Quantum game perspective on green product optimal pricing under emission reduction cooperation of dual-channel supply chain

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Abstract

Purpose – From the quantum game perspective, this paper aims to study a green product optimal pricing problem of the dual-channel supply chain under the cooperation of the retailer and manufacturer to reduce carbon emissions.

Design/methodology/approach – The decentralized and centralized decision-making optimal prices and profits are obtained by establishing the classical and quantum game models. Then the classical game and quantum game are compared.

Findings – When the quantum entanglement is greater than 0, the selling prices of the quantum model are higher than the classical model. Through theoretical research and numerical analysis results, centralized decision-making is more economical and efficient than decentralized decision-making. Publicity and education on carbon emission reduction for consumers will help consumers accept carbon emission reduction products with slightly higher prices. When the emission reduction increases too fast, the cost of emission reduction will form a significant burden and affect the profits of manufacturers and supply chain systems.

Originality/value – From the perspective of the quantum game, the author explores the optimal prices of green product and compares them with the classical game.

Keywords Quantum game, Quantum entanglement, Carbon emission reduction, Dual-channel supply chain, Investment cooperation

Paper type Research paper

1. Introduction

Over the past 20 years to today, one of the environmental problems about which the international community is most concerned is climate warming. To cope with global warming and find feasible solutions, government officials, scientists and environmentalists all over the world have conducted extensive discussions and cooperation. The carbon tax is proposed in these research and discussions. It is a tax on carbon dioxide emissions because carbon dioxide emissions have always been the main cause of climate warming. At present, many countries reduce carbon emissions by levying carbon taxes.

When climate change and air pollution have threatened the economies, ecosystems and sustainable lifestyles of people, we beings must make changes, improve our awareness of environmental protection, change our production and consumption habits, develop new technologies and reduce carbon emissions, so that the environmental changes brought about by human beings are within the scope of ecological resilience. Ecological resilience refers to the ability of an ecosystem to withstand external interference to maintain its original state. Ecological resilience determines the sustainable relationship within an ecosystem (Holling, 1973).

The ultimate solution to climate warming is to develop a low-carbon lifestyle. As individuals, we should avoid the waste of water, food, clothing, paper, electricity and other materials. In terms of transportation, we should try to take public transport or use new energy vehicles and bicycles should be used for short distances. Domestic water can be recycled, and energy-saving appliances and green products should be used. Garbage classification and waste materials recycling should be done well. Enterprises should use more environment-friendly and recyclable materials for product production, building and construction and should develop new technologies to reduce carbon emissions and other industrial wastes. The low-carbon lifestyle requires people to change their living and consumption habits and enterprises to invest in carbon emission reduction costs. Therefore, the government should establish some transmission mechanisms to promote people's green living awareness and green habits and formulate laws and incentive measures to encourage enterprises to green production.

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Government initiatives, government arrangements and economic incentives support are very important to develop a low-carbon lifestyle (Sampford, 2009). The carbon tax policy is a potentially significant tool to encourage the development of low-carbon life and prevent greenhouse gas emissions that lead to climate change.

Among emerging market countries, one of China's important strategies to deal with climate change is to select some important production cities as low-carbon pilot cities (LCPCs), accumulate low-carbon development experience through government initiatives and economic incentive support and better achieve the committed carbon emission reduction goals (Qiu *et al.*, 2021). The LCPCs strategy aims to establish a sustainable energy ecosystem, so as to reduce carbon emissions in the production and consumption process and promote green life awareness and green habits through the transmission mechanisms, which can play a role in inducing residents' low-carbon choices. The study found that LCPCs can promote the green transformation of the residents' lifestyle, reducing the carbon emissions of life by about 15.3% (Liu and Xu, 2022). The Brazilian Government is committed to developing bioenergy-livestock integrated (BLI) systems, which can not only meet future agricultural needs but also relieve land use pressure and reduce greenhouse gas emissions. According to the study, BLI systems have low greenhouse gas emissions and the mitigation rates of meat and ethanol production are 32% and 22%, respectively. However, BLI systems have many potential obstacles, including operational complexity, specific technical know-how and the need for economic incentives (Nariê *et al.*, 2021). Therefore, government initiatives and economic incentives support are very important in the development of the BLI systems in Brazil.

Even if the government takes action and adopts economic incentive support to reduce the negative impact of the carbon tax on the macroeconomy, enterprises themselves will still face the increase of carbon tax and carbon emission reduction production costs. These carbon emission reduction costs may lead to capital shortage and form the problem of financial constraints for enterprises. Enterprises' initiatives of green manufacturing may be reduced by financial constraints (Cao *et al.*, 2020). Therefore, seeking partners to invest in emission reduction costs to avoid capital shortage has become a solution to the problem of financial constraints. This also makes it possible for the manufacturer and the government, or the manufacturer and partners, to form a game situation, respectively.

