Determining the best ship loading strategy during military deployments

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Abstract

Purpose – This study aims to examine ship loading strategies during large-scale military deployments. Ships are usually loaded to a stowage goal of about 65% of the ship's capacity. The authors identify how much cargo to load onto ships for each sailing and propose lower stowage goals that could improve the delivery of forces during the deployment.

Design/methodology/approach – The authors construct several mixed integer programs to identify optimal ship loading strategies that minimize delivery timelines for notional, but realistic, problem variables. The authors study the relative importance of these variables using experimental designs, regressions, correlations and chi-square tests of the empirical results.

Findings – The research specifies the conditions during which ships should be light loaded, i.e. loaded to less than 65% of total capacity. Empirical results show cargo delivered up to 16% faster with a light-loaded strategy compared to fully loaded ships.

Research limitations/implications – This work assumes deterministic sailing times and ship loading times. Also, all timing aspects of the problem are estimated to the nearest natural number of days.

Practical implications – This research provides important new insights about optimal ship loading strategies, which were not previously quantified. More importantly, logistics planners could use these insights to reduce sealift delivery timelines during military deployments.

Originality/value – Most ship routing and scheduling problems minimize costs as the primary goal. This research identifies the situations in which ships transporting military forces should be light loaded, thereby trading efficiency for effectiveness, to enable faster overall delivery of unit equipment to theater seaports.

Keywords Ship loading, Stowage goals, Military logistics, Mixed integer programming, Correlation, Regression, Chi-square tests

Paper type Research paper

Introduction

During US military deployments, large amounts of unit equipment flow from the Continental US (CONUS) to overseas locations. About 90% of the deploying unit equipment is transported on sealift vessels or, more commonly termed, ships (US Headquarters of the Navy, 2020; US Joint Chiefs of Staff, 2013b). Although airlift can transport equipment anywhere in the world within several hours, only sealift can move the millions of square feet of military equipment during contingency operations (US Joint Chiefs of Staff, 2013b). The US Transportation Command is responsible for scheduling the military's unit equipment onto a limited fleet of sealift ships that must cycle between CONUS and theater ports to deliver the deploying ground forces. Determining the amount of unit equipment to load onto ships, frequently termed the stowage goal, is an important operational problem for the US Transportation

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Command. Analysts at the command refer to this problem as the Military Ship Loading Problem (MSLP). The focus of this research is solving the MSLP using several mixed integer program (MIP) variants to obtain exact solutions. Then, we provide statistical analyses of the empirical results.

Various ship characteristics are pertinent to the MSLP. First, military deployments involve several types of sealift ships, primarily roll-on/roll-off (RO/RO) ships and containerships (US Joint Chiefs of Staff, 2005). The MSLP focuses solely on RO/RO ships, because unit equipment is transported predominantly using government-owned and commercially available RO/RO ships (US Joint Chiefs of Staff, 2013b). Second, each ship has unique characteristics in terms of speed, capacity, loading rates (i.e. how much cargo can be loaded onto the ship over time as the ship is on berth) and starting location. Ship speed and capacity are known with certainty, but loading rates and starting location are less certain. Ship loading rates depend on cargo availability at the CONUS port and port infrastructure capabilities. Ship starting locations depend primarily on whether the ship is government- or commercially owned. Government-owned ships are layberthed on the East, Gulf and West Coast of the CONUS and are typically ready within 5–10 days after being notified of a large-scale contingency (US Department of Transportation, 2021). Conversely, ships owned by commercial partners are usually actively moving cargo and thus could be anywhere in the world when a large-scale contingency begins. After being notified of the deployment, commercial ships must offload their cargo at the nearest world-wide port prior to sailing to the nearest assigned CONUS seaport, which could take up to 30 days. In addition to ship characteristics, the MSLP requires additional problem data, including the amount of unit equipment deploying, the number of berths available at CONUS seaports, the distance between CONUS and theater seaports, and the number of ships available to transport the cargo. These remaining inputs for the MSLP are described later in the paper.

Military planners use doctrinal sources and operational experience as the basis for most logistics problems, including the MSLP. First, RO/RO ships are the primary ships used to deploy initial unit cargos (e.g. tanks, towed artillery, armored fighting vehicles) because these types of cargos are rolling stock, which can be driven (vice crane-loaded) onto the ship using side or end ramps (US Joint Chiefs of Staff, 2013b). No ship loading plan will occupy 100% of the ship capacity, because some empty space is needed for bracing, tie-down, maneuvering, shape of the cargo and contour of the ship (Kurinovich, 2005). The cargo space left unoccupied after the ship is considered fully loaded is termed broken stowage (US Joint Chiefs of Staff, 2016). However, planners usually refer to the percent of ship capacity that can be occupied with unit equipment, which we simply call the stow factor. Historical planning stow factors have ranged from 65 to 75% during the past 30 years (Kurinovich, 2005). Higher stow factors correlate with fewer ship voyages, whereas lower stow factors correlate with more ship voyages. In real-world sealift operations, such as Operation Enduring Freedom (OEF) and Operation Iraqi Freedom (OIF) in the early 2000s, planners used a ship load planning stow factor of 65% due to less-than-optimal cargo loading processes as well as the rolling stock being formed into task force packages (Kurinovich, 2005). The research provided in this paper largely supports doctrinal recommendations for the 65% stow factor as a general planning factor, given operational realities; however, we provide empirical evidence that supports deviations from the 65% stow factor. Specifically, we identify circumstances in which ships should load to less than 65% of the ship capacity.

The MSLP is most similar to ship loading and scheduling problems in the literature that focus on flexible ship cargo loads, i.e. instances in which the amount of cargo loaded onto ships is a decision variable. Research on flexible ship cargo sizes is comparatively rare in the literature compared to other ship loading and scheduling problems related to the MSLP, such as problems involving split cargo loads. Split cargo load problems allow the total cargo requirement to be broken into smaller requirements delivered by multiple transport assets over time. In addition to flexible and split cargo loads, the MSLP also considers a heterogeneous fleet of ships, constrained port infrastructure in terms of limited CONUS berth space and that ship loading and unloading times depend on the amount of cargo on the ship.

The primary difference between the MSLP described in this research and the various flexible cargo loading and split cargo load problems is that the focus of the MSLP is speed of delivery, whereas the focus of most flexible or split cargo problems is increased profits or decreased costs, i.e. cost-efficiency considerations. Weschler (1976) was the first to formally propose the primacy of combat effectiveness and, from a logistics standpoint, speed of delivery over financial, or cost, considerations during times of war. Indeed, military doctrinal sources continue to suggest the primary goal of military deployment operations is the fastest possible delivery of unit equipment given available transportation assets (US Joint Chiefs of Staff, 2013a, 2019), i.e. the measure of success is effectiveness in terms of faster deployments instead of measures of efficiency or cost. In fact, the US military is generally not concerned with monetary costs during a major deployment. Instead, delays to a unit's deployment timeline must be minimized to enable rapid mobility (US Joint Chiefs of Staff, 2013a). In terms of transport modes and speed of delivery, US Joint Chiefs of Staff (2013b) notes that sealift is the fastest transport mode for the delivery of large amounts of unit equipment, whereas airlift is the fastest mode for small amounts of deploying cargo. The priority for faster delivery over cost efficiency is lacking in the existing flexible cargo load and split cargo load literature. Thus, the present research adds the military's perspective to the body of literature for ship loading and scheduling.

The purpose of this research is to provide new insights about ship loading strategies during military deployments. The remainder of this paper is organized as follows. First, we review the pertinent literature most similar to the MSLP. Second, we describe the data required for the MSLP and then construct several math programs, specifically MIPs, to identify the optimal amount of unit equipment to load onto each ship for each sailing as the entirety of the unit equipment is delivered. Third, we provide an empirical analysis with notional, but realistic, problem data. Because the input data can vary, we construct a two-level experimental design and then solve the MIP for each experimental run. Fourth, we conduct statistical tests on the solution outputs, including regressions, correlations and chi-square (χ^2) tests. Next, we discuss our findings in the context of similar problems studied in the literature. Finally, we suggest possible extensions to the MSLP for future researchers.

