this study, the individual's utility is assumed to be additively time-separable, as follows:

$$U(W) = u(W_1) + \beta u(W_2), \tag{2.1}$$

where β is a discount factor.

In the first period, the DM purchases insurance with insurance premium Q to improve the utility of the second period. The income of the first period becomes $W_1 - Q$ with insurance. In the second period, the DM faces a loss, x, distributed on $[0, \bar{x}]$, and receives indemnity $I(x) = ax$. The DM's prior beliefs on loss distribution and density functions are represented by the conditional distribution $F(x;\theta)$ and $f(x;\theta)$, respectively, in which θ denotes the ambiguity parameter which implies the second-order belief. That is, the DM does not know the objective loss distribution precisely. The ambiguity parameter is a random variable with distribution $G(\theta)$, and a probability density function $g(\theta)$ on the support of $\left[\underline{\theta}, \overline{\theta}\right]$. Then distribution of loss is denoted as follows:

$$F(x) = \int_0^x \int f(y;\,\theta)g(\theta)d\theta dy = \int \int_0^x f(y;\,\theta)g(\theta)dyd\theta = \int \int dF(x;\,\theta)dG(\theta).$$

The income of the second period with a loss becomes $W_2 - x + ax$. We suppose that insurers are risk neutral, as well as ambiguity neutral. If the loading is λ, then the insurance premium is

$$Q = (1+\lambda)aE_G Ex = (1+\lambda)\int \int xdF(x;\,\theta)dG(\theta).$$

As noted above, numerous studies have discussed ambiguity and ambiguity aversion. In particular, Klibanoff *et al.* (2009) (KMM, hereafter) propose an intertemporal model to explain ambiguity distinguished from risk. We adopt KMM's "recursive smooth model" to discuss insurance demand in a two-period model. According to KMM, the attitude towards ambiguity is represented by the increasing function ϕ. The DM is ambiguity averse (neutral, loving) when ϕ is concave (linear, convex) in utility. We additionally suppose that the function ϕ is four times differentiable. The intertemporal decision problem to maximise the expected utility of the individual under the mixture distribution of loss is

$$\underset{a}{Max}\; U = u(W_1 - Q) + \beta\phi^{-1}\{E_G\phi(Eu(W_2 - x + ax))\}$$
$$= u(W_1 - Q) + \beta\phi^{-1}\left\{\int \phi\left(\int u(W_2 - x + ax)dF(x;\,\theta)\right)dG(\theta)\right\} \quad (2.2)$$

$$s.t. \quad Q = (1+\lambda)aE_G Ex = (1+\lambda)\int \int xdF(x;\,\theta)dG(\theta) \quad (2.3)$$

In (2.2), $\beta\phi^{-1}\{E_G\phi(Eu(W_2 - x + ax))\}$ is ϕ-certainty equivalence with respect to the expected utility of the second period. Insurance premium is interpreted as a DM's current willingness to pay to hedge not only the risk of the second period but also the uncertain distribution of risk. Ambiguity aversion affects one's willingness to pay for insurance through the certainty equivalent. Meanwhile, KMM define the notion of the degree of ambiguity aversion. Similar to absolute risk aversion, a DM with ambiguity preference ϕ_2 is more ambiguity averse than a DM with ϕ_1 in the Arrow–Pratt sense when $-\frac{\phi_2''}{\phi_2'}(U) \geq -\frac{\phi_1''}{\phi_1'}(U)$ under the same belief. If $A(U)$ is decreasing (constant, increasing) in utility U where $A(U) = -\frac{\phi''}{\phi'}(U)$, $A(U)$ refers to decreasing (constant, increasing) absolute ambiguity aversion, DAAA (CAAA, IAAA) in utility.

