A VECM analysis of Bitcoin price using time-varying cointegration approach

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Abstract

This study proposed an optimal model to examine the relationship between the Bitcoin price and six macroeconomic variables – the Bitcoin price, Standard and Poor's 500 volatility index, US treasury 10-year yield, US consumer price index, gold price and dollar index. It also examined the effectiveness of the vector error correction model (VECM) in analyzing the interrelationship among these variables. The authors employed the following approach: first, the authors sampled the period August 2010–February 2022. This is because Bitcoin achieved a market capitalization of more than US$1 tn over this period, gaining market attention and acceptance from retail, corporate and institutional investors. Second, the authors employed a VECM with the six macroeconomic variables. Finally, the authors expanded the long-run equilibrium relationship (time-invariant cointegration)-based VECM to develop a time-varying cointegration (TVC) VECM. The authors estimated the TVC VECM using the Chebyshev polynomial specification based on various information criteria. The results showed that the Bitcoin price can be modeled with the VECM ($p = 1, r = 1$). The TVC approach generated more explanatory power for Bitcoin pricing, indicating the effectiveness of the approach for modeling the long-run relationship between Bitcoin price and macroeconomic variables.

Keywords Bitcoin price, Vector error correction model, Johansen test, Time-varying cointegration, Chebyshev polynomials

Paper type Research paper

1. Introduction

Bitcoin exceeded a market capitalization of US$1 tn in February 2021, thereby attracting the interests of institutional investors and listed corporations in the cryptocurrency market (Coinmarketcap https://coinmarketcap.com/). Its market capitalization can be compared to the market capitalization rate of gold, silver, Apple and Samsung Electronics at 6.7, 58.6, 29.7, and 208%, respectively, as of March 2022 (8marketcap https://www.8marketcap.com/). In addition to this appreciation in the Bitcoin's market capitalization since 2010, there has been growing debate on Bitcoin's price, which is fundamental to the analysis of Bitcoin's price model.

In this context, while some studies use the security market analysis theory, such as the capital asset pricing model (CAPM) and the Fama–French multi-factor model, to predict the Bitcoin Price, some others use historical time series-based models, such as the univariate autoregressive integrated moving average (ARIMA) model and the multivariate vector autoregressive (VAR) models to explain Bitcoin's price dynamics.

From the perspective of the Bitcoin price metric, there has been a rise in the Bitcoin price since the mining of the genesis block in early 2009. Specifically, the price increased from $1 to $10 to $100 to below $1000 by April 2011, August 2012, April 2013, and the end of 2016,

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respectively, and it has been hovering at $10,000 since July 2020. From the market capitalization perspective, Bitcoin has staged only 10% of the time horizon from our study period. This study captures the period when Bitcoin exceeded a market capitalization of US$100 bn computed based on the total number of mined coins, which holds significance for the financial market.

Despite Bitcoin's price and market capitalization history and its higher return/volatility characteristics as an emerging asset class (Figure 1), previous studies have failed to analyze Bitcoin price in relation to the macro, financial and economic variables. Most of the Bitcoin price analysis, in mid-2010, have reported the significant relationship between the Bitcoin price and the factors (e.g. search volume and media exposure) extracted from the social network service-based statistics. However, they have paid little attention to the causality between the macroeconomic variables. Despite functioning as an alternative investment asset class facilitating portfolio diversification and yield enhancement, studies have produced inconsistent outcomes on the relationship between the Bitcoin price and the macroeconomic variables explaining the rapid price appreciation and market adoption (see Figure 2).

Based on the previous Bitcoin research, we adopt the following approach to analyze the Bitcoin price. First, we extend the data observation period to early 2022 when Bitcoin was more exposed to financial market interactions. Second, we interpret data using the VAR and vector error correction model (VECM) and the impulse response function (IRF) to analyze the explanatory power among the macroeconomic variables. Given that most of the explanatory variables are integrated of order one (I(1)), we conduct a cointegration test to determine the applicability of VECM. The results lead to the selection of the VECM over the VAR. Third, under the general VECM structure, the cointegration equation shows a long-run equilibrium with a constant coefficient (termed "time-invariant cointegration" [TIC]). However, this study presents a VECM model based on a time-varying coefficient (termed "time-varying cointegration" [TVC]). To check the significance of the TVC VECM model against the general TIC VECM, we test the null hypothesis using the likelihood ratio (LR) test across multiple TVC parameters. Based on the results, we propose an optimal TVC parameter based on information criteria.

The remaining paper is organized as follows. Chapter 2 reviews the literature review, and Chapter 3 presents the data and methodology. Chapter 4 estimates the TIC VECM and TVC VECM and explains the hypothesis used to report the optimal model parameter. Chapter 5 concludes the study.

2. Literature review

Based on the correlation between the Bitcoin price and the macroeconomic variables, Son and Kim (2019) presented Bitcoin as a safe-haven asset during high-volatility periods. The study considered the period from January 2019 to June 2019. It sourced data on the Bitcoin price (Korean Won) from Bithumb and on the macroeconomic variables from the Federal Reserve Economic Data of the Federal Reserve Bank of St. Louis. The results showed a low correlation between gold and the Bitcoin price. They did not show Granger causality between the price and the six macroeconomic variables – the Bitcoin price, Standard and Poor's 500 volatility index, US treasury 10-year yield, US consumer price index, gold price and dollar index.

Lee *et al.* (2019a, b) empirically analyzed the determinants of the Bitcoin price in the Korean market for the period July 2015–May 2018. They classified the pricing factors into three groups composed of 12 variables – supply/demand (7), real economic (4), and psychological factors (1) – and examined the effect of these factors on the Bitcoin price. They employed five regression analysis models. The results showed a significant relationship of the price only with one psychological variable extracted from the Naver Trend Index. They also showed a significant negative relationship between the net stock purchase from the retail

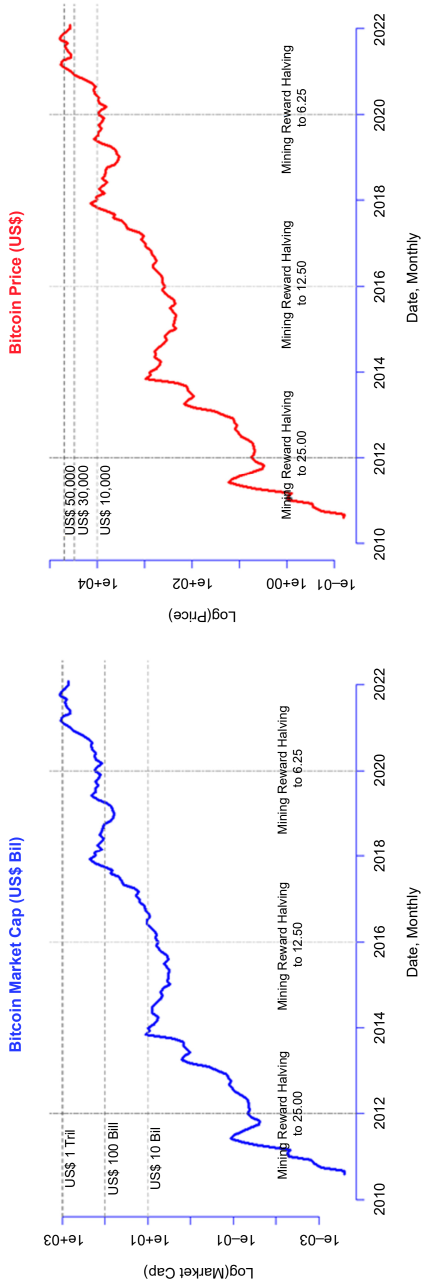

Figure 1.
Bitcoin market capitalization (left) and price (right). Chart (*Y*-axis: log scale)

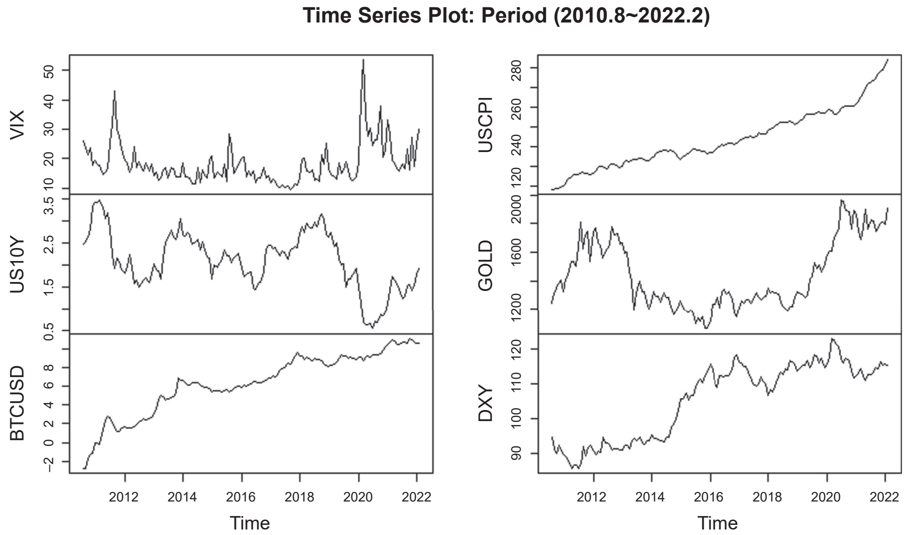

Figure 2.
Underlying variables – historical chart (data period: August 2010 to February 2022; monthly data: 139 observations). Bitcoin price (BTCUSD) is denoted by log (BTCUSD)

investors and the price. This finding implied the role of Bitcoin as an alternative investment from retail investors' standpoint.