In terms of business operation, managers must consider the cost of the carbon tax and the social responsibility of green production and then invest in carbon emission reduction. However, to avoid the financial risk caused by the high cost of carbon emission reduction, a successful manager should explore and understand how to find investment partners for carbon emission reduction and how to conduct investment cooperation on carbon emission reduction. Therefore, this paper wants to discuss how business managers should make decisions when facing carbon emission reduction investment cooperation by establishing a game model.

Supply chain management is the focal research topic in industrial marketing management. Many studies focused on buyer–supplier relationships and other procurement and supply management topics (Ellram and Murfield, 2019). Golgeci and Gligor (2017) studied the relationship between key marketing and supply chain management capabilities, as well as the integration mechanism to form the basic mechanism of capacity relationship. They found that the relationship between strategic marketing and supply chain management capabilities follows a specific pattern. Applying organizational dynamics to key marketing and supply chain management can better understand the relationship between these capabilities. Therefore, the study of the supply chain will help us to formulate industrial marketing strategies.

With e-commerce booming, the dual-channel supply chain is becoming popular. It is a supply chain in which retail channels and online channels coexist. The manufacturer and retailer in the dual-channel supply chain are in the relationship of coexistence of competition and cooperation, so the competitive and cooperative relationship between the two sides forms a game situation.

Is it better for manufacturers and retailers in the dual-channel supply chain to do investment cooperation in carbon emission reduction? Or is it better not to do investment cooperation? It is a problem worthy of discussion. When the two sides conduct carbon emission reduction investment cooperation, how to set the online sales price and the retailer's sales price so as to maximize the benefits of the dual-channel supply chain system is the focus of the managers of both sides. For business managers, these problems are decision-making problems for choosing the optimal business strategies, and they should be deeply understood and explored by every manager. From the perspective of game theory, when both sides of the dual-channel supply chain begin to compete in price or cooperate in investment, the relationships of competition and cooperation between the two sides form a quantum entanglement phenomenon. The faster the reaction of competition and cooperation is, the higher the quantum entanglement degree is. Therefore, such a problem is very suitable for starting from the perspective of the quantum game.

In this paper, we shall explore the emission reduction cooperation decision of the dual-channel supply chain from the perspective of the quantum game. Section 2 presents a literature review, including the theoretical background of game theory, theories and applications of quantum games, and the green product and emission reduction in channel cooperation. Section 3 presents the basic assumptions of the research model. Section 4, we construct the carbon tax-constrained emission reduction investment cooperation model in the dual-channel supply chain from the perspective of the quantum game, and the optimal decision strategy is deduced. In addition, it also includes the management significance of our theoretical model. Section 5 presents the numerical simulation results of the models. Section 6 offers the conclusion and suggestion, which not only compares the quantum game model with the classical game model but also concludes this paper. Then, the research results and relevant recommendations on business and management are summarized.

2. Literature review

2.1 Theoretical background of game theory

In this subsection, we will introduce the game theory methods used in this paper. Relevant theoretical knowledge can refer to

Nash (1950), Vives (1990), Fudenberg and Tirole (1991) and Lã *et al.* (2016).

2.1.1 Basic concepts of game theory

Game theory is a decision-making method, which originates from economic problems in society and is the process and method for game players to formulate their optimal strategies in the interaction and mutual influence. The basic elements of the game mainly include players, information, action, utility function (payoff), order and equilibrium.

The player is the main body that can make the game strategy and can maximize his payoff by making rational decisions. Information refers to the knowledge that game players can recognize and master in the game process. Before making decisions, each player will analyze the information that he has and then make decisions that are most beneficial to him, so as to optimize their game returns. Action refers to the collection of all strategies that can be made by the players under the existing game conditions. After each game player rationally formulates his strategies, he will get an expected benefit, which is the payoff of this game player and it is the optimization goal of each game player. In the process of the game, the strategies formulated by the players have an order. Each player has his own optimal game strategy, and equilibrium refers to the combination of optimal game strategies of each player.

It is assumed that the process of each player making strategies is rational, and each player can obtain all the game information. Use $G = \{N, \mathbb{S}, (U_i)_{i \in N}\}$ to represent a game with N players, where $\mathbb{S} = \boldsymbol{S}_1 \times \ldots \times \boldsymbol{S}_N$ is the strategy space defined as the Cartesian products of all individual strategy sets $\boldsymbol{S}_i$, and the payoff U_i of player i is a function of the strategy. In the game G, for any strategy $S'_i \in \boldsymbol{S}_i$, there is a strategy $S_i^* \in \boldsymbol{S}_i$ so that $U_i(S_i^*, S_{-i}) \geq U_i(S_i', S_{-i})$ holds, where S_{-i} refers to the joint strategy adopted by the opponents of player i, we say that S_i^* is the optimal response strategy of player i. The domain of S_{-i} is denoted by $\boldsymbol{S}_{-i}$.