When the DM is ambiguity neutral, the term $\beta\phi^{-1}\{E_G\phi(Eu(W_2 - x + ax))\}$ can be transformed using Jensen's inequality:

$$\beta\phi^{-1}\{E_G\phi(Eu(W_2 - x + ax))\} \leq \beta\phi^{-1}\phi\{E_G(Eu(W_2 - x + ax))\} = \beta E_G(Eu(W_2 - x + ax)). \quad (2.4)$$

The optimal insurance coverage for an ambiguity-neutral DM is obtained by solving the following problem:

$$\underset{a}{Max} \quad U = u(W_1 - Q) + \beta E_G Eu(W_2 - x + ax) \tag{2.5}$$

$$s.t. \quad Q = \beta(1+\lambda)aE_G Ex = \beta(1+\lambda)\int\int x dF(x;\theta)dG(\theta) \tag{2.6}$$

The first-order condition for the above problem is

$$U_a = -u'(W_1 - Q)(\beta(1+\lambda)E_G Ex) + \beta E_G Eu'(W_2 - x + ax)x \tag{2.7}$$

We rule out the case of over-insurance, since this is generally not allowed in the real world. Let us denote the insurance premium at $a = 1$ as $Q_{a=1}$. Then, the first-order condition, evaluated at $a = 1$ and 0, are, respectively,

$$U_{a|a=1} = -u'(W_1 - Q_{a=1})(\beta(1+\lambda)E_G Ex) + \beta u'(W_2)E_G Ex \tag{2.8}$$

$$U_{a|a=0} = -u'(W_1)(\beta(1+\lambda)E_G Ex) + \beta E_G Eu'(W_2 - x)x \tag{2.9}$$

As highlighted by Seog and Hong (2022), the DM may purchase partial insurance, even when the insurance premium is actuarially fair. We consider the case that the optimal coverage is partial for an ambiguity-neutral DM. That is, the following conditions hold:

$$u'(W_2) \le (1+\lambda)u'(W_1 - Q_{a=1}) \tag{2.10}$$

$$E_G Eu'(W_2 - x)x \ge u'(W_1)((1+\lambda)E_G Ex) \tag{2.11}$$

3. The two-period model

3.1 Insurance demand

Let us first investigate insurance demand under ambiguity aversion. For notational simplicity, we denote that $V = Eu(W_2 - x + ax)$ and $V_a = Eu'(W_2 - x + ax)x$. Then, the first-order condition for problem (2.2) at the optimum is

$$U_a = -u'(W_1 - Q)(\beta(1+\lambda)E_G Ex) + \beta\phi^{-1\prime}(E_G\phi(V))E_G\{\phi'(V)V_a\} = 0 \tag{3.1.1}$$

(3.1.1) [1] is transformed as follows:

$$-u'(W_1 - Q)(\beta(1+\lambda)E_G Ex) + \beta\frac{E_G\{\phi'(V)V_a\}}{\phi'\left(\phi^{-1}(E_G\phi(V))\right)} = 0 \tag{3.1.2}$$

In (3.1.2), the second term of the left-hand side (LHS) is transformed as follows:

$$\frac{E_G\{\phi'(V)V_a\}}{\phi'\left(\phi^{-1}(E_G\phi(V))\right)} = \frac{E_G\{\phi'(V)V_a\}}{E_G\phi'(V)}\frac{E_G\phi'(V)}{\phi'\left(\phi^{-1}(E_G\phi(V))\right)}$$

$$= \int V_a\frac{\phi'(V)}{E_G\phi'(V)}g(\theta)d\theta\frac{E_G\phi'(V)}{\phi'\left(\phi^{-1}(E_G\phi(V))\right)}. \tag{3.1.3}$$

Let us denote $\frac{\phi'(V)}{E_G\phi'(V)}g(\theta)$ as $\hat{g}(\theta)$. The term $\frac{\phi'(V)}{E_G\phi'(V)}$ is a Radon–Nikodym derivative that represents the distortion in the second-order belief via ambiguity aversion. Note that Gollier (2011) explains that ambiguity aversion causes pessimism of the second-period belief with MLR order [2].