Literature review

As noted previously, the lines of research most related to the MSLP are flexible cargo loads and split cargo loads for ship scheduling problems. Table 1 compares the attributes of the literature most related to the MSLP, including the solution method (e.g. exact or heuristic), objective function and key problem characteristics such as flexible cargo loads, split cargo loads, heterogeneous vehicles, constrained infrastructure and load/unload times dependent on the amount of cargo loaded. For additional details, the interested reader is directed to the summary papers by Song (2021), Christiansen *et al.* (2013) and Christiansen *et al.* (2004), which collectively cover the previous 30 years of research for ship routing and scheduling problems.

Flexible cargo loads have been studied for over 20 years. Fagerholt and Christiansen (2000a, b) examined a ship scheduling problem in which the ship holds could be configured for flexible amounts of cargo. The authors used a set partitioning approach to limit ship wait time and under-used cargo capacity while recognizing that time at port depends on the amount of cargo loaded. Then, Brønmo *et al.* (2007) provided the first definition of "flexible cargo sizes" for a short-term tramp ship scheduling problem. The authors developed a MIP, which they solved with a set partitioning approach to determine how much extra cargo to load onto a fleet of heterogeneous ships to increase profits for the company. Next, Al-Khayyal and Hwang (2007)

Table 1. Vehicle scheduling and routing literature most related to the MSLP

Relevant research	Solution: Exact (E), heuristic (H)	Objective: Minimize costs	Objective: Minimize late deliveries	Vehicle type: Ships (S), trucks (T)	Flexible cargo loads	Split cargo loads	Heterogeneous vehicle fleet	Infrastructure limits (port, depot)	Load/unload times based on cargo amount
Fagerholt and Christiansen (2000a, b)	E	X		S	X		X		X
Brønmo *et al.* (2007)	E	X		S	X		X		
Al-Khayyal and Hwang (2007)	E	X		S	X		X	X	X
Nowak *et al.* (2008)	H	X		T		X			
Brønmo *et al.* (2010)	E	X		S	X		X		X
Korsvik and Fagerholt (2010)	H	X		S	X				X
Korsvik *et al.* (2011)	H	X		S		X			X
Andersson *et al.* (2011)	E	X		S		X	X		
Stålhane *et al.* (2012)	E	X		S		X	X		
Sahin *et al.* (2013)	H	X		T		X			
Chen *et al.* (2014)	H	X		T		X			
Hennig *et al.* (2015)	E	X		S	X	X	X		
Lee and Kim (2015)	E, H	X		S		X	X		
Rodrigues *et al.* (2016)	E, H	X		S	X		X	X	
Haddad *et al.* (2018)	E, H	X		T		X			
Stanzani *et al.* (2018)	E, H	X		S	X		X	X	
Santos *et al.* (2020)	E, H	X		S	X	X	X	X	
Wolfinger (2021)	E, H	X		T		X	X		X
Wang *et al.* (2021)	H	X		T		X			
Wolfinger and Salazar-González (2021)	E	X		S		X	X		
Santos and Borenstein (2022)	E	X	X	S	X	X	X	X	
This work	E		X	S	X	X	X	X	X

used a network flow model while allowing a heterogeneous fleet of ships to be partially loaded. Similarly, Brønmo *et al.* (2010) expanded on previous work with flexible cargo loads while specifically studying the balance between extra profits from additional cargos versus the additional loading and unloading time incurred by ships taking on larger cargo loads. Most notably, the researchers highlighted that the time a ship spends in port depends on the amount of cargo loaded, which is central to the MSLP. Similarly, Korsvik and Fagerholt (2010) noted that load and unload times depend on quantity loaded in their study of ship routing and scheduling with flexible cargo quantities. The authors employed a heuristic using a tabu search technique to solve the problem. Hennig *et al.* (2015) considered both flexible and split cargo loads in their path flow solution for a crude oil tanker routing and scheduling problem. Rodrigues *et al.* (2016) considered port infrastructure restrictions, specifically ship draft limits at certain ports, and flexible cargo loads in their solution for a maritime oil transportation problem. Next, Stanzani *et al.* (2018) provided exact and heuristic solutions for how much crude oil to carry on tankers for a Brazilian oil company while considering a heterogeneous fleet of ships and a limited number of berths. More recently, Santos *et al.* (2020) formulated exact and heuristic solutions for a deep-sea maritime cargo routing problem for a heterogeneous fleet of ships with draft limits, flexible cargo sizes and split loads.

Although flexible cargo loading is the key component of the MSLP, the assumption that cargos can be split between multiple ships is another important aspect of the MSLP. The heterogeneous fleet of ships will conduct multiple sailings, or cycles, to deliver the immense amount of unit equipment for the deployment. The literature on split loads for transportation problems is more extensive than for flexible cargo loads. Nowak *et al.* (2008) provided seminal work on pickup and delivery problems with split loads by quantifying benefits based on load size, cost and frequency of common origins and destinations. As with the MSLP, Korsvik *et al.* (2011) noted for split cargo load problems that the time to load and unload cargo depends on the quantity loaded. Andersson *et al.* (2011) noted that split loads are most common in land-based logistics problems. The authors provided an exact method for shipping problems and concluded that load splitting results in more port calls and port costs, but also increased profits. Then, Stålhane *et al.* (2012) considered a split load shipping problem with a heterogeneous fleet via an exact, path-flow formulation solved with a branch-and-price algorithm. Next, Sahin *et al.* (2013) provided a heuristic solution for the multi-vehicle pickup and delivery problem and concluded a 32% cost savings with split loads, but noted that the savings depend on the spatial distribution of pickup and delivery locations. Chen *et al.* (2014) provided a heuristic solution using variable neighborhood search for a truck transportation problem involving split loads. Lee and Him (2015) provided exact and heuristic solutions for a shipping problem with a heterogeneous fleet of ships with split cargo loads. Similarly, Haddad *et al.* (2018) provided exact and heuristic solutions to multi-vehicle pickup and delivery problems with split loads for homogenous vehicles. Wolfinger (2021) noted that load and unload times depend on the amount of cargo while providing exact and heuristic solutions for a trucking problem assuming split cargo loads. Likewise, Wang *et al.* (2021) used a generic algorithm with tabu search to solve a trucking problem with split loads. Finally, Wolfinger and Salazar-González (2021) provided an exact solution for a shipping problem with a heterogenous fleet and split cargo loads.

The previously mentioned research with flexible or split cargo loads each minimized costs as the objective, whereas the objective of the MSLP is to minimize delivery timelines. The notable exception is the work of Santos and Borenstein (2022), who provided an exact solution via a fuzzy weighted max-min method for a shipping problem with flexible cargo loads, split loads, heterogenous fleet of ships and infrastructure constraints. More importantly, this recent work minimized late deliveries for some cargo in addition to minimizing costs. As such, this work most closely aligns with the MSLP, although the MSLP allows the load and unload times to vary based on the amount of cargo loaded on the ships. In addition, several other researchers have incorporated time delays or minimized late deliveries similar to the MSLP without considering flexible or split cargo loads. For instance, Campbell and Savelsbergh (2004) noted that increasing the amount of cargo on vehicles, and thereby taking more time to load the vehicle, could result in delivery delays. Also, Vélez-Gallego *et al.* (2020) focused on a truck loading problem and allowed for a minimum capacity on trucks (as opposed to full cargo loads) and stressed that early delivery was desired. Similarly, the primary goal of the MSLP is the earliest possible delivery of unit equipment to theater. Despite the diverse body of work related to the MSLP, to the best of our knowledge, no previous research has incorporated all aspects of the MSLP, including flexible cargo loads, split cargo loads, heterogeneous fleet of ships with different sizes and speeds, ships executing multiple cycles between CONUS and the theater during the deployment, load and unload times that depend upon the amount loaded and ships competing for limited CONUS berth space.