Definition 1. The (pure) strategy profile $(S_1^*, \ldots, S_N^*) \in \mathbb{S}$ is a pure-strategy Nash equilibrium if and only if $U_i(S_i^*, S_{-i}^*) \geq U_i(S'_i, S_{-i}^*)$ for all $S'_i \in \mathbf{S}_i$ and $i \in N$.

Definition 2. Given $\boldsymbol{S}_i$, a mixed strategy ρ_i is a probability distribution over $\boldsymbol{S}_i$. The symbol $\overline{\Delta}$ represents the Cartesian products $\Delta_1 \times \ldots \times \Delta_N$ where each Δ_i refers to the set of all probability distributions over $\boldsymbol{S}_i$, with $\rho_i \in \Delta_i$.

Definition 3. The mixed strategy profile $(\rho_1^*, \ldots, \rho_N^*)$ is a mixed-strategy Nash equilibrium if and only if $U_i(\rho_i^*, \rho_{-i}^*) \geq U_i(\rho'_i, \rho_{-i}^*)$ for all $\rho'_i \in \Delta_i$ and $i \in N$.

Nash equilibrium is the most important concept in game theory. When the players of a game are at a Nash equilibrium, there is no one player can obtain the gain by deviating from this Nash equilibrium point. The relevant theorems of Nash equilibrium are as follows. For proof, see Fudenberg and Tirole (1991), pp. 29–30, 34–35 and 35–36.

Nash equilibrium existence theorem (Nash, 1950). Every finite strategy game has a Nash equilibrium in mixed strategies.

Pure-strategy Nash equilibrium existence theorem (Debreu, 1952; Fan, 1952; Glicksberg, 1952). A game has a pure-strategy Nash equilibrium if, for all $i \in N$, the strategy set $\boldsymbol{S}_i$ is a nonempty, convex and compact subset of the Euclidean space and the utility function U_i is continuous and quasi-concave in each S_i.

Mixed-strategy Nash equilibrium existence theorem (Glicksberg, 1952). A game has a mixed-strategy Nash equilibrium if, for all $i \in N$, the strategy set $\boldsymbol{S}_i$ is a nonempty compact subset of a metric space and the utility function U_i is continuous.

2.1.2 Problem of unconstrained maximization

The problem of unconstrained maximization is that of choosing values of n variables such that the objective function has a maximum.

Theorem on first-order derivative conditions (Arrow and Michael, 1981). Let $F(x)$ be a real-valued differentiable function on X, where X is a given subset of Euclidean space $\boldsymbol{E}^n$. If $F(\boldsymbol{x}^*)$ is a local maximum $\boldsymbol{x}^* \in X$, then:

$$\frac{\partial F}{\partial \mathbf{x}}(\boldsymbol{x}^*) = \left(\frac{\partial F}{\partial x_1}(\boldsymbol{x}^*), \cdots, \frac{\partial F}{\partial x_n}(\boldsymbol{x}^*)\right) = \mathbf{0}. \quad (1)$$

Theorem on second-order derivative conditions (Arrow and Michael, 1981). Let $F(\boldsymbol{x})$ be a twice differentiable real-valued function with continuous second-order partial derivatives. If $F(\boldsymbol{x}^*)$ is a local maximum $\boldsymbol{x}^* \in X$, then the $n \times n$ Hessian matrix of second-order partial derivatives of $F(x)$:

$$\frac{\partial^2 F}{\partial \boldsymbol{x}^2}(\boldsymbol{x}) = \left[\left(\frac{\partial^2 F}{\partial x_i \partial x_j}(\boldsymbol{x})\right)_{ij}\right]_{n \times n} \quad (2)$$

is negative semidefinite at $\boldsymbol{x}^*$, that is, $h^t \frac{\partial^2 F}{\partial \boldsymbol{x}^2}(\boldsymbol{x})h \leq 0$ for all $n \times 1$ column vectors h.

2.1.3 Noncooperative game analysis

In the noncooperative game model, each player makes decisions to aim for his optimal payoff. When making strategies, each player in the game only cares about whether his own payoff can reach the optimal level but not whether the payoffs of other players are affected (Vives, 1990).

If there are only two manufacturers T_1 and T_2 producing the same product in the market, the demand D_i is a function of the sell price P_i, and D_i decreases with the increase of sell price P_i, for each $i = 1, 2$. The profit function of the manufacturer T_i is $\pi_i = P_i \times D_i(P_i)$, $i = 1, 2$. The profit function of the system is $\pi_s = \pi_1 + \pi_2$.