Let us suppose that V and V_a are anti-comonotone in θ. This implies that if V is increasing (decreasing) in θ, then V_a is decreasing (increasing) in θ. Using the covariance rule, (3.1.2) becomes

$$-u'(W_1-Q)(\beta(1+\lambda)E_G Ex)+\beta\frac{\text{cov}(\phi'(V),\ V_a)}{\phi'\left(\phi^{-1}(E_G\phi(V))\right)}+\beta\frac{E_G\phi'(V)}{\phi'\left(\phi^{-1}(E_G\phi(V))\right)}E_G EV_a=0 \tag{3.1.4}$$

In (3.1.4), the second term of the LHS is the effect that comes from the distorted second-order belief. If V and V_a are anti-comonotone, then the covariance term in (3.1.4) is positive since ϕ' is decreasing. This effect leads the DM to purchase more insurance.

In addition, let us suppose that ϕ exhibits non-increasing absolute ambiguity aversion, DAAA or CAAA. That is, $-\frac{\phi'''}{\phi''}\geq-\frac{\phi''}{\phi'}$. In this case, we have $E_G\phi'(V)\geq\phi'(\phi^{-1}(E_G\phi(V)))$ since the certainty equivalence of ϕ' is greater than or equal to that of ϕ. Thus, the third term of the LHS in (3.1.4) is greater than or equal to $E_G EV_a$. Osaki and Schlesinger (2014) point out that the effect of this term is a "timing of uncertainty effect". Note that under DAAA (IAAA), the DM regards the consumption of period 2 (period 1) as more valuable compared to that of period 1 (period 2). Hence, this term captures the importance of consumption smoothing evaluated certainty equivalence of the inter-period, whereas the second term illustrates the importance of uncertainty that hedges between loss states. As such, when the above two assumptions hold, the demand for insurance increases. In the case that ϕ exhibits IAAA, insurance demand also increases when the effect of the second term is greater than that of the third term. Following Proposition 1 summarises all of these observations:

Proposition 1. An ambiguity-averse individual purchases more insurance than that of an ambiguity-neutral individual when the following conditions hold:

(1) $Eu(W_2-x+ax)$ and $Eu'(W_2-x+ax)x$ are anti-comonotone in θ.

(2) ϕ exhibits non-increasing absolute ambiguity aversion.

Proof See the text above.

Meanwhile, Corollary 1 summarises additional conditions closely related to first-order stochastic dominance (FSD) or second-order stochastic dominance (SSD), under which $Eu(W_2-x+ax)$ and $Eu'(W_2-x+ax)x$ are anti-comonotone in θ.

Corollary 1. $Eu(W_2-x+ax)$ and $Eu'(W_2-x+ax)x$ are anti-comonotone in θ if one of the following conditions holds:

(1) For any distribution function $F(x;\ \theta_i)$ and $F(x;\ \theta_j)$, $F(x;\ \theta_i)$ first order stochastically dominates $F(x;\ \theta_j)$ for every θ_i and θ_j, where $\theta_i<\theta_j$ and $\theta_i,\theta_j\in\left[\underline{\theta},\overline{\theta}\right]$, and $-\frac{u''(W_2-x+ax)}{u'(W_2-x+ax)}(-x+ax)\leq 1$.

(2) For any distribution function $F(x;\ \theta_i)$ and $F(x;\ \theta_j)$, $F(x;\ \theta_i)$ second order stochastically dominates $F(x;\ \theta_j)$ for every θ_i and θ_j, where $\theta_i<\theta_j$ and $\theta_i,\theta_j\in\left[\underline{\theta},\overline{\theta}\right]$, $-\frac{u''(W_2-x+ax)}{u'(W_2-x+ax)}(-x+ax)\leq 1$ and $0\leq-\frac{u'''(W_2-x+ax)}{u''(W_2-x+ax)}(-x+ax)\leq 2$.

Proof See Appendix.

Note that the critical value of relative risk aversion, $-\frac{u''(W_2 - x + ax)}{u'(W_2 - x + ax)}(W_2 - x + ax)$, is known to be 1. Fishburn and Porter (1976) demonstrate that the demand for risky assets increases when relative risk aversion is less than 1 as the return distribution of such assets is improved in the sense of FSD. It is also noted that in Hadar and Seo (1990), the critical value of relative prudence, $-\frac{u'''(W_2 - x + ax)}{u''(W_2 - x + ax)}(W_2 - x + ax)$, is known to be 2. They demonstrate that the demand for risky assets increases when relative prudence is less than 2 as the asset's return distribution is improved in the sense of SSD.