Kim *et al.* (2019) analyzed the mutual relationship between Bitcoin and real, speculative and currency assets using VECM and daily data from July 2010 to April 2018. They found that Bitcoin is more closely associated with the speculative than with real or currency assets.

Bouoiyour and Selmi (2015) analyzed the Bitcoin pricing factors using the autoregressive distributed lag (ARDL) bounds test with data spanning from May 2010 to July 2014. They showed Bitcoin's strong speculative characteristics and its weak role as a safe-haven asset.

Zhu *et al.* (2017) performed Bitcoin analysis using VECM. From 2011 to 2016, they used monthly data on macroeconomic variables such as the price index, dollar index, Dow Jones industrial average, index, Fed fund rate, gold price and bitcoin price. They showed that the economic variables have explanatory power to Bitcoin price on a long-term basis. They also that the dollar index most significantly influenced the Bitcoin price movement, while gold had the least influence. They concluded that the Bitcoin as a speculative asset may not be a stable currency.

Thaker and Mand (2021) examined the relationship between the Bitcoin price and the Asian stock market index using the VECM, generalized autoregressive conditional heteroskedasticity in the mean-GARCH and wavelet analysis. They found a long-term significant relationship between the Bitcoin price and the stock market index.

Syafiqah and Mohamad (2021) included various cryptocurrencies (e.g. Bitcoin, Monero and Stella) to model the long-term relationship between cryptocurrencies and economic variables, using VECM and monthly data from January 2016 to December 2020. They found that only Monero had a significant long-run relationship with macroeconomic variables.

Yang *et al.* (2020) analyzed the price discovery relationship between the Bitcoin spot and futures price employing the time-varying Granger causality, cointegration and information sharing approaches. They found that the Bitcoin Futures price granger causes the Bitcoin spot price and that the former contributes more to the price discovery function than the latter.

This study is different from the previous research in the following aspects. First, it takes a longer data observation period of 10 years covering the US$1 trillion market capitalization period. Second, it expands the general VECM used in the initial stage of the research to include the TVC VECM in order to capture the dynamic nature of the long-term equilibrium relationship embedded in the VECM.

In summary, until 2015, the research used the GARCH and ARDL to determine the relationship between the Bitcoin price and the macroeconomic variables. Since 2017, studies have been using the VECM and TVC for exploring the long-term relationship between the variables of interest. However, this study expands the Bitcoin price model based on the TVC VECM to explain the dynamic long-run equilibrium between the Bitcoin price and the macroeconomic variables.

3. Methodology

3.1 Analysis flow

Despite the short history and idiosyncratic nature of its functioning, Bitcoin has been significantly influencing the financial markets. This background motivates our Bitcoin price analysis based on the VECM approach. In this context, we consider the dynamic nature of the macroeconomic variables and examine the long-run relationship by expanding the VECM to include the TVC approach. To test the significance of the TVC approach, we set a null hypothesis. This is because of the same cointegration coefficient relationship between the TIC VECM and the TVC VECM. We estimate the coefficient of the TVC VECM using the Chebyshev polynomials. We also propose an optimal TVC VECM under any Chebyshev polynomial specification, based on various information criteria outcomes (Bierens and Martins, 2010).

We employ the following steps to apply the VECM. First, we check the stationarity of the VAR variables using the augmented Dickey–Fuller (ADF) tests. Based on the test, we configure the variables as an I(0) or I(1) variable. For an appropriate lag selection for the ADF test, we use the Akaike information criterion (AIC) test. Second, we apply the VAR estimation with numerous information criteria outcomes to select a VAR lag model including the intercept adoption. Third, we estimate the VAR (p = lag) model. Fourth, given that most of the macroeconomic variables are I(1), we conduct Johansen tests. This helps us to set up the model correctly in the presence of cointegration among I(1) variables. Fifth, after confirming the presence of cointegration, we set the VECM model as the analysis model. We validate the model from various standpoints by employing additional tests such as the test for the significance of the model coefficient and the test for the characteristics of the residuals generated from the model. The test for residuals included the tests for heteroscedasticity, autocorrelation of residuals and normality of residuals. Sixth, based on the estimated VECM model IRF and the forecast error variance decomposition (FEVD), we explain the relationship among the selected variables. Finally, we adjust for the time-varying long-run relationship of the cointegration equation by using the Chebyshev polynomials. This included the following steps. First, we estimate the TVC VECM model (Bierens and Martins, 2010). Second, we run a hypothesis of the TIC VECM over the TVC VECM using an LR test. Third, we choose optimal the TVC VECM model with some Chebyshev polynomials of (m) to illustrate the trend of each TVC VECM long-run cointegration parameter alongside the polynomial (m).

3.2 Data

The dataset comprised six macroeconomic variables (https://data.nasdaq.com) sampled monthly from August 2010 to February 2022. Table 1 presents the summary statistics of these variables.

The six economic variables are the Bitcoin price (in US dollars; BTCUSD), Standard and Poor's (S&P) 500 volatility index (VIX), US treasury 10-year yield (US 10Y), US consumer price index (US CPI), gold price and dollar index. VIX derived from option price of the S&P500 index is a volatility index representing broad market uncertainties moving in the opposite direction of the stock market or risky asset. US10Y standing for the US treasury's 10-year yield is regarded as the constant maturity treasury (CMT). Rate operates as a safe asset during high volatility periods and absorbs the inflationary pressure with rising yields. BTCUSD is the log of the Bitcoin price, USCPI stands for the US CPI, GOLD stands for the gold price in US$, and DXY represents the dollar index.

3.3 VECM and cointegration

As the starting point for the multivariate analysis, we use VAR, where this model assumes the stationarity of the times series dataset. If there are k variables for the analysis with time lag = p (VAR (lags = p)), the VAR model can be defined as below (Sims, 1980).

$$Y_t = c + \Gamma_1 Y_{t-1} + \Gamma_2 Y_{t-2} + \cdots + \Gamma_p Y_{t-p} + \varepsilon_t \ where\ t = 1, \ldots, T \tag{1}$$

From Equation (1), Y_t denotes the value of Y at t as the vector of kx1; Y_{t-1} depicts the value of Y_t with the time lag = 1; c denotes the constant value with vector kx1; Γ_i represents a parameter vector of kxk with tune lag of $i = 1, \ldots, p$ and ε_t is a white noise with vector of kx1.

Given that VAR(p) model assumes the stationarity of the time series variables, if these variables fail to achieve stationarity at the raw level during the analysis, we differentiate the time series until the time series achieves stationarity and facilitates the application of VAR(p) model. However, the stationarity of the underlying variables can be obtained by differentiating the nonstationary time series at the expense of the cointegration information among variables. Hence, we use the Johansen test to check for the presence of cointegration among variables. This helps in validating the eligibility of the VECM.

Using the test results in Table 2, we conduct the ADF to test the time series stationarity (ADF lags were chosen based on the AIC).

According to ADF test result, all variables, except for USCPI, were reported as nonstationary, under no intercept condition (test regression: none). When we included the Trend term, the ADF test reported USCPI and DXY as nonstationary time series. The first differencing of all variables led to stationarity in the time series, except for USCPI with the Trend condition.

From Equation (1), if the solution of $\det(I_k - \Gamma_1 z - \ldots - \Gamma_p z^p) \neq 0$ for $|z| \leq 1$ with $z = 1$ case have an unit root, all or part of variables of VAR(p) model can be I(1) variables. Under this case, the VECM could be appropriate (Pfaff, 2008).

The long-run relationship between the Bitcoin and the macroeconomic variables under VECM can be explained by the linear cointegration vector. In this case (transitory), the VECM can be represented as follows (Engle and Granger, 1987).