Methods

The MSLP is to assign unit equipment to a fleet of heterogeneous ships cycling between CONUS seaports and theater seaports to minimize the delivery timeline. The methods applied in this research include formulating and solving MIPs followed by statistical tests of correlation and association. As presented in the literature review, the use of MIPs to find exact solutions for the MSLP is consistent with the methods used by other researchers to solve similar problems. The problem assumptions, notation, MIP formulations and outcome measures are provided next.

Assumptions

The following simplifying assumptions for the MSLP are used for this research. First, all timing aspects of the MSLP are in full day increments, i.e. fractional days are not allowed. Second, this research does not distinguish between CONUS seaports, which would add complexity to this initial examination of the MSLP. Instead, the total number of berths on the dominant coast are aggregated across the available CONUS seaports. The dominant coast is defined as the CONUS coast (West or East) nearest the overseas theater. Several operational realities support the aggregation of berths on the dominant coast, including (1) the transit time between CONUS and the theater seaports is nearly identical (less than a day difference in transit time) for each seaport on the dominant coast and (2) ships can be diverted to an alternative seaport on the dominant coast if the primary seaport has no available berth space. The third simplifying assumption for the MSLP is that sail times going from CONUS to theater are identical (less than a day difference in transit time) to sail times going from theater to CONUS, i.e. the direction of sailing has no effect on transit time. In reality, prevailing winds and weather disruptions could result in different sail times between CONUS and the theater depending on the direction of transit. Next, the time required to unload a ship in theater is assumed to be identical (within the full day increment) to the time to load the ship in CONUS, e.g. if a ship is loaded for two days in CONUS, then the unload time is two days in theater. In addition, we assume unit equipment is readily available at the CONUS seaport, i.e. ships are not waiting for equipment to arrive to the seaport. Finally, we assume berth space at theater seaports is not a constraint during the deployment. Future researchers are encouraged to adjust the above assumptions, which could increase the accuracy of the problem. However, the stated simplifying assumptions noted here are appropriate for the first iteration of the MSLP with future extensions provided in the Conclusion section.

Notation

The indices for the MSLP are defined first. Let S be the set of ships with a specific ship $s \in S$. Next, let U be the set of positive integers representing the deployment day with u and v

specific days such that u, $v \in U$. In this paper, we restrict the set of deployment days to $U = \{1,2,\ldots,150\}$; however, in practice, the number of deployment days typically exceed 300. Finally, let W be the set of positive integers representing the number of days a ship can be loaded while on berth with w and z specific load days such that $w, z \in W$. In this paper, we limit the set of load days to $W = \{1,2,3\}$. In our analysis, we focus on three types of RO/RO ships: large/medium speed RO/RO (LMSR), fast sealift ship (FSS) and standard RO/RO. Not all ship types require three days of loading time. In fact, SDDCTEA Pamphlet 700-2 (2011) notes that planning factors for load times of an LMSR or an FSS are up to three days while loading times for a standard, and generally smaller, RO/RO is up to two days. The maximum loading time, in days, for each ship will be provided as input data for the problem.

Next, we define the parameters for the input data. Several ship characteristics are important for the MSLP. First, let cap_s be the total ship cargo capacity in terms of square feet of unit equipment. Square feet of deploying cargo is the standard measure of sealift requirements for military deployments (US Joint Chiefs of Staff, 2005). Second, let spd_s be the ship speed as measured in knots. Next, let $load_s$ be the maximum number of days ship s can be loaded such that $load_s \in W$. Then, let a_s be the first day that ship s is available for loading at a port on the dominant CONUS coast. Also, let $c_{s,w}$ represent the amount of unit equipment, measured in square feet, that can be loaded onto ship s while berthed for w days. Finally, let d_s represent the positive integer number of days representing the one-way transit time between the dominant CONUS coast and theater seaports. In addition to the ship-based parameters, additional input data are required for the MSLP. First, let b_v be the available number of ship berths on the dominant CONUS coast on day v. Second, let the total amount of unit equipment to transport be represented by r, which will be stated as a positive number of square feet.

Mixed integer program (light loads) formulation

The MIP (light loads) formulation for the MSLP requires a single decision variable and one binary variable. Let $x_{s,u,w}$ be a non-negative decision variable representing the amount of square feet of unit equipment loaded onto ship s that berths on the dominant CONUS coast starting on day u and is then loaded for w days. Let $y_{s,u,w}$ be a binary intermediate variable that will be used to control ship taskings, i.e. to prevent a tasked ship from being tasked while executing its current mission. Let $y_{s,u,w} = 1$ if $x_{s,u,w} > 0$, and $y_{s,u,w} = 0$ otherwise. The objective function for the MIP (light loads) minimizes the weighted arrival time of unit equipment to theater, as given in equation (1):

$$minimize \sum_{s \in S} \sum_{u \in U} \sum_{w=1}^{load_s} (u + w + d_s) * x_{s,u,w} \tag{1}$$

We assume ship loading begins on the first day of berthing (u), which would suggest the ship arrives in theater on day ($u + w + d_s - 1$). However, SDDCTEA Pamphlet 700-2 (2011) notes that ships require at least 12 additional hours for piloting, docking and other non-loading port activities at the CONUS port. To account for this additional time and avoid a fractional number of days, the scalar ($u + w + d_s$) in equation (1) represents a conservative estimate for the expected arrival day at theater seaports. Finally, this arrival day in theater is weighted by the amount of cargo on the ship given by $x_{s,u,w}$.

The constraints for the MIP (light loads) formulation are given in equations (2)–(10):

$$\sum_{s \in S} \sum_{u \in U} \sum_{w=1}^{load_s} x_{s,u,w} = r \tag{2}$$

$$x_{s,u,w} \leq c_{s,w} \quad \forall s \in S \ \ \forall u \in U \ \ \forall w \in W \tag{3}$$

$$x_{s,u,w} \le BigM * y_{s,u,w} \quad \forall s \in S \;\; \forall u \in U \;\; \forall w \in W \tag{4}$$

$$\sum_{s \in S} \sum_{u=1}^{(a_s-1)} \sum_{w=1}^{load_s} y_{s,u,w} = 0 \tag{5}$$

$$\sum_{w=1}^{load_s} y_{s,u,w} \le 1 \quad \forall s \in S \;\; \forall u \in U \tag{6}$$

$$\sum_{s \in S} \sum_{v=(u-load_s+1)}^{u} \sum_{z=(u-v+1)}^{load_s} y_{s,v,z} \le b_u \quad \forall u \in U \tag{7}$$

$$\sum_{v=u+1}^{\left(\substack{u+2*w+\\2*d_s}\right)} \sum_{z=1}^{load_s} y_{s,v,z} \le BigM * \left(1 - y_{s,u,w}\right) \quad \forall s \in S \;\; \forall u \in U \;\; \forall w \in W \tag{8}$$

$$x_{s,u,w} \ge 0 \quad \forall s \in S \;\; \forall u \in U \;\; \forall w \in W \tag{9}$$

$$y_{s,u,w} \in \{0,\, 1\} \quad \forall s \in S \;\; \forall u \in U \;\; \forall w \in W \tag{10}$$

Equation (2) ensures all deploying requirements given by r are exactly met. Equation (3) limits the amount of unit cargo loaded on the ship to the associated load quantities for each ship and number of days on berth. Next, equation (4) includes a large positive value (*BigM*) and forces the binary intermediate variable to 1 for any positive value of $x_{s,u,w}$. Equation (5) prevents any assignment until the ship is available for onloading on the dominant coast. Equation (6) limits each ship onloading to a unique number of load days w. Then, equation (7) limits the number of ships berthed simultaneously on the dominant CONUS coast to the number of berths available by day, which requires accounting for ships that berthed prior to the current day (u) and are still being loaded. Equation (8) prevents a previously tasked ship from being available for a subsequent sailing until the ship has completed its previous sailing. The upper bound on the first summation in equation (8) accounts for the time to load, sail to theater, unload, sail back to CONUS and one additional combined day for piloting, docking and other non-loading port activities during the ship cycle (SDDCTEA Pamphlet 700-2, 2011). Finally, equations (9) and (10) ensure the decision variables are non-negative values and binary, respectively.