The conditions of Corollary 1 are similar to those in Gollier (2011). Gollier identifies the sufficient conditions to decrease the demand for risky assets as ambiguity aversion increases. This corollary also illustrates that relative risk aversion and relative prudence can affect the increase in insurance demand, along with ambiguity aversion, when both the expected utility and the expected marginal utility for insurance coverage can be ranked according to FSD or SSD.

3.2 The effect of an increase in ambiguity aversion on insurance demand

In the previous subsection, we compared the optimal insurance demand of an ambiguity-averse DM with that of an ambiguity-neutral DM. In this subsection, we find the condition to increase insurance demand as ambiguity aversion increases. Let us denote the function ϕ_2 as $\phi_2 = k \circ \phi_1$, where k is an increasing and concave function, to depict the increase in ambiguity aversion according to KMM (2005). Then, ϕ_2 is a uniformly more ambiguity-averse utility function than ϕ_1. Using (3.1.3), the second-order beliefs under ϕ_1 and ϕ_2 are denoted as $\widehat{g}_1(\theta) = \frac{\phi_1'(V)}{E_G\phi_1'(V)}g(\theta)$ and $\widehat{g}_2(\theta) = \frac{\phi_2'(V)}{E_G\phi_2'(V)}g(\theta)$, respectively. The following proposition shows that an increase in ambiguity aversion will not always increase insurance demand. The condition to increase such demand is as follows:

Proposition 2. A uniform increase in the individual's degree of ambiguity aversion leads to an increase in insurance demand when the following conditions hold:

(1) $Eu(W_2 - x + ax)$ and $Eu'(W_2 - x + ax)x$ are anti-comonotone in θ.

(2) Ambiguity preference exhibits CAAA.

(3) $\widehat{g}_2(\theta)$ dominates $\widehat{g}_1(\theta)$ in the sense of the MLR order.

Proof See Appendix.

An increase in ambiguity aversion implies that a DM is more inclined to prefer the lottery with certain probabilities to that with uncertain probabilities. However, an increase in ambiguity aversion does not always lead to increase in insurance demand. Although the distorted second-order belief can lead a DM to purchase more insurance, the relative effect of AAA in between ϕ_1 and ϕ_2 is not clear. Osaki and Schlesinger (2014) find similar conditions, whereby an increase in ambiguity aversion under CAAA leads to increase in precautionary savings. We will identify the results of Proposition 2 in the section of numerical examples.

3.3 The effects of an increase in ambiguity and downside ambiguity on insurance demand

We assume two loss distributions *H(x)* and *L(x)*, where *L(x)* is a first-degree increase in the risk of *H(x)*. Furthermore, as in Jindapon and Neilson (2007), we assume that the loss distribution is linear in θ: $F(x;\theta) = \theta H(x) + (1-\theta)L(x)$, where $\theta \in [0,1]$. Thus, $\frac{dF(x;\theta)}{d\theta} = F_\theta(x;\theta) = H(x) - L(x) \le 0$ and $\frac{dF^2(x;\theta)}{d\theta^2} = F_{\theta\theta}(x;\theta) = 0$.

Next, we define an increase in ambiguity as in Ekern (1980).

Definition 1. Following Ekern (1980), $T(\theta)$ is an increase in second-degree ambiguity over $G(\theta)$ if $E_T[\phi(V(\theta))] \leq E_G[\phi(V(\theta))]$ for all ϕ, where $\phi^{''} < 0$, for $\int_0^\theta T(t)dt \geq \int_0^\theta G(t)dt$.