	VIX	US10Y	BTCUSD	USCPI	GOLD	DXY
# of Observation	139	139	139	139	139	139
Max	9.5100	0.5500	−2.7806	218.3120	1060.0000	85.5999
Min	53.5400	3.4700	11.0305	283.7160	1964.9000	122.8165
Mean	18.3071	2.1023	6.3133	243.5492	1438.6662	105.3285
Median	16.4800	2.1600	6.4841	240.2360	1326.5000	110.0848
Stdev	6.9079	0.6448	3.2873	14.7723	238.3072	10.9168
Skewness	1.9509	−0.2706	−0.7036	0.5648	0.5271	−0.3695
Kurtosis	5.2782	−0.2170	−0.2192	−0.2483	−1.0416	−1.4860

Table 1.
Summary statistics

Table 2. Results of augmented Dickey–Fuller and test regression with none condition

	VIX	US10Y	BTCUSD	USCPI	GOLD	DXY
A. Level						
Test regression, None						
ADF(*t*-value)	−0.09	−0.49	0.74	2.53	0.39	1.00
P-value	0.928740	0.625000	0.463180	0.012767**	0.698000	0.322400
Lag (from AIC)	7	2	10	4	2	11
Test regression, Trend						
ADF(*t*-value)	−4.46	2.08	−4.44	0.53	−2.82	−1.64
P-value	0.0000212***	0.0399**	0.000024***	0.597698	0.005723***	0.1046
Lag (from AIC)	2	2	9	4	2	8

	VIX	US10Y	BTCUSD	USCPI	GOLD	DXY
B. 1st Difference						
Test regression, None						
ADF(*t*-value)	−6.69	−6.53	−2.83	0.60	−7.64	−2.83
P-value	0.0000000013***	0.0000000024***	0.005598***	0.553300	0.00000000001***	0.00571***
Lag (from AIC)	6	2	8	11	2	10
Test regression, Trend						
ADF(*t*-value)	−6.65	−6.47	−3.10	−4.44	−8.03	−3.34
P-value	0.0000000016***	0.0000000033***	0.00253***	0.000022***	0.00000000000***	0.00121***
Lag (from AIC)	6	2	9	3	2	10

Note(s): *** significant at 1% level, ** significant at 5% level and * significant at 10% level

$$\Delta Y_t = \Pi Y_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta Y_{t-j} + \varepsilon_t \textit{ where } \Delta Y_t = Y_t - Y_{t-1} \tag{2}$$

$$\Pi = \alpha\beta^{\top} = -(I - \Gamma_1 - \ldots - \Gamma_p) \tag{3}$$

From Equation (2), Γ_j stands for the transitory relationship, under the reduced rank of $\alpha\beta^{\top}$ matrix in the presence of cointegration. α means loading matrix (or weights), and β of k x r dimension matrix implies the long-run relationship among variables. r represents the number of cointegration equations.

3.4 Time varying cointegration

Utilizing the VECM from Equation (2), the TVC VECM can be modeled as follows.

$$\Delta Y_t = \Pi_t Y_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta Y_{t-j} + \varepsilon_t \textit{ where } \Pi_t = \alpha\beta_t^{\top} \tag{4}$$

Π_t is subjected to β_t , contrary to Π from Equation (2), where it depends on the time(t) and is defined as $\beta_t = f(\beta_{VIX,t}, \beta_{US10Y,t}, \beta_{BTCUSD,t}, \beta_{USCPI,t}, \beta_{GOLD,t}, \beta_{DXY,t})$. From this expression, we know that β presents TIC, while β_t includes the TVC information.

Our study uses the Chebyshev polynomials with different m (dimension) to estimate smoothly changing β_t, in line with Bierens and Martins (2010). Specifically, we approximate TVC β_t by $\beta_t(m)$, and this can be presented as follows:

$$\beta_t(m) = \sum_{i=0}^{m} \xi_i P_{i,T}(t) \tag{5}$$

where ξ_i stands for the Fourier coefficient, m denotes the order of the Chebyshev polynomials and β_t with $i(= 0, \cdots, \text{�})$ are estimated as smoothed values [1]. Substituting the above information into Equation (4), we obtain Equation (6).

$$\Delta Y_t = \alpha\left(\sum_{i=0}^{m} \xi_i P_{i,T}(t)\right)^{\top} Y_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta Y_{t-j} + \varepsilon_t \tag{6}$$

We use the LR test to test for the effectiveness of the TVC VECM over the TIC VECM. This test is expressed as below in Equation (7).

$$LR\ TVC = LR^{tvc} = -2\left[\hat{l}_T(r,0) - \hat{l}_T(r,m)\right] \tag{7}$$

We use the following values for the hypothesis test: $\hat{l}_T(r,0)$ from the TIC VECM(p), the log-likelihood value [2] from Equation (2), $\hat{l}_T(r,m)$ from the TVC VECM(p) and the log-likelihood value from Equation (4) [3].

4. Empirical analysis

4.1 VECM and cointegration

To study the Bitcoin pricing model using macro-economic variables, we test the VAR lag selection procedure without and with the constant + Trend term. We also use the following information criteria: the AIC, final prediction error (FPE), Schwarz information criterion (SIC) and Hannan–Quinn (HQ) information criterion. The AIC and FPE produce Lag = 2, while the HQ and SC produce Lag = 1 (see Table 3).

Our analysis adopted the AIC of Lag = 2 and VAR ($p = 2$) with Trend model. According to the ADF test, most of variables were I(1). Additionally, we use the Johansen test to check for the presence of cointegration. This helps in the selection of a suitable model between VAR and VECM. The Johansen test utilized the trace and Eigen values. The test confirmed the existence of a single cointegration equation; this presence is regardless of Trend term being under the 5% significance level. Table 4 summaries the outcomes of the Johansen test.

Based on the results of the Johansen test, we select the VECM ($p = 1, r = 1$) [4]. In the following sections, we present the weights (or loading matrix) and coefficient of cointegration from the VECM (see Table 5).

Concerning the significance of the loading matrix (or weights), VIX, BTCUSD and GOLD were significant at the 5% level with the Trend term. In the absence of the constant term, only VIX and GOLD were significant at the 5% level (see Table 6).

With the loading matrix and the cointegrating coefficient, we present $\Pi Y_{t-1} = \alpha\beta^{\top} Y_{t-1}$ part of the Bitcoin ($\Delta BTCUSD_t$) equation using Equation (2) (with Trend model).

None Lag	AIC(n)	HQ(n)	SC(n)	FPE(n)	Both (=Constant + Trend) Lag	AIC(n)	HQ(n)	SC(n)	FPE(n)
1	5.8999	*6.2210*	*6.6901*	365.1590	1	5.7723	*6.2004*	*6.8258*	321.5622
2	*5.7691*	6.4112	7.3494	*321.2424*	2	*5.6384*	6.3875	7.4820	*282.3844*
3	5.9645	6.9277	8.3349	393.4763	3	5.7789	6.8491	8.4126	328.0954
4	6.2795	7.5638	9.4400	547.0851	4	6.0630	7.4543	9.4869	443.6370
5	6.6174	8.2227	10.5680	786.0142	5	6.4024	8.1147	10.6164	640.8920
6	6.7399	8.6663	11.4807	922.3374	6	6.6185	8.6520	11.6227	829.9974
7	6.7850	9.0324	12.3159	1,017.9781	7	6.5402	8.8946	12.3345	814.7621
8	6.8309	9.3995	13.1520	1,147.4201	8	6.6051	9.2807	13.1896	943.1980

Table 3. Results of the VAR order selection criteria

None (lag = 2)	Null hypothesis	Test statistics	Critical value 1%	5%	Trend (lag = 2)	Null hypothesis	Test statistics	Critical value 1%	5%
Trace	$r \leq 2$	31.77	55.43	48.28	Trace	$r \leq 2$	37.62	70.05	62.99
	$r \leq 1$	52.07	78.87	70.6		$r \leq 1$	71.18	96.58	87.31
	$r = 0$	*96.37*	104.2	90.39		$r = 0$	*115.77*	124.75	114.9
Eigen	$r \leq 2$	15.87	32.14	32.14	Eigen	$r \leq 2$	16.67	36.65	31.46
	$r \leq 1$	20.31	38.78	33.32		$r \leq 1$	33.56	42.36	37.52
	$r = 0$	*44.3*	44.59	39.43		$r = 0$	*44.59*	49.51	43.97