Mixed integer program (full loads) formulation

The MIP (full loads) formulation is similar to the MIP (light loads) formulation. In fact, both formulations use the same input data. The differences stem from the decision variables in the MIP (full loads) formulation not needing to specify the number of days ships are loaded, because all ships achieve full loads by berthing for the maximum number of load days.

Let $x_{s,u}$ be a non-negative decision variable representing the amount of square feet of unit equipment loaded onto ship s that berths on the dominant CONUS coast starting on day u. Let $y_{s,u}$ be a binary intermediate variable that will be used to control ship taskings. Let $y_{s,u} = 1$ if $x_{s,u} > 0$, and $y_{s,u} = 0$ otherwise. The objective function for the MIP (full loads) formulation minimizes the weighted arrival time of unit equipment to theater, as given in equation (11):

$$\text{minimize} \sum_{s\in S}\sum_{u\in U}(u + load_s + d_s) * x_{s,u} \tag{11}$$

In equation (11), the scalar $(u + load_s + d_s)$ represents the arrival day based on the first day of berthing (u), the number of days berthed ($load_s$) to reach 65% stowage (i.e. fully loaded) and sail time to theater (d_s). This arrival day in theater is weighted by the amount of cargo on the ship given by $x_{s,u}$.

The constraints for the MIP (full loads) formulation are given in equations (12)–(19):

$$\sum_{s\in S}\sum_{u\in U} x_{s,u} = r \tag{12}$$

$$x_{s,u} \le c_{s,load_s} \quad \forall s \in S \ \ \forall u \in U \tag{13}$$

$$x_{s,u} \le BigM * y_{s,u} \quad \forall s \in S \ \ \forall u \in U \tag{14}$$

$$\sum_{s\in S}\sum_{u=1}^{(a_s-1)} y_{s,u} = 0 \tag{15}$$

$$\sum_{s\in S}\sum_{v=(u-load_s+1)}^{u} y_{s,v} \le b_u \quad \forall u \in U \tag{16}$$

$$\sum_{v=u+1}^{\left(\substack{u+2*load_s+\\ 2*d_s}\right)} y_{s,v} \le BigM * \left(1 - y_{s,u}\right) \quad \forall s \in S \ \ \forall u \in U \tag{17}$$

$$x_{s,u} \ge 0 \quad \forall s \in S \ \ \forall u \in U \tag{18}$$

$$y_{s,u} \in \{0,\ 1\} \quad \forall s \in S \ \ \forall u \in U \tag{19}$$

Measures

The key measure for the MSLP is the expected arrival time of unit equipment to theater seaports (US Joint Chiefs of Staff, 2013a). Let T be the average arrival time (given in days) of unit equipment to theater seaports. Thus, T represents an important effectiveness measure for the MSLP. Equation (20) provides the calculation for T depending on the MIP variant being used given the different subscripts in the primary decision variable.

$$T = \begin{cases} \dfrac{\sum_{s\in S}\sum_{u\in U}\sum_{w=1}^{load_s}(u + w + d_s) * x_{s,u,w}}{r} & \text{for MIP (light loads)} \\[2ex] \dfrac{\sum_{s\in S}\sum_{u\in U}(u + load_s + d_s) * x_{s,u}}{r} & \text{for MIP (full loads)} \end{cases} \tag{20}$$

The next measure of interest is the fleet-wide proportion of light-loaded sailings during the deployment, which we designate as L and calculate using equation (21). There is no associated measure for light-loaded proportions for MIP (full loads), because all ships are required to be fully loaded to the 65% stow factor.

$$L = \frac{\sum_{s \in S} \sum_{u \in U} \sum_{w < load_s} y_{s,u,w}}{\sum_{s \in S} \sum_{u \in U} \sum_{w=1}^{load_s} y_{s,u,w}} \tag{21}$$

In addition, the proportion of light-loaded sailings for each ship s across all MIP results may be of interest in the post-analysis to assess correlations between light loadings and various ship characteristics, such as speed and capacity. Let L_s be the proportion of light-loaded sailings for ship s across all sailings of the MIP results, as given in equation (22).

$$L_s = \frac{\sum_{u \in U} \sum_{w < load_s} y_{s,u,w}}{\sum_{u \in U} \sum_{w=1}^{load_s} y_{s,u,w}} \quad \forall s \in S \tag{22}$$

Finally, the number of shiploads required to deliver all requirements would be of interest to decision-makers, primarily as an efficiency measure for comparing the MIP solutions. Let N be the number of shiploads, which is calculated using equation (23) depending on the MIP being used given the different subscripts in the primary decision variable:

$$N = \begin{cases} \sum_{s \in S} \sum_{u \in U} \sum_{w=1}^{load_s} y_{s,u,w} & \text{for MIP (light loads)} \\ \sum_{s \in S} \sum_{u \in U} y_{s,u} & \text{for MIP (full loads)} \end{cases} \tag{23}$$

Results

This section includes empirical results for the MSLP. First, we define test data representing deployment requirements, ships with differing characteristics (including fleet size, capacity, transit time between CONUS and theater and load rates) and dominant coast berth restrictions. Two levels of realistic input data, one reflecting a low level and one reflecting a high level, are provided and then we construct two experimental designs to test the MSLP solutions. The two data levels are structured such that the low level (-1) represents a more stressing deployment and the high level ($+1$) represents a less stressing deployment. A fractional factorial design (FFD) is used for the MIP (light loads), and the same design is used for the MIP (full loads). The design variables are regression predictors, and the measures are outcomes (Bruce and Bruce, 2017, p. 129). Each experimental run is solved to optimality using a commercial MIP solver. Various statistical analyses are conducted on the results, including regressions, correlations and χ^2 tests.

Test data

The problem input data for this empirical analysis was derived from values based on historical, or plausible, deployment operations. Unit equipment deployment requirements depend on the scale of the contingency operation. Two recent, large-scale sealift operations were used as the low and high levels for deployment requirements. The low level was set at 31.5M sq ft. based on Operation Desert Shield/Desert Storm (Matthews and Holt, 2003, p. 116), and the high level was set at 21M sq ft. based on the initial four-month surge for OIF (Kennedy, 2003). The high level thus reflects a less stressing deployment with less unit equipment deploying compared to the low level. In terms of ship characteristics, a fleet of 60 ships were used for the low level (more stressing), and a fleet of 74 ships were used for the high level (less stressing). These fleet values represent realistic ranges of available shipping capacity based on past military deployments (Kennedy, 2003). Table 2 provides the notional, but realistic, ship characteristics for all ships in terms of ship capacity, speed and first day of availability on the dominant CONUS coast. We selected the first 60 ships of Table 2 ($s = 1,2,\ldots,60$) for the low level fleet of ships. Available berth space on the dominant coast was notionally set at six berths for the low level (more stressing) and ten berths as the high level (less stressing). Transit times were based on deploying to notional, theater seaports with the low level set as a far theater (more stressing) and the high level set as a near theater (less stressing), with transit times for each ship s as in Table 3. Finally, load rates were estimated by engineers from the Ports for National Defense with low load rates (more stressing) and high load rates (less stressing) for each ship type (LMSR, FSS, RO/RO), as shown in Figure 1.