Then, we have the following proposition:

Proposition 3. In the sense of Ekern (1980), let us suppose that $T(\theta)$ is more ambiguous than $G(\theta)$, and the optimal coverages under $T(\theta)$ and $G(\theta)$ are denoted as a_T* and a_G*, respectively. In a two-period model, the increase in ambiguity leads to increase in insurance demand when the following conditions hold at a_G*:

(1) $-\frac{u^{''}(W_2 - x + ax)}{u'(W_2 - x + ax)} ax \leq 1$ and

(2) $-\frac{\phi^{'''}(V)}{\phi^{''}(V)} V \frac{V_a V_\theta}{V_{a\theta} V} \geq 2$, where $V = Eu(W_2 - x + ax)$.

Proof See Appendix.

Note that condition (1) in Proposition 3 satisfies when the relative risk aversion is less than unity. Proposition 3 suggests a sufficient condition to increase insurance demand as ambiguity increases. The term $\frac{V_a V_\theta}{V_{a\theta} V}$ can be expressed as follows:

$$\frac{V_a V_\theta}{V_{a\theta} V} = \frac{\frac{\partial V}{\partial \theta}}{\frac{V}{\theta}} \frac{\frac{V_a}{\theta}}{\frac{\partial V_a}{\partial \theta}} \tag{3.3.1}$$

The outcome of this proposition seems similar to the findings of Huang and Tzeng (2018) and Peter (2019). However, in this two-period setting, the term (3.3.1) may or may not be less than 1, whereas it is always less than 1 in the aforementioned studies. This term can be interpreted as the elasticity of the expected utility for the second period with respect to θ, divided by the elasticity of the expected marginal utility of insurance for the second period with respect to θ.

Meanwhile, Baillon *et al.* (2018) define ambiguity prudence as a DM's preference, whereby the DM prefers to attach ambiguity to states with a high rather than a lower chance of a good outcome. They highlight that a DM is ambiguity prudent if $\phi^{'''} > 0$ in the smooth model. Peter (2019) also defines ambiguity prudence as $P(V) = -\frac{\phi^{'''}(V)}{\phi^{''}(V)} V$. According to the definition, Proposition 3 (2) illustrates that if the sensitivity of the expected utility for the second period is less than that of the expected marginal utility for the second period with respect to the ambiguity parameter, then ambiguity prudence should be greater than 2 to increase insurance demand as ambiguity increases. This Proposition 3 shows that an increase in ambiguity does not always lead to increase in insurance demand.

Next, we investigate the effect of an increase in downside ambiguity on insurance demand according to Ekern (1980). Such an increase implies an increase in third-degree ambiguity. Note that increase in downside risk refers to the distribution of risk becoming more skewed towards the left. Analogous to this, an increase in downside ambiguity can be defined as follows:

Definition 2. Following Ekern (1980), $T(\theta)$ is an increase in third-degree ambiguity over $G(\theta)$ if $E_T[\phi(V(\theta))] \leq E_G[\phi(V(\theta))]$ for all ϕ at every $V(\theta)$, where $\phi^{'''} > 0$, for $\int_0^\theta T^{(3)}(t)dt \geq \int_0^\theta G^{(3)}(t)dt$.

Using the above definition, we have Proposition 4.

Proposition 4. In the sense of Ekern (1980), let us suppose that $T(\theta)$ is more downside ambiguous than $G(\theta)$ and the optimal coverages under $T(\theta)$ and $G(\theta)$ are denoted as $a_T{}^*$ and $a_G{}^*$, respectively. In a two-period model, the increase in downside ambiguity leads to increase in insurance demand when the following conditions hold at $a_G{}^*$:

(1) $-\frac{u''(W_2-x+ax)}{u'(W_2-x+ax)}ax \leq 1,$

(2) $-\frac{\phi''''(V)}{\phi'''(V)}V\frac{V_a V_\theta}{V_{a\theta}V} \geq 3$, where $V = Eu(W_2 - x + ax)$.

Proof See Appendix.