In the noncooperative game model, T_i decides the selling price P_i to aim for the optimal profit π_i. If π_i has a maximum at P_i^*, from the first derivative condition we have $\frac{\partial \pi_i}{\partial P_i}(P_i^*) = 0$, for each $i = 0, 1$. From the second derivative condition, we have $\frac{\partial^2 \pi_1}{\partial^2 P_1}(P_1^*) \leq 0$ and $\frac{\partial^2 \pi_2}{\partial^2 P_2}(\ P_2^*) \leq 0$.

2.1.4 Cooperative game analysis

The cooperative game, also known as a positive-sum game, means that the interests of both sides of the game have increased, or at least the interests of one side have increased, whereas the interests of the other side are not damaged, so the interests of the whole system have increased. In the cooperative game model, each player aims to maximize the gain of the system (Ichiishi and Idzik, 1990).

In the cooperative game model, T_1 and T_2 decide the selling price P_1 and P_2 together, with the goal of maximizing the system profit π_s. If π_s has a maximum at (P_1^{c*}, P_2^{c*}), from the first derivative condition we have $\frac{\partial \pi_s}{\partial P_i}(P_1^{c*}, P_2^{c*}) = 0$, for all $i = 0, 1$.

From the second derivative condition, we have the 2×2 Hessian matrix:

$$\begin{bmatrix} \frac{\partial^2 \pi_s}{\partial^2 P_1}(P_1^{c*}, P_2^{c*}) & \frac{\partial^2 \pi_s}{\partial P_2 \partial P_1}(P_1^{c*}, P_2^{c*}) \\ \frac{\partial^2 \pi_s}{\partial P_1 \partial P_2}(P_1^{c*}, P_2^{c*}) & \frac{\partial^2 \pi_s}{\partial^2 P_2}(P_1^{c*}, P_2^{c*}) \end{bmatrix}$$

is negative semidefinite, that is, $h^t \frac{\partial^2 \pi_s}{\partial P_2 \partial P_1}(P_1^{c*}, P_2^{c*}) h \leq 0$ for all 2×1 column vectors h. Choose h = (1,0) and h = (0,1), we have $\frac{\partial^2 \pi_s}{\partial^2 P_i}(P_1^{c*}, P_2^{c*}) \leq 0$ for all $i = 1, 2$.

2.2 Quantum game theory

In 1999, Meyer explored the classical game theory from the perspective of a quantum algorithm and found that an optimal quantum player always beats an opponent which uses an optimal mixed strategy (Meyer, 1999). Eisert *et al.* (1999) introduced quantum strategy into the prisoner's dilemma and found that the game will no longer constitute a dilemma if the quantum strategy is allowed. There is a lot of research evidence that quantum game solves some of the difficult problems encountered in classical game theory and that quantum game is more flexible than classical game (Flitney and Hollenberg, 2007; Hong *et al.*, 2008).

2.2.1 Quantum game with n *players*

A quantum game with n players has the structure shown in Figure 1. This game starts from quantum state $|vac\rangle_1 \otimes |vac\rangle_2 \otimes \ldots |vac\rangle_n$. The quantum entangled state:

$$|\psi\rangle = \hat{\mathcal{J}}(\gamma)\left(|vac\rangle_1 \otimes |vac\rangle_2 \otimes \cdots \otimes |vac\rangle_n\right) \tag{3}$$

can be transformed from the initial state through a unitary operator:

$$\hat{\mathcal{J}}(\gamma) = \exp\left\{-\gamma\left[\sum\nolimits_{i,j=1,\ i\neq j}^{n}\left(\hat{a}_j^+ \hat{a}_i^+ - \hat{a}_j \hat{a}_i\right)\right]\right\}, \tag{4}$$

where γ is called the quantum entanglement degree and $\hat{a}_j^+$ and $\hat{a}_j$ are the creation operator and annihilation operator, respectively. For the player j, the final observation is $\hat{X}_j = \left(\hat{a}_j^+ + \hat{a}_j\right)/\sqrt{2}$, which is the position operator. Let $\mathbf{i} = \sqrt{-1}$ and $\hat{P}_j = \mathbf{i}\left(\hat{a}_j^+ - \hat{a}_j\right)/\sqrt{2}$, $\hat{P}_j$ represents the momentum operator of j. If $\hat{X}_j$ can be accurately measured, let $\hat{x}_j$ be the measurement result. When the decision is production quantity q_j, we can get $q_j = \hat{x}_j$. When the decision is selling price p_j, we can get $p_j = \hat{x}_j$.