Mixed integer program implementation

We implemented the MIPs in the General Algebraic Modeling System v24.3.3 software. Exploratory test runs showed that solutions with optimality tolerances set at 1% could be obtained within about 24 h of runtime. Thus, all MIP solutions were produced using the GAMS CPLEX solver with settings to halt the solution after achieving a 1% optimality gap. All MIP solutions were obtained on a Dell Precision T7500 computer, which was running Windows 7 with 3.33 GHz and 48 GB of RAM.

Design of experiments

The focus of this paper is identifying the circumstances in which ships should be light loaded, i.e. loaded to less than the 65% maximum stowage goal. Montgomery (2005) notes that experimenters often leverage their domain knowledge of the problem under study when selecting potential factors for the design of experiments (p. 21). As such, we conducted exploratory analysis with the MIP (light loads) by changing various problem variables, including size of the requirement (*Reqt*), size of the shipping fleet (*Fleet*), number of CONUS berth spaces (*Berth*), ship transit times (*Transit*) and ship load rates (*Load*). The exploratory analysis suggested that the outcome measures were somewhat sensitive to each of these five problem variables. Therefore, we constructed a $2^{(5-1)}$ FFD with 16 runs and solved the MIP (light loads) with the prescribed two-level combinations of the five problem variables. The selected FFD is a Resolution V design (Montgomery, 2005, p. 305), which permitted identifying main effects as well as two-way interactions while assuming higher-level interactions between variables were negligible. From a computational standpoint, the one-half fraction design allowed us to identify the significant variables and interactions while saving 16 computationally expensive runs compared to the associated full factorial design.

Next, we used the same experimental design for the MIP (full loads) runs. All ships were loaded to 65% of the ship's capacity for this MIP, so the *Load* variable was not relevant in the design. We could have used a $2^{(4-1)}$ FFD with only four variables and eight runs. However, we decided to use the same $2^{(5-1)}$, or 16 runs, as a full factorial design on the four variables: *Reqt*, *Fleet*, *Berth* and *Transit*. Additionally, the MIP (full loads) solutions were generally obtained within about a minute of GAMS CPLEX runtime. Table 4 shows the experimental design variables and settings for both MIPs.

Table 5 shows the MIP results with outcome measures and solution runtimes (in seconds) reported. First, the MIP (light loads) results show faster overall delivery across all runs with T values about 1.69 days earlier, on average, compared to the MIP (full loads) results. Conversely, the N values show that, on average, the MIP (full loads) solution resulted in about 15 fewer ship sailings to deliver the same requirements. The T and N values thus reflect useful effectiveness and efficiency measures, respectively. Across all 16 runs, the average

Table 2. Ship characteristics for MSLP

s	Type	cap_s	spd_s	a_s	s	Type	cap_s	spd_s	a_s	s	Type	cap_s	spd_s	a_s
1	RO/RO	150,000	20	7	26	RO/RO	180,478	15	7	51	RO/RO	165,120	20	5
2	RO/RO	150,000	20	7	27	RO/RO	180,478	15	7	52	RO/RO	165,125	20	7
3	RO/RO	150,000	20	7	28	RO/RO	180,478	15	9	53	RO/RO	112,471	21	10
4	RO/RO	141,843	18	8	29	RO/RO	295,958	14	9	54	RO/RO	198,385	20	5
5	RO/RO	161,372	13	6	30	RO/RO	295,958	14	8	55	RO/RO	198,385	20	9
6	RO/RO	167,338	13	9	31	RO/RO	118,780	15	8	56	RO/RO	170,143	20	5
7	RO/RO	167,338	13	6	32	RO/RO	131,265	13	8	57	RO/RO	114,934	21	5
8	FSS	206,963	27	7	33	RO/RO	131,265	13	7	58	RO/RO	114,934	21	9
9	FSS	199,362	27	10	34	LMSR	387,662	20	10	59	RO/RO	150,195	20	5
10	LMSR	321,831	20	9	35	LMSR	387,662	20	6	60	RO/RO	165,632	20	7
11	RO/RO	167,338	13	5	36	RO/RO	262,252	20	9	61	RO/RO	150,000	20	6
12	LMSR	321,831	20	7	37	RO/RO	208,965	19	10	62	RO/RO	150,000	20	6
13	FSS	202,998	27	5	38	RO/RO	211,071	19	6	63	RO/RO	150,000	20	6
14	FSS	202,998	27	9	39	LMSR	392,615	20	8	64	RO/RO	150,000	20	7
15	FSS	199,362	27	8	40	LMSR	387,662	20	9	65	RO/RO	150,000	20	7
16	FSS	206,963	27	5	41	RO/RO	208,989	19	5	66	RO/RO	150,000	20	7
17	RO/RO	148,665	16	9	42	RO/RO	208,989	19	10	67	RO/RO	150,000	20	8
18	RO/RO	148,665	16	7	43	RO/RO	211,148	19	7	68	RO/RO	150,000	20	8
19	RO/RO	148,665	16	9	44	LMSR	387,662	20	7	69	RO/RO	150,000	20	8
20	RO/RO	176,312	16	8	45	LMSR	387,662	20	5	70	RO/RO	150,000	20	9
21	RO/RO	176,312	16	8	46	LMSR	392,615	20	7	71	RO/RO	150,000	20	9
22	RO/RO	117,888	13	7	47	LMSR	303,000	24	7	72	RO/RO	150,000	20	9
23	RO/RO	117,888	13	9	48	RO/RO	112,471	21	5	73	RO/RO	150,000	20	10
24	RO/RO	115,618	13	5	49	RO/RO	144,012	20	7	74	RO/RO	150,000	20	10
25	RO/RO	176,312	16	6	50	RO/RO	160,271	20	7					

Table 3.
Transit time in days for each ship between dominant CONUS coast and Far and Near theaters

s	d_s Far	d_s Near	s	d_s Far	d_s Near	s	d_s Far	d_s Near	s	d_s Far	d_s Near	s	d_s Far	d_s Near
1	14	8	16	10	6	31	18	11	46	14	8	61	14	8
2	14	8	17	16	10	32	20	12	47	11	7	62	14	8
3	14	8	18	16	10	33	20	12	48	13	8	63	14	8
4	15	9	19	16	10	34	14	8	49	14	8	64	14	8
5	20	12	20	16	10	35	14	8	50	14	8	65	14	8
6	20	12	21	16	10	36	14	8	51	14	8	66	14	8
7	20	12	22	20	12	37	14	8	52	14	8	67	14	8
8	10	6	23	20	12	38	14	8	53	13	8	68	14	8
9	10	6	24	20	12	39	14	8	54	14	8	69	14	8
10	14	8	25	16	10	40	14	8	55	14	8	70	14	8
11	20	12	26	18	11	41	14	8	56	14	8	71	14	8
12	14	8	27	18	11	42	14	8	57	13	8	72	14	8
13	10	6	28	18	11	43	14	8	58	13	8	73	14	8
14	10	6	29	19	11	44	14	8	59	14	8	74	14	8
15	10	6	30	19	11	45	14	8	60	14	8			

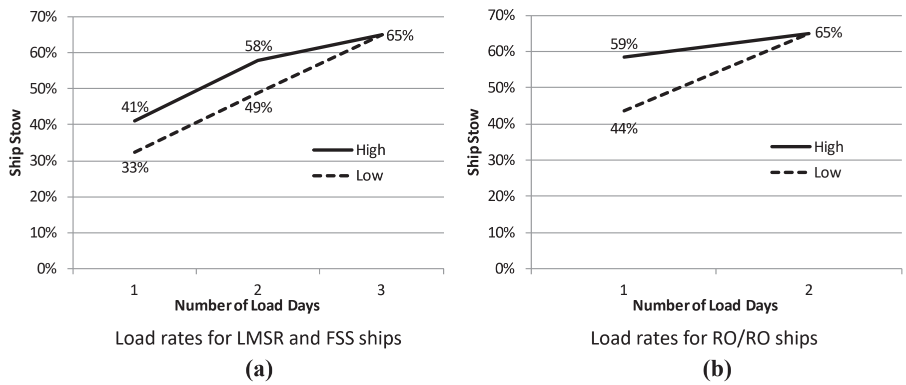

Figure 1.
High and low ship load rates based on ship type and number of load days