Table 4. Results of the Johansen test

		Trend* Coefficient	t-value	Pr(>|t|)	None Coefficient	t-value	Pr(>|t|)
	VIX	*−0.31694*	*−4.13220*	*0.00010*	−0.36320	−4.42370	0.00003
	US10Y	0.00101	0.32600	0.75200	0.00076	0.22920	0.82400
ECT1**	*BTCU SD*	*0.01108*	*2.29080*	*0.02796*	0.00894	1.69930	0.10160
	USCPI	−0.00804	−0.84160	0.41600	−0.00961	−0.93280	0.36700
	GOLD	*2.88630*	*2.80290*	*0.00743*	3.22430	2.90770	0.00554
	DXY	−0.00893	−0.34750	0.73600	−0.01685	−0.60810	0.55600

Note(s): * VECM with Trend adopted from the analysis ** ECT1: error correction term (α)

Table 5. VECM loading matrix (or weights) and statistical significance test

$$\Pi Y_{t-1}\ of\ \Delta BTCUSD_t = 0.0110 * |0.2281,\ 1.0000,\ -4.5776,\ -2.7270,\ 0.3617,\ -0.0443,\ 0.6762| \begin{vmatrix} Trend_{t-1} \\ VIX_{t-1} \\ US10Y_{t-1} \\ BTCUSD_{t-1} \\ USCPI_{t-1} \\ GOLD_{t-1} \\ DXY_{t-1} \end{vmatrix}$$

F-statistics of $\Delta BTCUSD_t$ equation was estimated to be 4.322 (*P*-value: 0.0001). It implies the significance of the model equation for the Bitcoin price is significant. However, we noted a variance in the individual coefficients. For example, the constant, 1st differenced BTCUSD (lag = 1) and 1st differenced DXY (lag = 1) were significant at the 5, 1 and 10% levels, respectively (see Table 7).

Table 8 summarizes the residual analysis for the VECM (p = 1, r = 1). The heteroscedasticity test failed to reject the null hypothesis, implying the absence of an ARCH effect. The Portmanteau test (PT) for serial correlation also failed to reject the null hypothesis, implying the absence of a serial correlation. The Jarque–Bera (JB) test also rejected the null hypothesis, rejecting the normality of residuals.

4.2 Granger causality, impulse response function and forecast error variance decomposition analyses

To understand the economic relationship between variables, we employed the Granger causality test. According to the analysis, the USCPI and DXY granger causes the Bitcoin price

Table 6. VECM cointegration relationship: the coefficient (β) of cointegrating equation estimated under Trend and none assumption

		Trend*			None		
		Coefficient	*t*-value	Pr(>|*t*|)	Coefficient	*t*-value	Pr(>|*t*|)
ßeta	VIX	1.0000			1.0000		
	US10Y	*−4.5776*	*−2.0368*	*0.0420*	−3.6978	−1.8067	0.0710
	BTCUSD	*−2.7270*	*−2.4217*	*0.0150*	−1.4801	−1.9976	0.0460
	USCPI	0.3617	0.9594	0.3370	0.5122	2.3408	0.2190
	GOLD	−0.0443	−5.0238	0.0000	−0.0376	−5.1166	0.0070
	DXY	*−0.6762*	*−1.7510*	*0.0800*	−0.3418	−1.9713	0.1730
	Trend	0.2281	0.9121	0.3620			

Note(s): * VECM with Trend adopted from the analysis

Table 7. VECM (p = 1, r = 1) representation

D.BTCUSD	Trend				
	Coefficient	*t*-value	Pr(>|*t*|)		
ECT1	0.0111	2.2230	0.0280		
Const	*0.5023*	*2.5960*	*0.0105*		
D.VIX(l1)	−0.0081	−1.3130	0.1915		
D.US10Y(l1)	0.0571	0.3750	0.7085		
D.BTCUSD(l1)	*0.2279*	*2.6890*	*0.0081*	Adj *R* square	0.1625
D.USCPI(l1)	0.0469	1.2530	0.2126	*F*-statistics	4.3220
D.GOLD(l1)	−0.0007	−1.4620	0.1463	*Pr(>|t|)*	*0.0001*
D.DXY(l1)	*−0.0422*	*−1.9740*	*0.0505*		

Note(s): ECT1: error correction term (α); coefficient: short-term relationship; adjusted R square: 0.1625; F-statistics: 4.322; p-value: 0.001 (significance of ΔBTCUSD equation)

(10 and 5% significance levels, respectively). When analyzing the opposite direction, we find that only DXY granger causes Bitcoin at a 5% significance level. This outcome contradicts the previous results in Kim *et al.* (2019). Specifically, they reported a reciprocal Granger causality between VIX and the Bitcoin, while we found a mutual Granger causality between DXY and Bitcoin [5] (Table 9).

The IRF results with lag ($n = 12$) are summarized by $n = 1, 3, 5$ and 12 (left axis = impulse variable). In case of the impulse from the VIX, the Bitcoin price first moved negatively and gradually mitigated negative shocks over time. Concerning the impulse from the USCPI and US10Y, they provided a (+) positive impulse to the Bitcoin price. Concerning GOLD and DXY, these safe assets exerted a persistent (−) negative impact until $n = 5$ (roughly 5 months). In summary, Bitcoin has moved positively toward the inflation linked variables, like USCPI and USCPI, while it moved against the GOLD and DXY impulse shock (Table 10).

Concerning the outcomes of FEVD with lag ($n = 12$), they are summarized as follows. In case of VIX, GOLD and Bitcoin accounted for 19.3 and 2.2%, respectively, each at $n = 12$ (at the 12th month). For Bitcoin at $n = 12$, Bitcoin, DXY and GOLD accounted for 89.6, 3.7 and 2.1%, respectively. For DXY, when n increases from 1 to 12, the explanatory weights of GOLD decreased from 21 to 12.8%. However, the explanatory power of DXY, USCPI and GOLD increased from 52 to 55.9, 2.0–2.9, and 0.2–1.7%, respectively. The explanatory power of GOLD declined from 85.6 to 57.5%, when n moved from 1 to 12. However, US10Y to GOLD increased from 8.3 to 16.6%, USCPI to GOLD increased from 4.6 to 11.0% and Bitcoin to GOLD also increased from 1.0 to 2.7% (Table 11).

4.3 Time-varying cointegration test

Given the dynamic relationship between the Bitcoin price and the macroeconomic time series, it would be appropriate to consider the varying degree of their long-run relationship. This means that a time-varying approach should replace a constant coefficient equation from the general VECM. Hence, we develop the TVC VECM and test the feasibility of this expansion using the LR test. The cointegration rank for both the TIC and TVC VECM was set at $r = 1$ (see Table 12).

(1) Null hypothesis: The TIC VECM and TVC VECM share the same cointegration coefficient relationship across the Chebyshev polynomials $m = 1, 2, 3$ and 4

VECM($r = 1$)	Trend (Y/N)	Multivariate ARCH (lags = 5)	*P* value	Portmanteau test (asymptotic, lags = 16)	*P* value	JB normality test	*P* value
$p = 1$	Y	2251.6	0.23990	527.33	0.28860	370.2	2.20E-16

Table 8. Model diagnostic statistics using ARCH, PT and JB

	Caused	BITCUSD *F*-value	Pr(>|*t*|)		Causing	BITCUSD *F*-value	Pr(>|*t*|)
Causing	VIX	1.38310	0.25440	*Caused*	VIX	0.34470	0.70910
	US10Y	1.21360	0.30040		US10Y	1.11040	0.33250
	USCPI	2.38990	*0.09559*		USCPI	0.25840	0.77260
	GOLD	0.79880	0.45200		GOLD	0.07260	0.93000
	DXY	3.39360	*0.03654*		DXY	2.52590	*0.08384*

Table 9. Granger causality test based on *F*-statistics with lag = 2 (Wald test comparing the unrestricted with the restricted model)

Table 10. Impulse response function

	Response Variable							Response Variable					
$n = 1$	VIX	US10Y	*BTCUSD*	USCPI	GOLD	DXY	$n = 3$	VIX	US10Y	*BTCUSD*	USCPI	GOLD	DXY
VIX	4.945	−0.052	*−0.059*	−0.064	4.801	0.818	VIX	2.586	−0.071	*−0.087*	−0.408	16.131	0.883
US10Y	–	0.192	*−0.020*	0.101	−19.110	−0.097	US10Y	−0.486	0.220	*0.029*	0.310	−25.838	−0.077
BTCUSD	–	–	*0.306*	*−0.066*	*−6.621*	*−0.081*	*BTCUSD*	*0.202*	*0.021*	*0.385*	*−0.174*	*−8.916*	*−0.237*
USCPI	–	–	–	0.600	14.244	−0.234	USCPI	0.113	−0.013	*0.042*	1.148	20.474	−0.275
GOLD	–	–	–	–	61.415	−0.759	GOLD	1.039	−0.023	*−0.048*	0.118	48.074	−0.612
DXY	–	–	–	–	–	1.195	DXY	0.133	0.004	*−0.072*	−0.111	−11.656	1.276