Baillon *et al.* (2018) define ambiguity temperance as a preference that a DM dislikes facing two informationally symmetric ambiguities at the same time and prefers to disaggregate them. Baillon *et al.* (2018) also state that the DM is ambiguity temperate if the sign of the fourth-order derivative of ϕ is negative, $\phi'''' < 0$, in the smooth model. In line with Baillon *et al.* (2018), we define ambiguity temperance as $T(V) = -\frac{\phi''''(V)}{\phi'''(V)}V$. Analogous to Proposition 3 (2), if the sensitivity of the expected utility for the second period is less than that of the expected marginal utility for the second period with respect to the ambiguity parameter, then ambiguity prudence should be greater than 3 to increase insurance demand as downside ambiguity increases.

4. Conclusion

Using a two-period model, this study investigates the demand for insurance under ambiguity aversion compared to ambiguity neutrality. The model reflects not only the interactions between the present and the future but also ambiguity in terms of future loss distribution. The findings are as follows. We first find that the demand of the ambiguity-averse DM is greater than that of an ambiguity-neutral DM when (1) the expected utility and the expected marginal utility of insurance for the second period are anti-comonotone in terms of ambiguity parameter and (2) the ambiguity preference exhibits non-decreasing absolute ambiguity aversion. Second, we also derive sufficient conditions that an increase in ambiguity aversion increases insurance demand. If (1) holds, the ambiguity preference exhibits constant absolute ambiguity aversion, and the second-order belief has deteriorated via the MLR order, then the increase in ambiguity aversion leads to increase in insurance demand. Lastly, ambiguity prudence increases insurance demand as ambiguity increases, whereas ambiguity temperance increases insurance demand as downside ambiguity increases.

Notes

1. The second-order condition is $U_{aa} = u''(W_1 - Q)(\beta(1+\lambda)E_G Ex)^2 + \beta\phi^{-1''}(E_G\phi(V))\ E_G\{\phi'(V)\ V_a\}^2 + \beta\phi^{-1'}(E_G\phi(V))E_G\{\phi''(V)V_a^2 + \phi'(V)V_{aa}\} < 0$. Under the assumption that the function ϕ is increasing concave function, this second-order condition always holds.

2. Note that the MLR order is a subset of FSD.

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Appendix

Proof of Corollary 1.

(1) The derivative of V_a with respect to x is

$$V_{ax} = Eu''(W_2 - x + ax)(-1 + a)x + Eu'(W_2 - x + ax) > 0,\ \text{when}\ a \le 1. \tag{4}$$

In addition, V is decreasing in x when $a \le 1$. Thus, V and V_a are anti-comonotone in θ, in which the loss distribution can be ranked according to FSD when V_a is increasing in x. If $-\frac{u''(W_2-x+ax)}{u'(W_2-x+ax)}(-x + ax) \le 1$, the sign of (4) is positive.

(2) The derivative of V_{ax} with respect to x is

$$V_{axx} = Eu'''(W_2 - x + ax)(-1 + a)^2 x + 2Eu''(W_2 - x + ax)(-1 + a) \tag{5}$$

V is decreasing and concave in x when $a \le 1$. Thus, V and V_a are anti-comonotone in θ, in which the loss distribution can be ranked according to SSD when V_{ax} is increasing and concave in x. If $0 \le -\frac{u'''(W_2-x+ax)}{u''(W_2-x+ax)}(-x + ax) \le 2$, the sign of (5) is negative.

Proof of Proposition 2.
Let a^* denote the optimal level of coverage under the ambiguity preference ϕ_1. In addition, let ϕ_2 denote a function that is uniformly more ambiguity averse than ϕ_1, such that $\phi_2(.) = k(\phi_1)(.)$, where $k' > 0, k'' < 0$. Under ϕ_2, the first-order condition evaluated at a^* is as follows:

$$\begin{aligned} &-u'(W_1 - Q^*)(\beta(1+\lambda)E_G Ex) + \beta\phi_2^{-1\prime}(E_G\phi_2(V^*))E_G\{\phi_2'(V^*)V_a^*\} \\ &= -u'(W_1 - Q^*)(\beta(1+\lambda)E_G Ex) + \beta\frac{E_G\{\phi_2'(V^*)V_a^*\}}{\phi_2'\left(\phi_2^{-1}(E_G\phi_2(V^*))\right)}. \end{aligned} \tag{6}$$