Table 4.
Design of experiment settings for coded and natural variables

	Coded variables					Natural variables				
Run	*Reqt*	*Fleet*	*Berth*	*Transit*	*Load*	*Reqt*	*Fleet*	*Berth*	*Transit*	*Load*
1	−1	−1	−1	−1	1	31.5M	60	6	Far	High
2	1	−1	−1	−1	−1	21.0M	60	6	Far	Low
3	−1	1	−1	−1	−1	31.5M	74	6	Far	Low
4	1	1	−1	−1	1	21.0M	74	6	Far	High
5	−1	−1	1	−1	−1	31.5M	60	10	Far	Low
6	1	−1	1	−1	1	21.0M	60	10	Far	High
7	−1	1	1	−1	1	31.5M	74	10	Far	High
8	1	1	1	−1	−1	21.0M	74	10	Far	Low
9	−1	−1	−1	1	−1	31.5M	60	6	Near	Low
10	1	−1	−1	1	1	21.0M	60	6	Near	High
11	−1	1	−1	1	1	31.5M	74	6	Near	High
12	1	1	−1	1	−1	21.0M	74	6	Near	Low
13	−1	−1	1	1	1	31.5M	60	10	Near	High
14	1	−1	1	1	−1	21.0M	60	10	Near	Low
15	−1	1	1	1	−1	31.5M	74	10	Near	Low
16	1	1	1	1	1	21.0M	74	10	Near	High

Run	T	MIP (light loads) L	N	IP time	MIP (full loads) T	N	IP time	Deltas ΔT	ΔN
1	79.64	0.26	243	11,393	81.60	235	24	1.96	−8
2	58.88	0.08	158	311	59.01	153	8	0.12	−5
3	73.24	0.09	253	9,422	73.69	241	18	0.45	−12
4	51.22	0.57	178	20,649	54.19	162	10	2.97	−16
5	77.84	0.01	236	1,646	77.86	235	2	0.02	−1
6	54.43	0.34	163	134	55.47	156	4	1.03	−7
7	67.68	0.35	260	3,532	69.21	248	7	1.53	−12
8	49.84	0.04	168	702	49.90	167	3	0.05	−1
9	58.03	0.17	250	66,872	58.66	233	57	0.64	−17
10	39.87	0.63	167	192	43.15	151	16	3.28	−16
11	48.84	0.77	271	29,677	57.01	232	31	8.18	−39
12	40.44	0.61	201	144,473	42.47	150	7	2.03	−51
13	52.90	0.54	250	5,416	54.53	235	2	1.63	−15
14	39.42	0.09	160	59	39.81	153	4	0.39	−7
15	48.61	0.12	258	284	48.98	244	7	0.37	−14
16	33.87	0.75	185	69	36.26	164	3	2.39	−21
Avg	54.67	0.34	213	18,427	56.36	198	13	1.69	−15

Note(s): GAMS/CPLEX optimality tolerance set at 1% for each MIP solution

Table 5. Design of experiment results for each MIP and comparisons

proportion of light-loaded sailings was about 0.34, so the optimal results suggest that about a third of all sailings could be light loaded to improve the rate of delivery.

Runs with higher proportions of light-loaded ships generally suggest a faster overall delivery of forces albeit with additional shiploads required. Run 11 is particularly interesting as it has the highest proportion of light-loaded sailings, about 0.77, and likewise has the largest decrease in the weighted average delivery time measure T with more than eight days earlier delivery compared to the optimal solution with fully loaded ships. Although the difference in T measures is about eight days, Figure 2 compares the cumulative arrival of the 31.5M sq ft. of unit equipment for the light- and full-load solutions. The full-load solution delivers the unit equipment up to about 14 days later than the light-load solution, which would represent a significant delay of critical combat capability to the commander in theater. Finally, Figure 3 offers a visual comparison of ship berthing activity in CONUS for the 74

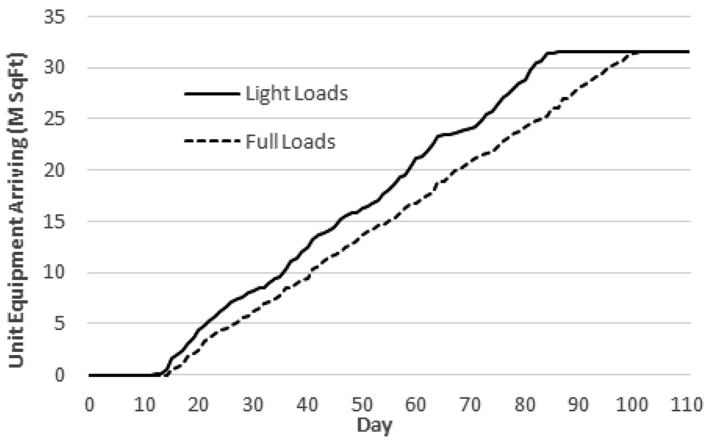

Figure 2. Cumulative delivery to theater, given light- or full-load results for Run 11

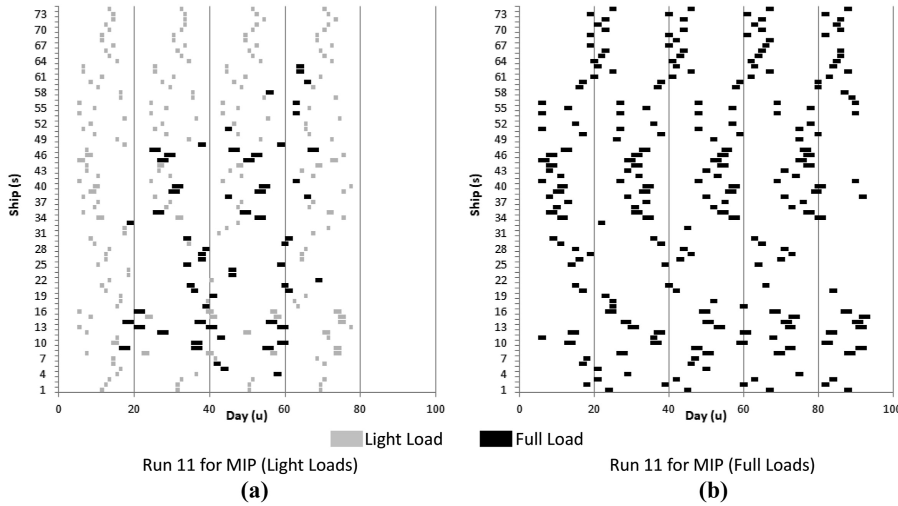

Figure 3.
Ship berthing activity by day for Run 11 with gray boxes representing ships with light loads and black boxes representing ships with full loads

ships in Run 11. The graphic shows no berthing in CONUS after Day 77 in the light-load solution, whereas the last ship berths on Day 91 in the full-load solution, which accounts for the 14-day delay in equipment delivered to theater.

Statistical analyses

We conducted various statistical analyses on the MSLP outputs, including multiple linear regression, to identify significant predictors for the outcome measures T and L, correlation analyses of ship speed and size against L and χ^2 tests for ship speed and size with L.