IRF outcome (vertical: impulse variables), n = number of time lapse

	Response Variable							Response Variable					
$n = 5$	VIX	US10Y	*BTCUSD*	USCPI	GOLD	DXY	$n = 12$	VIX	US10Y	*BTCUSD*	USCPI	GOLD	DXY
VIX	1.809	−0.058	*−0.054*	−0.505	19.529	0.891	VIX	1.433	−0.050	*−0.030*	−0.537	21.359	0.882
US10Y	−0.383	0.220	*0.037*	0.384	−26.297	−0.088	US10Y	−0.293	0.218	*0.036*	0.416	−26.499	−0.091
BTCUSD	*0.397*	*0.021*	*0.378*	*−0.201*	*−10.93*	*−0.239*	*BTCUSD*	*0.493*	*0.019*	*0.370*	*−0.214*	*−11.74*	*−0.234*
USCPI	0.226	−0.017	*0.055*	1.313	21.969	−0.298	USCPI	0.337	−0.019	*0.057*	1.382	22.184	−0.306
GOLD	1.240	−0.027	*−0.060*	0.140	47.007	−0.601	GOLD	1.304	−0.029	*−0.065*	0.146	46.743	−0.599
DXY	0.173	0.004	*−0.080*	−0.143	−12.647	1.291	DXY	0.161	0.004	*−0.082*	−0.157	−12.748	1.293

Table 11. Forecast error variance decomposition by dependent variables

Dependent variable	VIX						US10Y						BTCUSD					
n =	VIX	US10Y	*BTCUSD*	USCPI	GOLD	DXY	VIX	US10Y	*BTCUSD*	USCPI	GOLD	DXY	VIX	US10Y	*BTCUSD*	USCPI	GOLD	DXY
1	100.0%	0.0%	*0.0%*	0.0%	0.0%	0.0%	6.8%	93.2%	*0.0%*	0.0%	0.0%	0.0%	3.5%	0.4%	*96.0%*	0.0%	0.0%	0.0%
2	96.6%	0.5%	*0.0%*	0.0%	2.8%	0.1%	9.7%	89.6%	*0.3%*	0.1%	0.3%	0.0%	5.7%	0.2%	*92.0%*	0.3%	0.5%	1.4%
3	94.1%	1.0%	*0.1%*	0.0%	4.7%	0.1%	9.5%	89.3%	*0.5%*	0.2%	0.6%	0.0%	5.3%	0.3%	*90.9%*	0.6%	0.8%	2.1%
4	91.4%	1.2%	*0.3%*	0.1%	6.9%	0.2%	9.0%	89.5%	*0.6%*	0.2%	0.7%	0.0%	4.6%	0.4%	*90.4%*	0.9%	1.1%	2.5%
5	88.8%	1.3%	*0.5%*	0.2%	8.9%	0.2%	8.4%	89.7%	*0.6%*	0.3%	0.9%	0.0%	4.0%	0.5%	*90.1%*	1.1%	1.4%	2.8%
6	86.4%	1.4%	*0.8%*	0.3%	10.9%	0.2%	8.0%	90.0%	*0.7%*	0.3%	1.0%	0.0%	3.5%	0.6%	*90.0%*	1.3%	1.6%	3.1%
7	84.1%	1.5%	*1.1%*	0.4%	12.7%	0.3%	7.6%	90.3%	*0.7%*	0.4%	1.1%	0.0%	3.1%	0.6%	*89.9%*	1.4%	1.7%	3.3%
8	82.0%	1.5%	*1.4%*	0.5%	14.3%	0.3%	7.3%	90.5%	*0.7%*	0.4%	1.1%	0.0%	2.8%	0.7%	*89.8%*	1.5%	1.8%	3.4%
9	80.2%	1.6%	*1.6%*	0.6%	15.7%	0.3%	7.0%	90.6%	*0.7%*	0.5%	1.2%	0.0%	2.6%	0.7%	*89.7%*	1.5%	1.9%	3.5%
10	78.5%	1.6%	*1.8%*	0.7%	17.1%	0.3%	6.8%	90.8%	*0.7%*	0.5%	1.2%	0.0%	2.4%	0.7%	*89.7%*	1.6%	2.0%	3.6%
11	77.0%	1.6%	*2.0%*	0.8%	18.2%	0.3%	6.6%	90.9%	*0.7%*	0.5%	1.3%	0.0%	2.2%	0.7%	*89.7%*	1.6%	2.1%	3.7%
12	75.6%	1.6%	*2.2%*	0.9%	19.3%	0.3%	6.5%	91.0%	*0.7%*	0.5%	1.3%	0.0%	2.1%	0.7%	*89.6%*	1.7%	2.1%	3.7%

Dependent variable	USCPI						GOLD						DXY					
n =	VIX	US10Y	*BTCUSD*	USCPI	GOLD	DXY	VIX	US10Y	*BTCUSD*	USCPI	GOLD	DXY	VIX	US10Y	*BTCUSD*	USCPI	GOLD	DXY
1	1.1%	2.7%	*1.2%*	95.1%	0.0%	0.0%	0.5%	8.3%	*1.0%*	4.6%	85.6%	0.0%	24.4%	0.3%	*0.2%*	2.0%	21.0%	52.0%
2	5.8%	4.4%	*1.7%*	87.1%	0.7%	0.3%	1.0%	14.3%	*1.0%*	6.7%	74.9%	2.2%	26.1%	0.2%	*1.0%*	2.0%	16.2%	54.4%
3	8.1%	5.2%	*1.8%*	83.6%	0.8%	0.5%	2.9%	15.3%	*1.4%*	8.0%	69.8%	2.6%	26.3%	0.2%	*1.3%*	2.2%	15.1%	54.9%
4	9.3%	5.7%	*1.8%*	81.8%	0.8%	0.7%	4.1%	15.8%	*1.7%*	9.0%	66.4%	3.0%	26.5%	0.2%	*1.5%*	2.4%	14.3%	55.1%
5	10.0%	6.0%	*1.8%*	80.7%	0.8%	0.7%	5.2%	16.1%	*1.9%*	9.6%	64.0%	3.2%	26.5%	0.2%	*1.6%*	2.5%	13.9%	55.3%
6	10.4%	6.2%	*1.8%*	80.0%	0.8%	0.8%	6.1%	16.3%	*2.1%*	10.0%	62.2%	3.3%	26.5%	0.2%	*1.6%*	2.6%	13.6%	55.4%
7	10.7%	6.3%	*1.8%*	79.5%	0.9%	0.8%	6.7%	16.4%	*2.3%*	10.3%	60.9%	3.4%	26.5%	0.2%	*1.7%*	2.7%	13.4%	55.6%
8	10.8%	6.4%	*1.8%*	79.2%	0.9%	0.9%	7.3%	16.4%	*2.4%*	10.5%	59.9%	3.5%	26.5%	0.2%	*1.7%*	2.7%	13.2%	55.7%
9	11.0%	6.5%	*1.8%*	78.9%	0.9%	0.9%	7.7%	16.5%	*2.5%*	10.7%	59.1%	3.5%	26.5%	0.2%	*1.7%*	2.8%	13.1%	55.7%
10	11.1%	6.6%	*1.8%*	78.7%	0.9%	0.9%	8.0%	16.5%	*2.6%*	10.8%	58.5%	3.6%	26.4%	0.2%	*1.7%*	2.8%	13.0%	55.8%
11	11.1%	6.6%	*1.8%*	78.6%	0.9%	0.9%	8.3%	16.6%	*2.7%*	10.9%	57.9%	3.6%	26.4%	0.2%	*1.7%*	2.8%	12.9%	55.9%
12	11.2%	6.7%	*1.8%*	78.5%	0.9%	0.9%	8.5%	16.6%	*2.7%*	11.0%	57.5%	3.6%	26.4%	0.3%	*1.7%*	2.9%	12.8%	55.9%

The tests yield the following results. First, when Chebyshev polynomials (m) = 1, the null hypothesis was not rejected at a 10% significance level (P value = 0.18). Second, when Chebyshev polynomial of $m = 2$ or $m > 3$, the null hypothesis was rejected (P value = 0.00152 for $m = 2$, P value = 0.00233 for $m = 3$, P value = 0.00019 for $m = 4$), with TVC revealed visually. When we applied the HQ information criteria, the optimal Chebyshev polynomials m were 2 (Bierens and Martins, 2010).