The action strategies of player j are determined by the unitary operator $\hat{D}_j$. The set of strategies of the player j is:

$$S_j = \left\{\hat{D}_j(x_j) = \exp\left(-\mathbf{i}x_j\hat{P}_j\right) \;\middle|\; x_j \in (0, +\infty)\right\}, j = 1,\ 2, \cdots n. \tag{5}$$

The unitary operator $\hat{\mathcal{J}}^+$ can measure the state, the final state is expressed as:

$$|\psi_f\rangle = \hat{\mathcal{J}}(\gamma)^+ \left(\hat{D}_1 \otimes \hat{D}_2 \otimes \cdots \otimes \hat{D}_n\right)\hat{\mathcal{J}}(\gamma)\left(|vac\rangle_1 \otimes |vac\rangle_2 \otimes \cdots \otimes |vac\rangle_n\right). \tag{6}$$

For j = 1, 2, … n, it can be proved by mathematical induction that $\hat{\mathcal{J}}(\gamma)^+ \hat{D}_j(x_j)\hat{\mathcal{J}}(\gamma)$ is as follows (Zhou *et al.*, 2005):

$$\exp\left\{-\mathbf{i}x_j\left[\hat{P}_j \cdot \frac{1}{n}\left(e^{(n-1)\gamma} + (n-1)e^{-\gamma}\right) + \sum\nolimits_{i=1,\ i\neq j}^{n} \hat{P}_i \cdot \frac{1}{n}\left(e^{(n-1)\gamma} - e^{-\gamma}\right)\right]\right\}. \tag{7}$$

2.2.2 Quantum game with two players

Consider a quantum game with two players T_1 and T_2, they decide the selling price p_1 and p_2 to achieve their optimal profit π_i. The set of strategies of the player j is:

$$S_j = \left\{\hat{D}_j(x_j) = \exp\left(-\mathbf{i}x_j\hat{P}_j\right) \;\middle|\; x_j \in (0, +\infty)\right\}, j = 1,\ 2. \tag{8}$$

The final state is:

$$|\psi_f\rangle = \hat{\mathcal{J}}(\gamma)^+ \left(\hat{D}_1 \otimes \hat{D}_2\right)\hat{\mathcal{J}}(\gamma)\left(|vac\rangle_1 \otimes |vac\rangle_2\right). \tag{9}$$

From equation (7), we have:

$$\hat{\mathcal{J}}(\gamma)^+ \hat{D}_1(x_1)\hat{\mathcal{J}}(\gamma) = \exp\left\{-\mathbf{i}x_1\left[\hat{P}_1 \cdot \frac{1}{2}\left(e^{\gamma} + e^{-\gamma}\right) + \hat{P}_2 \cdot \frac{1}{2}\left(e^{\gamma} - e^{-\gamma}\right)\right]\right\}, \tag{10}$$

$$\hat{\mathcal{J}}(\gamma)^+ \hat{D}_2(x_2)\hat{\mathcal{J}}(\gamma) = \exp\left\{-\mathbf{i}x_2\left[\hat{P}_2 \cdot \frac{1}{2}\left(e^{\gamma} + e^{-\gamma}\right) + \hat{P}_1 \cdot \frac{1}{2}\left(e^{\gamma} - e^{-\gamma}\right)\right]\right\}. \tag{11}$$

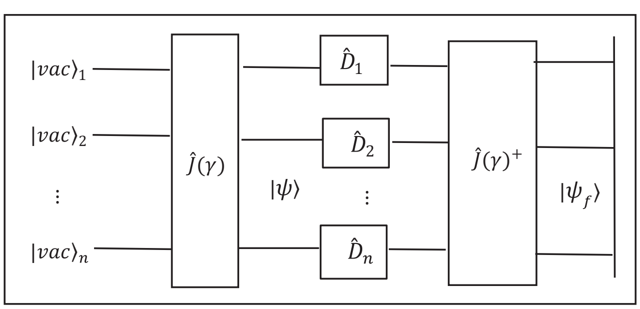
Because sinh $\gamma = (e^{\gamma} - e^{-\gamma})/2$ and cosh $\gamma = (e^{\gamma} + e^{-\gamma})/2$, we can obtain:

$$\hat{\mathcal{J}}(\gamma)^+ \hat{D}_1(x_1)\hat{\mathcal{J}}(\gamma) = \exp\left\{-\mathbf{i}x_1\left[\hat{P}_1 \cdot \cosh\gamma + \hat{P}_2 \cdot \sinh\gamma\right]\right\}, \tag{12}$$

$$\hat{\mathcal{J}}(\gamma)^+ \hat{D}_2(x_2)\hat{\mathcal{J}}(\gamma) = \exp\left\{-\mathbf{i}x_2\left[\hat{P}_2 \cdot \cosh\gamma + \hat{P}_1 \cdot \sinh\gamma\right]\right\}. \tag{13}$$