In (6), the superscript * denotes the value at a^*. The positive sign of (6) implies

$$\beta\frac{E_\theta\{\phi_2'(V^*)V_a^*\}}{\phi_2'\left(\phi_2^{-1}(E_G\phi_2(V^*))\right)} \ge \beta\frac{E_\theta\{\phi_1'(V^*)V_a^*\}}{\phi_1'\left(\phi_1^{-1}(E_G\phi_1(V^*))\right)}. \tag{7}$$

Using (3.1.3), (7) is transformed as follows:

$$\int V_a^*\frac{\phi_2'(V^*)}{E_G\phi_2'(V^*)}g(\theta)d\theta\frac{E_G\phi_2'(V^*)}{\phi_2'\left(\phi_2^{-1}(E_G\phi_2(V^*))\right)} \ge \int V_a^*\frac{\phi_1'(V^*)}{E_G\phi_1'(V^*)}g(\theta)d\theta\frac{E_G\phi_1'(V^*)}{\phi_1'\left(\phi_1^{-1}(E_G\phi_1(V^*))\right)}$$

$$\Rightarrow \int V_a^*\hat{g}_2(\theta)d\theta\frac{E_G\phi_2'(V^*)}{\phi_2'\left(\phi_2^{-1}(E_G\phi_2(V^*))\right)} \ge \int V_a^*\hat{g}_1(\theta)d\theta\frac{E_G\phi_1'(V^*)}{\phi_1'\left(\phi_1^{-1}(E_G\phi_1(V^*))\right)},$$

where

$$\hat{g}_1(\theta) = \frac{\phi_1'(V^*)}{E_G\phi_1'(V^*)}g(\theta)\ \text{and}\ \hat{g}_2(\theta) = \frac{\phi_2'(V^*)}{E_G\phi_2'(V^*)}g(\theta) \tag{8}$$

According to Gollier (2011), we have

$$\frac{\widehat{g}_2(\theta)}{\widehat{g}_1(\theta)} = \frac{\frac{\phi_2'(V^*)}{E_G\phi_2'(V^*)}g(\theta)}{\frac{\phi_1'(V^*)}{E_G\phi_1'(V^*)}g(\theta)} = k'(\phi_1(V^*))\frac{E_G\phi_1'(V^*)}{E_G\phi_2'(V^*)}. \tag{9}$$

The likelihood ratio $\frac{\widehat{g}_2(\theta)}{\widehat{g}_1(\theta)}$ is increasing (decreasing) in θ if V^* is decreasing (increasing) in θ. Let us suppose that V^* and $V_a{}^*$ are anti-comonotone in θ. As $V_a{}^*$ is increasing (decreasing) in θ, we have

$$\int V_a{}^*\widehat{g}_2(\theta)d\theta \geq \int V_a{}^*\widehat{g}_1(\theta)d\theta \tag{10}$$

In addition, in the case of CAAA, $\frac{E_G\phi_2'(V^*)}{\phi_2'(\phi_2^{-1}(E_G\phi_2(V^*)))} = \frac{E_G\phi_1'(V^*)}{\phi_1'(\phi_1^{-1}(E_G\phi_1(V^*)))} = 1$. As a result, (8) holds.

Proof of Proposition 3.

Let us denote the optimal coverages under $G(\theta)$ and $T(\theta)$ as $a_G{}^*$ and $a_T{}^*$, respectively. Then by (3.1.2), $a_G{}^* \leq a_T{}^*$ when the following conditions hold at $a_G{}^*$:

$$E_G(\phi'(V)V) = \int \phi'(V)V_a dG(\theta) \leq E_T(\phi'(V)V) = \int \phi'(V)V_a dT(\theta), \tag{11}$$

$$\phi'\left(\phi^{-1}\left(\int \phi(V)dG(\theta)\right)\right) \geq \phi'\left(\phi^{-1}\left(\int \phi(V)dT(\theta)\right)\right). \tag{12}$$