When we used the Chebyshev polynomial of $m = 1$, the null hypothesis was not rejected. As shown in Figure 3, the TVC parameters have shown like constant numbers across the time horizon. At this point, we believe that they are similar to the TIC VECM cointegration parameters (Beta chart with $m = 1$). Hence, the null hypothesis holds.

When we use the Chebyshev polynomial of $m = 2$, the null hypothesis was rejected, implying the significance of the TVC parameters. Figure 4 shows the smooth and gradual

Table 12.
Summary results on time-varying cointegration

Model	*VECM(p = 1, r = 1)* $m = 1$	$m = 2$	$m = 3$	$m = 4$
LR TVC statistics	8.87395	31.73887	39.65449	56.63646
P-value	0.18079	*0.00152****	*0.00233****	*0.00019****
Log likelihood	−1511.23162	−1499.79916	−1495.84136	−1487.35037
AIC	22.70411	22.59561	22.82980	22.76424
BIC	23.64192	23.61867	24.27913	24.29883
HQC	23.08521	23.01135	23.41877	23.38786

Note(s): *** significant at 1% level, ** significant at 5% level and * significant at 10% level

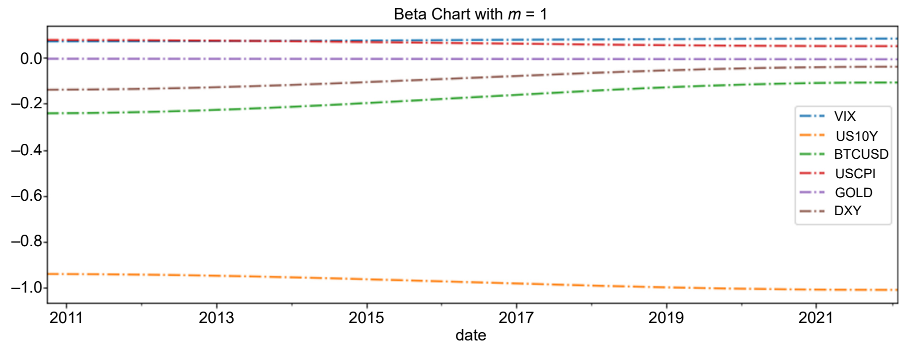

Figure 3.
Chebyshev polynomials with $m = 1$

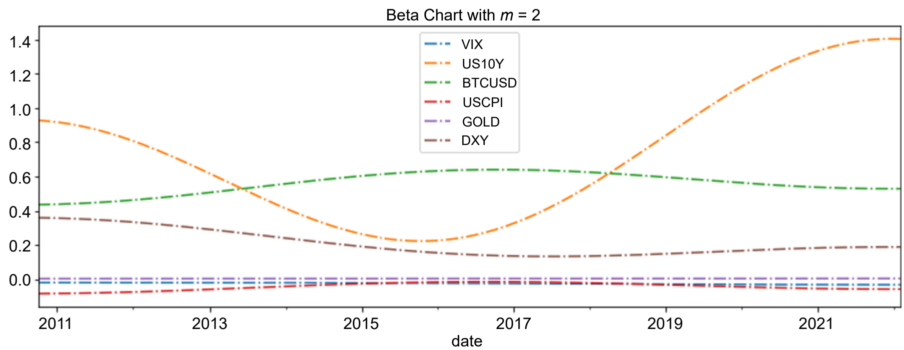

Figure 4.
Chebyshev polynomials with $m = 2$

evolution of the long-run relationship coefficient of three economic variables, including US10Y.

While we have shown the coefficient of the long-run relationship using the Chebyshev polynomials of m (see Appendix 1 for $m = 3$ and 4), the following Figures 5–7 exhibit the $\Pi Y_{t-1} = \alpha\beta^{\top} Y_{t-1}$ (cointegration relationship) under the VECM. Figure 5 (cointegration relation with nondeterministic term) shows a cointegration relationship under the TIC VECM, and Figures 6 and 7 show a cointegration relationship ($m = 1$,2) under the TVC VECM [6].

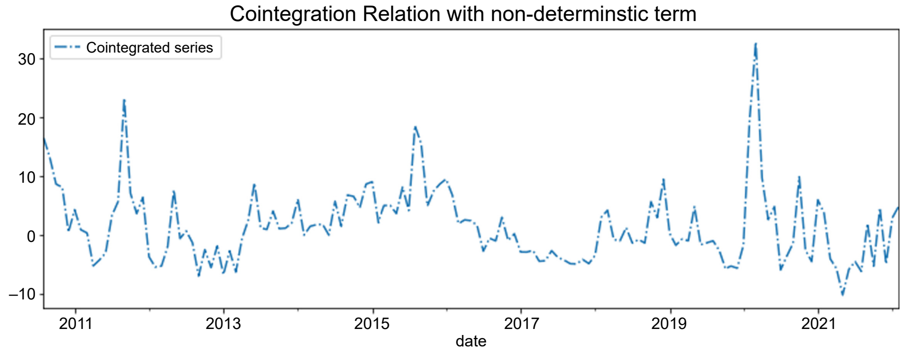

Figure 5. Cointegration relationship of Chebyshev polynomials with $m = 0$

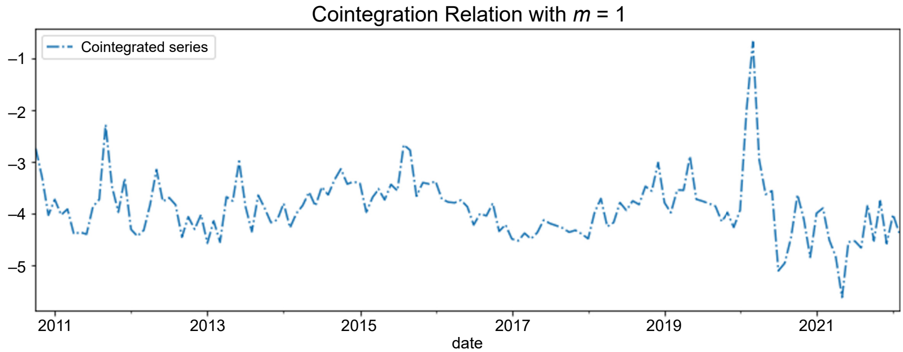

Figure 6. Cointegration relationship of Chebyshev polynomials with $m = 1$

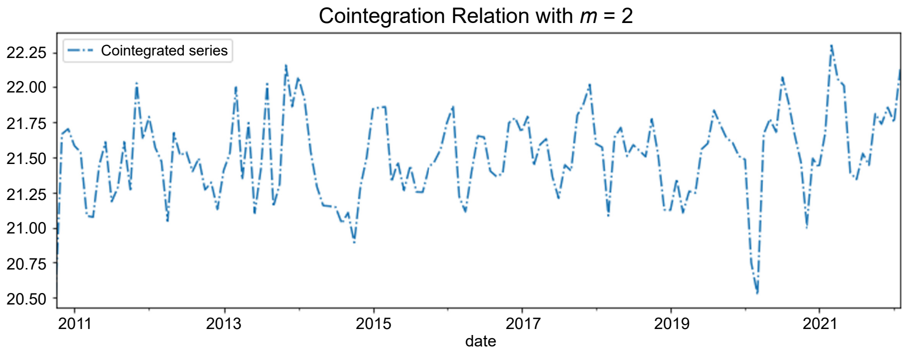

Figure 7. Cointegration relationship of Chebyshev polynomials with $m = 2$

5. Conclusion

This study analyzes the Bitcoin price in relation to six macroeconomic variables studied over a 10-year time horizon, using the VECM. Most studies have reported the speculative characteristics of Bitcoin owing to its short history and association with the financial market. Hence, we use a period when Bitcoin was more exposed to the financial market interactions, that is, when it achieved and crossed a market capitalization of US$1 tn. We also construct a VAR/VECM model suitable for capturing the interrelationship among the sampled macroeconomic variables. We validate these relationships using the time series stationary test and an optimal lag selection. We also test for the long-run cointegration both under the TIC VECM ($p = 1, r = 1$) and TVC VECM. Given the dynamic nature of the macroeconomic variables, based on the results of the LR test, we use a TVC VECM for Bitcoin modeling.

This study has implications in demonstrating that the emerging digital assets, including Bitcoin, would prevail and consolidate. Given this, it is important to analyze the dynamic long-run relationship between digital assets and macroeconomic variables from a modeling perspective. This study has used the time-varying parameter approach for long-run cointegration from the VECM. However, the future studies may apply the time-varying approach to loading factor.