Therefore:

Figure 1 Quantum game with *n* players



$$
\begin{aligned}
|\psi_f\rangle &= \hat{\mathcal{J}}(\gamma)^+\left[\hat{D}_1(x_1)\otimes\hat{D}_2(x_2)\right]\hat{\mathcal{J}}(\gamma)\\
&= \exp\{-\mathbf{i}x_1[\hat{P}_1\cdot\cosh\gamma+\hat{P}_2\cdot\sinh\gamma]\}\\
&\quad\times\exp\{-\mathbf{i}x_2[\hat{P}_2\cdot\cosh\gamma+\hat{P}_1\cdot\sinh\gamma]\}.
\end{aligned} \tag{14}
$$

The final state becomes (Li *et al.*, 2002):

$$
\begin{aligned}
|\psi_f\rangle = {}& \exp\left\{-\mathbf{i}(x_1\cosh\gamma+x_2\sinh\gamma)\hat{P}_1\right\}|vac\rangle_1\\
&\otimes\exp\left\{-\mathbf{i}(x_2\cosh\gamma+x_1\sinh\gamma)\hat{P}_2\right\}|vac\rangle_2.
\end{aligned} \tag{15}
$$

When the decision is the selling price p_1 and p_2, we can get:

$$p_1=\hat{x}_1=x_1\cosh\gamma+x_2\sinh\gamma, \tag{16}$$

$$p_2=\hat{x}_2=x_2\cosh\gamma+x_1\sinh\gamma. \tag{17}$$

Then, the relationships between the quantum strategies and the prices are:

$$p_1(x_1,x_2)=x_1\cosh\gamma+x_2\sinh\gamma, \tag{18}$$

$$p_2(x_1,x_2)=x_2\cosh\gamma+x_1\sinh\gamma. \tag{19}$$

2.3 Research on the application of game theory

2.3.1 Application of game theory in supply chain

Many researchers explore the pricing problem and management strategies in the supply chain by using the game theory (Chiang *et al.*, 2003). Yao and Liu (2005) used Bertrand and Stackelberg model to solve the dual-channel price competition problem and obtained the optimal decision. Yan and Zhi (2009) found that online channels are not always harmful to retailers. Retailers can rise higher sales by negotiating with manufacturers to reduce wholesale prices and provide better retail services.

Hua *et al.* (2010) studied the impact of the delivery lead time of the direct channel on the channel members' pricing, they derived the optimal delivery lead time and optimal price in the dual-channel supply chain by using the two-stage optimization technique and Stackelberg game. Modak and Kelle (2019) further studied the optimal ordering and pricing strategy of a dual-channel supply chain with price and delivery time dependent on stochastic customer demand. They examined the impact of delivery lead time and customer channel preference on the optimal operation. And then use the hybrid all-unit quantity discount in the franchise fee contract to coordinate the supply chain.

Sharma and Jain (2020) established a retailer fairness model to discuss the impact of retailer's fairness concerns on partners' pricing strategies and profits. The optimal pricing strategy of channel members is derived by using the established model.

2.3.2 Application of game theory in carbon reduction

Feng *et al.* (2017) established a government and manufacturer conflict of interest game model to discuss the responses of a manufacturer to government low-carbon regulations. They found the strategies that a manufacturer and government should adopt to maximize profits under the reducing carbon emissions situation. Zhi *et al.* (2019) established a collaborative carbon emission reduction (CCER) evolutionary game model to study the role of Chinese Government policies in promoting cooperative emission reduction between suppliers and manufacturers. They found that the CCER evolution process is affected by the initial state of the supply chain, cost, additional income and investment risks related to CCER.

Adetutu *et al.* (2020) found that energy efficiency is the least response of enterprises to the UK climate change levy (CCL), whereas factor substitution and technological progress are the main response of enterprises to CCL. Liu *et al.* (2020) established a model of auto parts low-carbon supply chain to explore the optimal decision strategy. In the agricultural supply chain, if the retailers participate in the investment plan to help manufacturers to purchase equipment or develop technologies to reduce carbon emissions, the overall supply chain profit can be improved. Therefore, the environment-protecting goal and the profit-increasing goal can both be achieved, when manufacturers and retailers form a centralized supply chain (Liu *et al.*, 2021).