Table 6 shows the fitted multiple linear regression models for outcome measures T and L for the MIP (light loads) as well as the multiple linear regression model for outcome measure T for the MIP (full loads). Each fitted model was statistically significant at the $\alpha = 0.001$ level, with the majority of variance accounted for based on adjusted R^2 values (Field, 2013, p. 312). In terms of the delivery measure T for the MIPs, the statistically significant predictors were *Reqt*, *Fleet*, *Berth* and *Transit*, along with one or more two-way interactions, as depicted in Table 6. Two-way interactions not included in the table were insignificant with p-values > 0.05. The coded design variables were structured such that positive values should decrease expected delivery times, i.e. fewer requirements to move, more ships available, more berth space in CONUS and shorter transit times. Therefore, the negative B values for each predictor (*Reqt*, *Fleet*, *Berth*, *Transit*) support the decrease in T for increases in the predictors. In terms of the light-loaded proportion measure L for the MIP (light loads) results, the statistically significant predictors were *Fleet*, *Berth*, *Transit* and *Load*. The predictor *Reqt* was not significant at the $\alpha = 0.01$ level. Again, the corresponding signs of the B values suggest higher proportions of light-loaded ships with more ships, fewer berths, shorter transit times and higher rates of cargo loaded while berthed.

Next, we computed for each ship s the number of sailings (n) and the proportion of light-loaded sailings (L_s) as an average across the ship's n sailings for the MIP (light loads), as shown in Table 7. We then computed pairwise Pearson correlations between ship capacity (cap_s), speed (spd_s) and L_s, as provided in Table 8. The correlation between cap_s and spd_s was not statistically significant, but the correlations between cap_s and L_s and between spd_s and L_s were statistically significant at the $\alpha = 0.001$ level and the $\alpha = 0.01$ level, respectively. For the correlation between cap_s and L_s, the negative value suggests that ships with a higher capacity

Table 6. Multiple linear regression results for delivery time (T) and proportion of light loads (L)

MIP	Response	*F* statistic (*p*-value)	Adjusted R^2	Standard error	B0 (*p*-value)	Predictor	*B* value	*t*-statistic	*p*-value
Light loads	T^1	143.10 (<0.001)	0.979	1.999	54.672 (<0.001)	*Reqt*	−8.674	−17.353	*<0.001*
						Fleet	−2.955	−5.912	*<0.001*
						Berth	−1.597	−3.196	*<0.01*
						Transit	−9.426	−18.857	*<0.001*
						*Reqt * Transit*	1.829	3.659	*<0.01*
	L^2	18.98 (<0.001)	0.827	0.111	0.338 (<0.001)	*Fleet*	0.074	2.657	*0.02*
						Berth	−0.060	−2.162	*0.05*
						Transit	0.121	4.341	*<0.01*
						Load	0.187	6.734	*<0.001*
Full loads	T^3	441.50 (<0.001)	0.996	0.878	56.362 (<0.001)	*Reqt*	−8.830	−40.231	*<0.001*
						Fleet	−2.399	−10.931	*<0.001*
						Berth	−2.360	−10.754	*<0.001*
						Transit	−8.752	−39.876	*<0.001*
						*Reqt * Fleet*	0.572	2.604	*0.04*
						*Reqt * Transit*	1.645	7.495	*<0.001*
						*Fleet * Berth*	−0.516	−2.351	*0.05*
						*Fleet * Transit*	0.970	4.420	*<0.01*

Note(s): B values represent unstandardized coefficients; Shapiro–Wilk tests failed to show evidence of non-normality of the residuals ([1]W = 0.985, *p*-value = 0.992; [2]W = 0.988, *p*-value = 0.998; [3]W = 0.965, *p*-value = 0.758)

Table 7.
Number of sailings and average light-loaded proportions for MIP (light loads) runs

s	n	L_s	s	n	L_s	s	n	L_s	s	n	L_s	s	n	L_s
1	55	0.545	16	72	0.431	31	40	0.225	46	52	0.212	61	26	0.577
2	56	0.482	17	43	0.209	32	37	0.297	47	59	0.203	62	26	0.538
3	55	0.400	18	44	0.205	33	36	0.250	48	55	0.418	63	27	0.556
4	46	0.239	19	44	0.295	34	51	0.196	49	51	0.412	64	24	0.417
5	37	0.324	20	45	0.267	35	55	0.218	50	57	0.439	65	26	0.462
6	37	0.243	21	47	0.277	36	58	0.345	51	58	0.448	66	24	0.458
7	39	0.333	22	37	0.243	37	54	0.296	52	57	0.421	67	24	0.500
8	65	0.415	23	36	0.222	38	58	0.328	53	52	0.385	68	24	0.417
9	61	0.246	24	36	0.250	39	52	0.250	54	59	0.373	69	27	0.519
10	49	0.143	25	46	0.283	40	50	0.180	55	56	0.393	70	27	0.630
11	40	0.325	26	40	0.175	41	59	0.356	56	59	0.441	71	22	0.364
12	50	0.260	27	41	0.244	42	54	0.333	57	58	0.483	72	25	0.520
13	72	0.361	28	41	0.220	43	58	0.397	58	52	0.327	73	22	0.318
14	61	0.262	29	41	0.220	44	51	0.235	59	58	0.466	74	24	0.500
15	65	0.385	30	42	0.214	45	56	0.321	60	58	0.379			

Table 8.
Correlations between ship capacity, speed and proportion of light-loaded sailings for MIP (light loads)

Variable	*cap*	*spd*	*L*
cap	1	0.189 p-value = 0.106	−0.459 p-value <0.001 95% CI [−0.600, −0.290]
spd		1	0.355 p-value < 0.01 95% CI [0.184, 0.526]
L			1

Note(s): CI = Confidence interval

are less likely to be light loaded. The strength of correlation, as reported in the 95% confidence interval (CI), ranged from medium to large (Cohen, 1988, p. 413). For the correlation between spd_s and L_s, the positive value suggests that ships with a higher speed are more likely to be light loaded. Finally, the strength of correlation reported in the 95% CI ranged from small to large per Cohen's criteria.

The final statistical analysis we conducted was χ^2 tests to evaluate the relationship between categorical groups (Bruce and Bruce, 2017, p. 111), specifically ship speed, ship capacity and light-loaded sailings. We intended the χ^2 tests to confirm associations between certain groups of these key ship characteristics and the light-loaded proportions from the MIP results. The groupings we selected were as follows: ship speed (<20 knots, ≥ 20 knots), ship capacity (≤ 150K sq ft., 150–200K sq ft., ≥ 200K sq ft.) and light-loaded sailings (Yes, No). Table 9 shows the χ^2 test results across the 16 runs for the MIP (light loads) results. Ship speed and light-loaded sailings had a statistically significant association at the $\alpha = 0.001$ level, with a test statistic of 33.505 with one degree of freedom; however, the strength of the association is small with a phi coefficient of about 0.1 (Field, 2013, p. 740). Similarly, the association between ship capacity and light-loaded sailings was statistically significant at the $\alpha = 0.001$ level, with a test statistic of 25.164 with two degrees of freedom. However, the effect size was small with a phi coefficient of about 0.09.

Analysis of the cross-tabulation results from Table 9 provides important insights about the nature of the association. For the ship speed and light-load χ^2 test, more of the fast ships (speed ≥ 20 knots) were light loaded (829) than expected (753) across all MIP runs, whereas

Table 9.
Cross tabulation of observed and expected values and chi-square (χ^2) test results from aggregated MIP (light loads) runs

	Group		*Light loaded* No	Yes	Total				
Speed	< 20	Observed	855	323	1,178				
		Expected	779	399					
	≥ 20	Observed	1,394	829	2,223				
		Expected	1,470	753		χ^2 statistic	*df*	*p-value*	φ
		Total	2,249	1,152	3,401	33.505	1	<0.001	0.099
Capacity	≤ 150K	Observed	726	455	1,181				
		Expected	781	400					
	150–200K	Observed	656	345	1,001				
		Expected	662	339					
	≥ 200K	Observed	867	352	1,219				
		Expected	806	413		χ^2 statistic	*df*	*p-value*	φ
		Total	2,249	1,152	3,401	25.164	2	<0.001	0.086

Note(s): φ = Phi coefficient (effect size)

fewer of the slow ships (speed < 20 knots) were light loaded (323) than expected (399). This insight further supports the correlational analysis results that faster ships are more likely to be light loaded than slower ships. For the ship capacity and light-load χ^2 test, more of the lower capacity ships (≤ 150K sq ft.) were light loaded (455) than expected (400) across all MIP runs, whereas fewer of the higher capacity ships (≥ 200 sq ft.) were light loaded (352) than expected (413). The observed and expected counts for medium capacity ships were nearly identical. This insight further supports the correlational analysis results that lower capacity ships are more likely to be light loaded than higher capacity ships.