Notes

1. See Appendix 2
2. This is the case when Chebyshev polynomials $m = 0$
3. For the maximum likelihood estimates and LR TVC, refer to Bierens and Martins (2010)
4. As the VAR model was estimated with lag = 2 (VAR ($p = 2$)), the VECM model is presented as lag = 1 (VECM ($p = 1$)).
5. Kim *et al.* (2019) used the daily data from July 2010 to April 2018, while used monthly data from August 2010 to Feb 2022.
6. Cointegration Relation with $m = 3,4$ were reported at Appendix 3 and normalization in charts was based on the coefficient of BTCUSD.

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Appendix 1

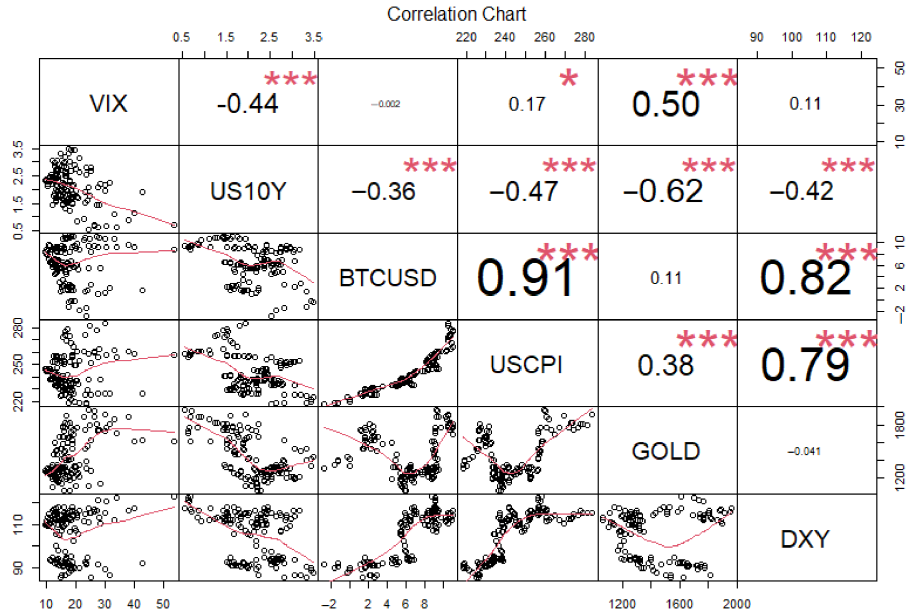

Figure A1.
Underlying variables – correlation statistics (Data period: Aug-2010 to Feb-2022, monthly data, 139 observations)

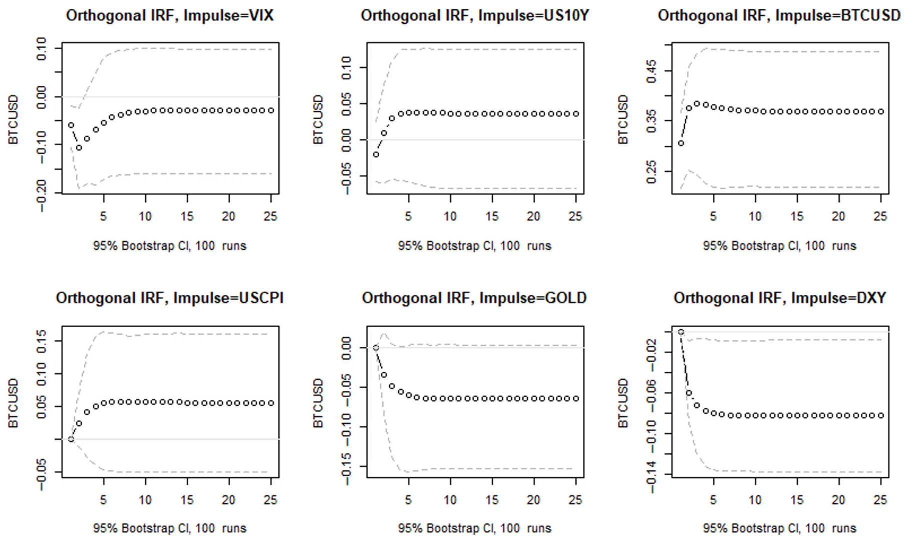

Figure A2.
VECM ($p = 1, r = 1$) impulse response function (Response = BTCUSD)

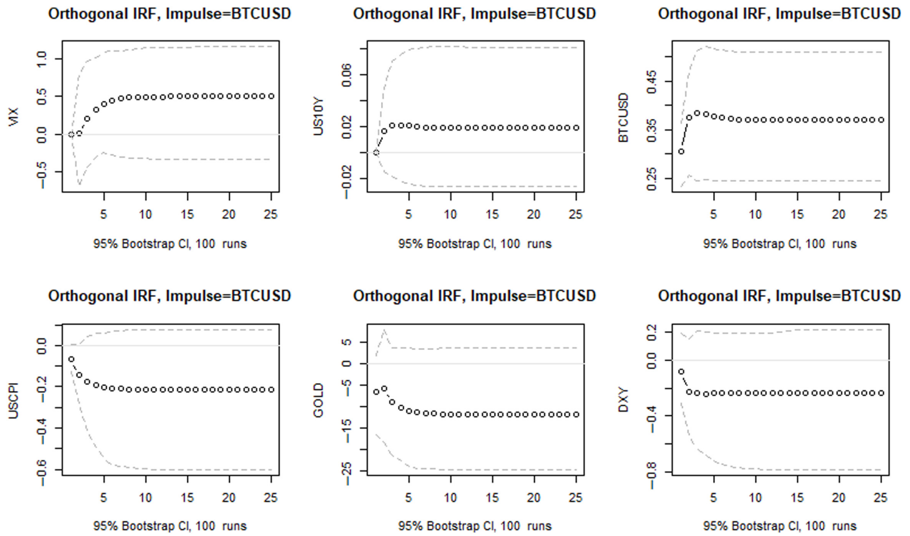

Figure A3.
VECM ($p = 1, r = 1$) impulse response function (Impulse = BTCUSD)

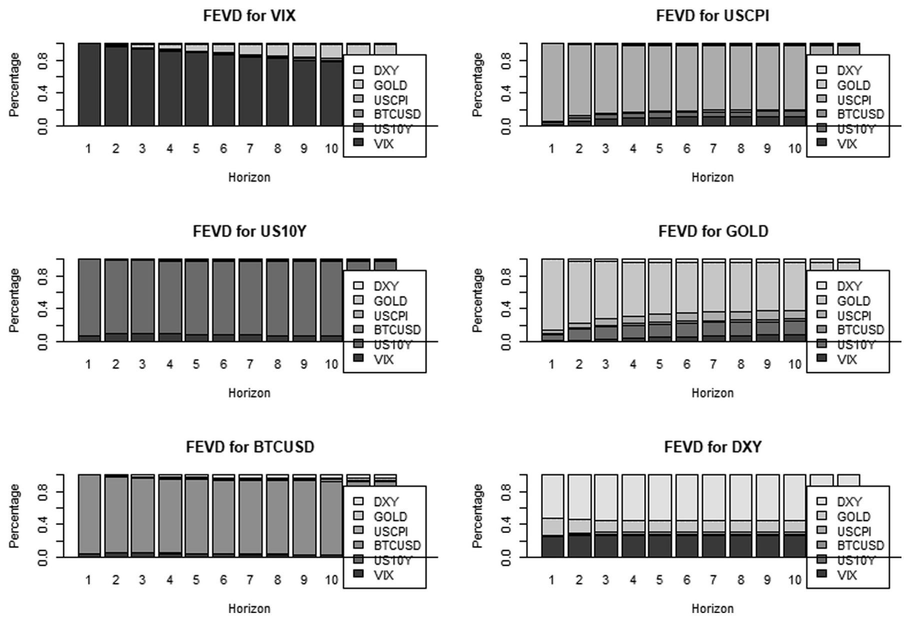

Figure A4.
VECM ($p = 1, r = 1$) FEVD chart representation from the main paper

Appendix 2

Orthonormal Chebyshev Polynomials $P_{i,T}(t)$ can be presented as below,

$$P_{i,T}(t) = \sqrt{2}\cos\left(\frac{i\pi(t-0.5)}{T}\right), \quad P_{0,T}(t) = 1, \quad t = 1,2,\ldots,T, \quad i = 1,2,3,.. \tag{A}$$