2.4 Impact of the carbon tax

In practical policy practice, carbon tax collection and related environmental policies may result in the spatial transfer of pollution. Fredriksson and Millimet (2002) found that there is strategic interaction in environmental regulation between regions. The asymmetry of regional environmental regulation policies will prompt enterprises to make site selection decisions, that is, to move from areas with strong environmental regulation to areas with weak environmental regulation, so as to reduce the production costs of enterprises. Enterprises may also move from areas with poor ecological resilience to areas with high ecological resilience. If the spatial transfer of pollution and ecological resilience do not be considered, only focus on the impact of the carbon tax on the economy; although the imposition of a carbon tax may have a negative impact on the economy, there is a lot of research evidence that the imposition of a carbon tax can indeed reduce greenhouse gas emissions and improve the impact of climate warming.

Lee *et al.* (2008) compared the Influence on the economy among different industrial sectors by carbon emission policies. They found that carbon tax is the only policy that has an adverse influence. The international competitive advantage of energy-intensive industries is affected by the carbon tax (Zhao, 2011). Through the comparison of the carbon tax schemes' macroeconomic effects and their influences, Liang *et al.* (2007) believe that although levy carbon tax has a negative influence, it can be alleviated by properly relieving or subsidizing. Marron and Toder (2014) point out that designing a reasonable carbon tax collection method can decrease the risk caused by climate change, minimize the emission reduction cost, incite the innovation of low-carbon technology and increase new public revenue. Murray and Rivers (2015) summed the experience and results in British Columbia. They found that greenhouse gas emissions have been reduced by 5%–15% under the levy of the carbon tax and only little influence on the economic activity of the whole. Over time, public support degree for carbon tax collection policy has gradually risen.

2.5 Green product and emission reduction cooperation

Since the first industrial revolution, with the development of technology and machines' mass production, massive waste pollution and carbon emission cause environmental degradation, global warming, sea level rise and ecological damage. The people of insight send out warnings and start planning for the earth's environmental protection strategies. New energy development, carbon emission reduction and green production have attracted the attention of all countries in the world (Chakravarty *et al.*, 2009; Wang and Feng, 2021). Governments of all countries gradually adjust their industrial structures and increase subsidies and assistance for green manufacturing.

With the implementation of the low carbon policy, how to change the marketing strategy is a realistic and serious problem faced by enterprises. Lei *et al.* (2018) studied the manufacturer's channel selection and emission reduction decisions under carbon emission constraints. They found that the key factors affecting the channel choice of manufacturers are the product attributes and the channel preferences of consumers. It is more effective and achievable to encourage the manufacturer to develop a dual-channel mode when the government set larger carbon quotas. With more and more consumers preferring online shopping, the government can formulate more stringent emission control.

Tan *et al.* (2021) studied developments and trends in research on green energy and environmental technologies. They found that publications on green energy and environmental technologies have grown exponentially, China, the USA and Italy are the most active countries. Through the bibliometric method of the research frontier identification, the categories of energy, wastewater, and performance remain stable, while the discusses trending up of catalyst and carbon dioxide emissions. The areas of technical indicators in green energy and environment technologies have gradually become research hotspots. Therefore, the importance of new energy, green production, and carbon emission reduction is increasing. With the development of e-commerce and the increase in online shopping, the dual-channel supply chain model is more able to achieve the goal of "emission reduction."

3. Basic assumptions

Figure 2 shows the research model structure, the cost of a single product produced by the manufacturer M is c, the wholesale price of this product purchased by the retailer T from the manufacturer is ω and the sales price is p_0. Manufacturer M sells this product at the price of p_1 through the online channel I. Assume that $0 < c < w < p_0$ and $0 < c < p_1$.

Assuming that θ is the amount of annual emission reduction and emission reduction cost is $f(\theta)$. The retailer T invests the proportion m of emission reduction cost, that is, the amount of the retailers' investment is $mf(\theta)$. The manufacturer M takes the proportion n of the profit from the total revenue R deducting the carbon tax payable T_c as the reward. The subsidy is based on the investment proportion of the retailer. Hence, the amount of subsidy is $(R - T_c)mn$.

3.1 Demand function and symbol description

Referring to the research of Banker *et al.* (1998), Huang and Swaminathan (2009), Mukhopadhyay *et al.* (2011) and Liu *et al.* (2021), based on prices of each channel and the consumers' sensitivity to the manufacturers' carbon emission reduction, the retailer and manufacturer's demand are Q_0 and Q_1:

$$Q_0 = a_0 - b_0 p_0 + \alpha p_1 + \beta\theta, \tag{20}$$

$$Q_1 = a_1 - b_1 p_1 + \alpha p_0 + \beta\theta, \tag{21}$$

where a_j is the potential market size, $j = 0, 1$, θ is the total annual carbon emission reduction, β is the consumers' sensitivity to the manufacturers' carbon emission reduction, $0 < \alpha < 1$ is the channel cross-elasticity coefficient between the channel of dual-channel supply chain and $b_0 > 1$ and $b_1 > 1$ are the direct price elasticity coefficients of the retail and online channel.