Let us define that $T^{(2)}(\theta) = \int_0^\theta T(t)dt$ and $G^{(2)}(\theta) = \int_0^\theta G(t)dt$. Then, we have the following expression from integration by parts:

$$\begin{aligned}\int \phi'(V)V_a d[G(\theta)-T(\theta)] &= \phi'(V)V_a[G(\theta)-T(\theta)]|_{\underline{\theta}}^{\overline{\theta}} - \int \left(\phi''(V)V_\theta V_a + \phi'(V)V_{a\theta}\right)[G(\theta)-T(\theta)]d\theta \\ &= \int \left(\phi'''(V)V_\theta^2 V_a + 2\phi''(V)V_\theta V_{a\theta}\right)\left[G^{(2)}(\theta) - T^{(2)}(\theta)\right]d\theta \\ &= \int -\phi''(V)V_\theta V_{a\theta}\left(-\frac{\phi'''(V)}{\phi''(V)}V\frac{V_\theta V_a}{V_{a\theta}V} - 2\right)\left[G^{(2)}(\theta) - T^{(2)}(\theta)\right]d\theta\end{aligned} \tag{13}$$

In (13), the sign of each term is as follows:

$$V_\theta = \int u(W-x+ax)dF_\theta(x;\theta) = -\int u'(W-x+ax)aF_\theta(x;\theta)d\theta \geq 0 \tag{14}$$

$$\begin{aligned}V_{a\theta} &= \int u'(W-x+ax)xdF_\theta(x;\theta) = -\int \left(u''(W-x+ax)ax + u\prime(W-x+ax)\right)F_\theta(x;\theta)d\theta \\ &= \int \left(-\frac{u''(W-x+ax)}{u'(W-x+ax)}ax - 1\right)u'(W-x+ax)F_\theta(x;\theta)d\theta\end{aligned} \tag{15}$$

Note that the sign of (15) is positive when $-\frac{u''(W_2-x+ax)}{u'(W_2-x+ax)}ax \leq 1$. Thus, the sign of (13) is positive if both signs of $-\frac{\phi'''(V)}{\phi''(V)}V\frac{V_a V_\theta}{V_{a\theta}V} - 2$ and (15) are positive. In addition, we have the following expression by transforming (12):

$$\int \phi(V)d[G(\theta)-T(\theta)] = \phi(V)[G(\theta)-T(\theta)]|_{\underline{\theta}}^{\bar{\theta}} - \int \phi'(V)V_\theta[G(\theta)-T(\theta)]d\theta$$
$$= \int \left(\phi''(V)V_\theta^2\right)\left[G^{(2)}(\theta)-T^{(2)}(\theta)\right]d\theta \geq 0 \quad (16)$$

Since ϕ^{-1} is an increasing function, (12) holds.

Proof of Proposition 4.
Like the proof of Proposition 3, let us define that $T^{(3)}(\theta) = \int_0^\theta T^{(2)}(t)dt$ and $G^{(3)}(\theta) = \int_0^\theta G^{(3)}(t)dt$. Then, we have the following expression from integration by parts:

$$\int \phi'(V)V_a d[G(\theta)-T(\theta)] = \int -\phi'''(V)V_\theta V_{a\theta}\left(-\frac{\phi''''(V)}{\phi'''(V)}V\frac{V_\theta V_a}{V_{a\theta}V}-3\right)\left[G^{(3)}(\theta) - T^{(3)}(\theta)\right]d\theta \quad (17)$$

The sign of (18) is positive if the signs of $-\frac{\phi''''(V)}{\phi'''(V)}V\frac{V_a V_\theta}{V_{a\theta}V}-3$ and (14) are positive. In addition, by transforming (12), we have the following expression:

$$\int \phi(V)d[G(\theta)-T(\theta)] = -\int \left(\phi'''(V)V_\theta^3\right)\left[G^{(3)}(\theta)-T^{(3)}(\theta)\right]d\theta \geq 0 \quad (18)$$

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