Using and substituting $\beta_t \sim \beta_t(m) = \sum_{i=0}^{m} \xi_i P_{i,T}(t)$ into TVC VECM equation (6) from main article results in

$$\Delta Y_t = \alpha\left(\sum_{i=0}^{m} \xi_i P_{i,T}(t)\right)^{\top} Y_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta Y_{t-j} + \varepsilon_t \tag{B}$$

With respect to arbitrary kxr matrix ξ_i (B) equation can be arranged as follows,

$$\Delta Y_t = \alpha \xi^{\top} Y_{t-1}^{(m)} + \Gamma X_t + \varepsilon_t \tag{C}$$

$$where\ \xi = \left(\xi_0^{\top}, \xi_1^{\top}, \ldots, \xi_m^{\top}\right)\ is\ an\ r*(m+1)*k\ matrix\ of\ rank\ r$$

Let $Y_{t-1}^{(m)} = \left(Y_{t-1}^{\top}, P_{1,T}(t)Y_{t-1}^{\top}, P_{2,T}(t)Y_{t-1}^{\top}, \ldots, P_{m,T}(t)Y_{t-1}^{\top}\right)^{\top}$ and $X_t = \left(\Delta Y_{t-1}^{\top}, \ldots, \Delta Y_{t-p+1}^{\top}\right)^{\top}$ then,

Coefficient β of TIC (time invariant cointegration) for null hypothesis is derived from $\xi^{\top} Y_{t-1}^{(m)} = \beta^{\top} Y_{t-1}^{(0)}\ with\ Y_{t-1}^{(0)} = Y_{t-1}$ which means $\xi^{\top} = \left(\beta^{\top},\ O_{r,k,m}\right)$

Appendix 3

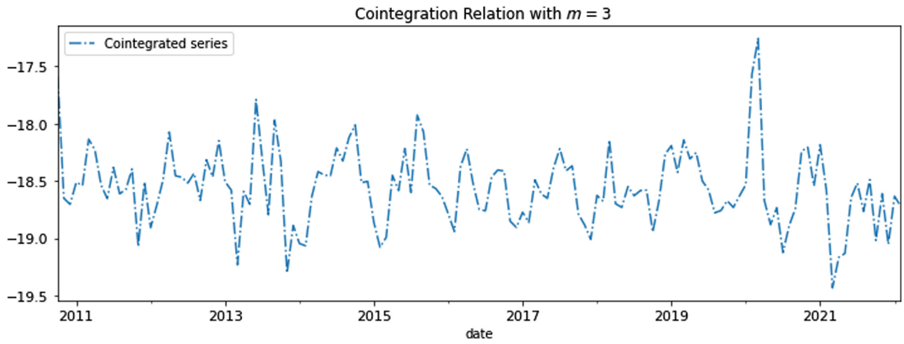

Figure A5. Cointegration relation (with Chebyshev polynomials of $m = 3$)

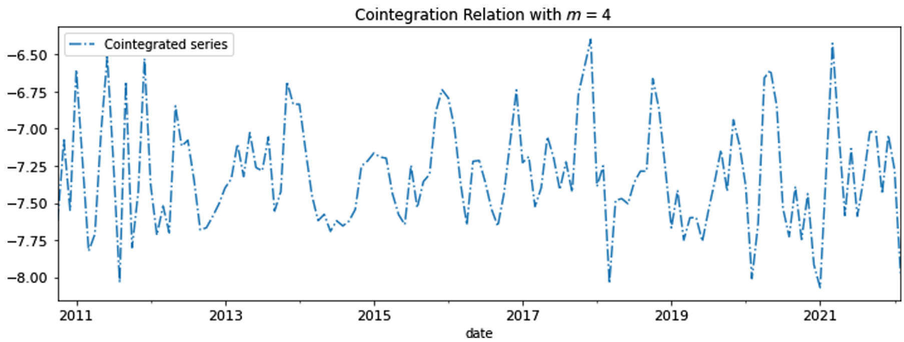

Figure A6. Cointegration relation (with Chebyshev polynomials of $m = 4$)

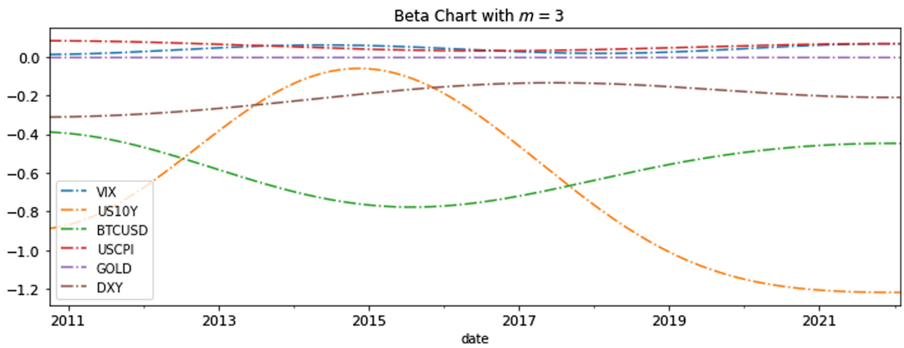

Figure A7. Cointegration beta (with Chebyshev polynomials of $m = 3$)

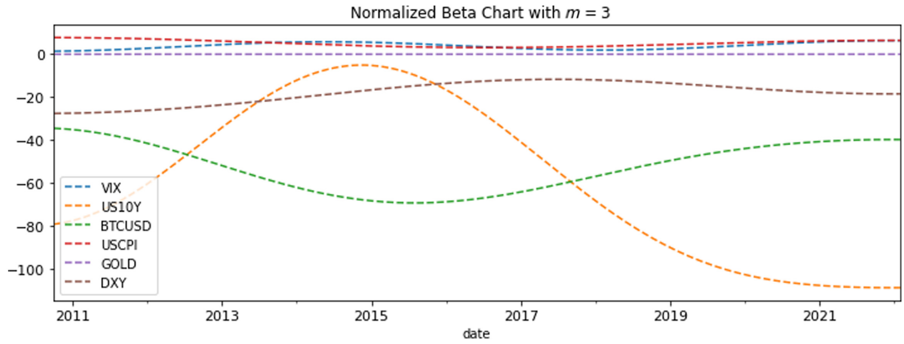

Figure A8. Cointegration beta – normalized on BTCUSD (with Chebyshev polynomials of $m = 3$)

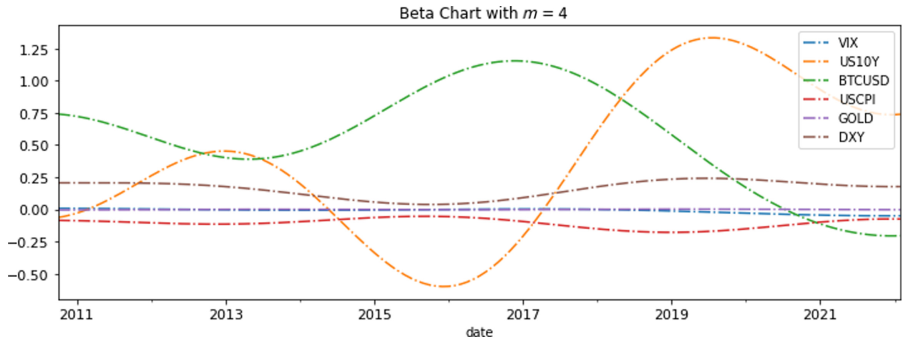

Figure A9. Cointegration beta (with Chebyshev polynomials of $m = 4$)

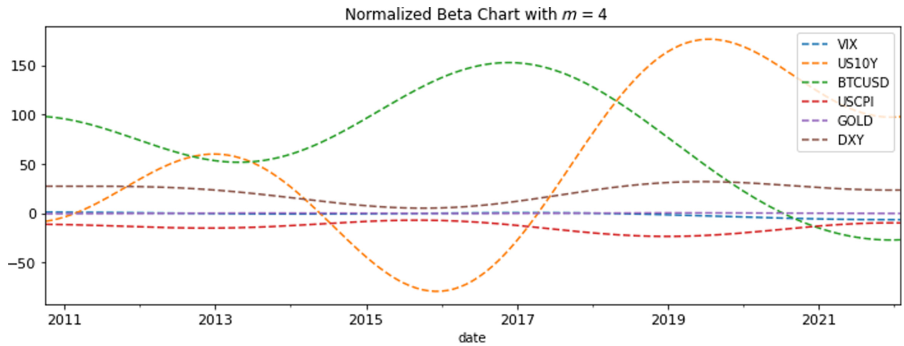

Figure A10. Cointegration beta – normalized on BTCUSD (with Chebyshev polynomials of $m = 4$)

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