Discussion

The empirical results in this paper identify operational circumstances and specific ship characteristics affecting the proportion of ships that should be light loaded during deployments, which we have shown to decrease overall delivery timelines. Limited CONUS berth space is one such operational circumstance identified in this empirical analysis. Military doctrine notes that ships require access to limited, militarily-useful berth space at CONUS seaports during large-scale contingency operations (US Chiefs of Staff, 2013a). The present research confirms this doctrinal reference by showing available berth space as a statistically significant predictor for the delivery measure T and light-loaded proportion measure L. More importantly, the analysis shows that a higher proportion of ships should be light loaded when berth space is constrained, which is a key insight for military logistics planners and decision-makers.

Furthermore, an examination of optimal ship stow factors provides additional insights for military logistics personnel. The optimal ship stow factors across the MIP (light loads) runs had a mean of 61.2%, with $n = 3{,}401$ sailings, and a 95% CI of [60.9%, 61.5%]. Comparing optimal ship stow factors by ship type is perhaps more insightful: FSS (mean 58.8%, $n = 396$, 95% CI [57.8%, 59.9%]), RO/RO (mean 61.6%, $n = 2{,}480$, 95% CI [61.3%, 61.8%]) and LMSR (mean 61.6%, $n = 525$, 95% CI [60.9%, 62.2%]. The average optimal stow factors for FSS and LMSR ships in our empirical analysis align well with actual ship stow factors of approximately 60% for FSS ships and approximately 59%–61% for LMSR ships during OEF and OIF (Kurinovich, 2005). The average RO/RO stow factors in our analysis are slightly higher than actual ship stow factors of approximately 57% during OEF and OIF (Kurinovich, 2005). Based on this aggregate stow factor analysis, loading ships to the 65% historical planning ship stow factor could lead to sub-optimal ship schedules, resulting in forces delivered about 3% later (1.69/54.67), on average, across the scenarios studied, but up to about 16% later (8.18/48.84) in the most extreme case (Run 11). Instead, average ship stow factors ranging from 59 to 62% are empirically shown here to improve the rate of delivery albeit at the expense of additional ship sailings required.

The preceding aggregate stow factor analysis covers a breadth of deployment circumstances, including situations with few light-loaded ships (e.g. Run 5 with approximately 1% light-loaded sailings) and situations with many light-loaded ships (e.g. Run 11 with approximately 77% light-loaded sailings). Not surprisingly, the average ship stow factors are approximately 65% for each ship type in Run 5; however, the average ship stow factors in Run 11 were: FSS (mean 57.4%, $n = 28$, 95% CI [54.4%, 60.3%]), RO/RO (mean 59.6%, $n = 204$, 95% CI [59.3%, 60.0%]) and LMSR (mean 57.5%, $n = 39$, 95% CI [54.6%, 60.1%]. As such, Run 11 represents somewhat of a lower bound in terms of recommended optimal ship stow factors for a stressing deployment with large amounts of equipment deploying and limited CONUS berth space. Based on this supposition, ship stow factors for FSS and LMSR ships should be no lower than approximately 54%–55% and RO/RO ships should be no lower than approximately 59%, which represent the respective 95% CI lower bounds. This lower bound on ship stow factors is consistent with several papers in the split load literature. Nowak *et al.* (2008) first quantified the benefits of split loads for land-based logistics stating that the highest benefits were achieved when load sizes were just over half of the vehicle capacity. Later, Sahin *et al.* (2013) showed experimentally a roughly 32% improvement in key output measures, with load sizes ranging between 51 and 60% of vehicle capacity. Similarly, the present research quantifies suggested optimal ship loads ranging from 54 to 65%, depending on the operational circumstances and specific ships available during the deployment.

Conclusion

The primary decision of the MSLP is the assignment of unit equipment to a heterogeneous fleet of sealift ships during the deployment, with the primary goal being the fastest possible delivery to theater seaports. The available literature most closely related to the MSLP seems to be ship scheduling that considers flexible cargo loads, i.e. scheduling or assignment problems in which the amount of cargo to load onto the ship is a decision variable. In addition, research on split cargo loads, which allow the cargo to be split into multiple loads across multiple ships, is also pertinent to the MSLP.

This research largely supports the doctrinal planning factor for ship stow factors (set at 65% of the ship's total capacity); however, we provide empirical evidence that suggests which ships should be light-loaded to improve the speed of delivery. The research presented in this paper suggests that light loading ships improve delivery timelines up to 16% under certain circumstances, such as when CONUS berth space is constrained, ship fleet size is robust (or has excess ships compared to the deployment requirements) and the transit time between CONUS and theater seaports is relatively short. Conversely, the empirical analysis in this paper shows that light loading ships occur infrequently when berth space is less constrained, fleet size is small and transit times are relatively long.

Our research adds the perspective of military deployers to the diverse literature on ship routing and scheduling. The military perspective differs from industry, in that the primary consideration during military deployments is speed of delivery, even at the cost of additional ship voyages. More importantly, this research provides important new insights that should be adopted by sealift planners at the US Transportation Command when assigning unit equipment to ships during future contingency operations. Specifically, our empirical analysis suggests optimal ship stow factors ranging from 54 to 65%, depending on various operational conditions. The proposed optimal stow factor goals align well with actual ship loads observed during sealift-intensive operations in support of OEF and OIF.

Future work

The present research assumes no distinction between specific seaports on the dominant CONUS coast, i.e. the number of berths on the dominant coast are aggregated. This assumption is valid in terms of transit times between ports on the dominant CONUS coast (East or West) and some theater locations, because the sail times are nearly identical regardless of the CONUS seaport selected. However, some real-world differences between seaports could affect the MSLP and is a suggested line of research for follow-on efforts. In particular, seaport infrastructure limits such as staging areas, pier strength and crane capabilities at the CONUS and theater seaports could be incorporated into future MSLP variants to improve the precision of the analysis. In addition, seaport access restrictions could affect the amount of unit equipment loaded onto vessels, e.g. shallow channel depths to the seaport could necessitate light loads or low bridges in the channel could necessitate heavy ship loads to clear the obstructions. Therefore, future work on the MSLP could incorporate distinct seaports to incorporate additional problem realities albeit at the expense of increased problem complexity. In addition, the exact MIP solutions in this paper were obtained within about 5 h, on average, for plausible input data. However, the MSLP complexity will inherently increase as distinct seaports are incorporated into the problem formulation. As such, future researchers should consider introducing heuristics to solve the MSLP in a reasonable amount of time.

Finally, an ancillary effect of light-loaded ships, although not explicitly studied in this paper, may be that less fuel is consumed and thereby fewer emissions produced. Previous research suggests that a ship's fuel consumption is directly related to sailing speed and payload (Gao and Hu, 2021; Andersson *et al.*, 2015; Psaraftis and Kontovas, 2014). Reductions in ship speed would be counter to the MSLP objective of faster cargo delivery; however, reduced payloads are a direct result of light loading ships during a military deployment. Therefore, the aggregate fuel and emission effects of light loading ships during deployments may indeed warrant further study.

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