3.2 Basic assumptions

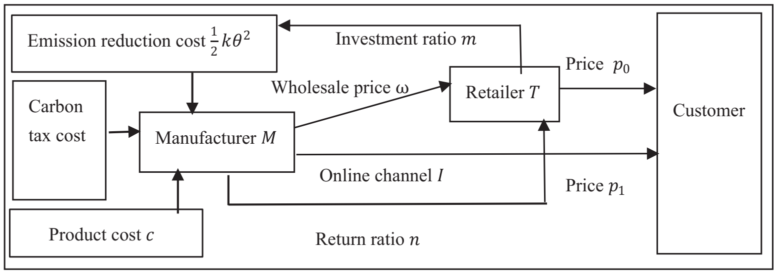
The following are the basic assumptions.

Assumption 1: Assume that the difficulty degree coefficient of reduction in carbon emissions is k and the cost of emission reduction is $f(\theta) = \frac{1}{2}k\theta^2$. The emission reduction costs borne by the manufacturer and retailer are $(1 - m)f(\theta)$ and $mf(\theta)$, respectively.

Assumption 2: Assume that λ is the rate of the carbon tax and C_e is the total original carbon emission. Because the total annual carbon emission reduction is θ, the total carbon emission is $(C_e - \theta)$. The manufacturer M must bear the cost of carbon tax $T_c = \lambda(C_e - \theta)$.

Assumption 3: Customers are generally more sensitive to price than to carbon emission reduction, so we assume that $b_j > \beta, j = 0, 1$.

Figure 2 Research model structure



3.3 Profit function

The retailer's profit π_0 and manufacturer's profit π_1 are:

$$\pi_0 = (p_0 - \omega)Q_0 - mf(\theta) + mn[(\omega - c)Q_0 + (p_1 - c)Q_1 - \lambda(C_e - \theta)], \quad (22)$$

$$\pi_1 = \left[(\omega - c)Q_0 + (p_1 - c)Q_1 - \lambda(C_e - \theta)\right](1 - mn) - (1 - m)f(\theta). \quad (23)$$

The dual-channel supply chain system profit π_{SC} is:

$$\pi_{SC} = (p_0 - c)Q_0 + (p_1 - c)Q_1 - \lambda(C_e - \theta) - \frac{1}{2}k\theta^2. \quad (24)$$

3.4 Classical model solutions

3.4.1 Decentralized decision-making in the classical model

Using the first derivative condition, if the manufacturer and retailer aim to maximize their profit, then the second derivative conditions of the profit functions π_0 and π_1 are $\frac{\partial^2 \pi_0}{\partial {p_0}^2} = -2b_0 < 0$ and $\frac{\partial^2 \pi_1}{\partial {p_1}^2} = -2b_1 < 0$, respectively. Hence, the optimal prices of the classical model exist. From the first derivative condition $\frac{\partial \pi_j}{\partial p_j} = 0$, $j = 0, 1$:

$$\frac{\partial \pi_0}{\partial p_0} = -2b_0p_0 + \alpha(1 + mn)p_1 + a_0 + \beta\theta + b_0\omega - mnb_0(\omega - c) - mnc\alpha, \quad (25)$$

$$\frac{\partial \pi_1}{\partial p_1} = \alpha p_0 - 2b_1p_1 + a_1 + \beta\theta + (\omega - c)\alpha + b_1c, \quad (26)$$

we have:

$$\begin{bmatrix} -2b_0 & \alpha(1 + mn) \\ \alpha & -2b_1 \end{bmatrix} \begin{bmatrix} p_0^* \\ p_1^* \end{bmatrix} = \begin{bmatrix} -(L_1 - mnL_2) \\ -L_3 \end{bmatrix}, \quad (27)$$

where $L_1 = a_0 + \beta\theta + b_0\omega$, $L_2 = b_0(\omega - c) + c\alpha$ and $L_3 = a_1 + \beta\theta + (\omega - c)\alpha + b_1c$. By using $b_j > 1 > n, m, \alpha, j = 0, 1$, the determinant:

$$|D| = \begin{vmatrix} -2b_0 & \alpha(1 + mn) \\ \alpha & -2b_1 \end{vmatrix} = 4b_0b_1 - (1 + mn)\alpha^2 > 0.$$

By Cramer's rule, the optimal prices are:

$$p_0^* = \frac{1}{|D|}\left[2b_1(L_1 - mnL_2) + \alpha(1 + mn)L_3\right], \quad (28)$$

$$p_1^* = \frac{1}{|D|}[\alpha(L_1 - mnL_2) + 2b_0L_3], \quad (29)$$

where $L_1 = a_0 + \beta\theta + b_0\omega$, $L_2 = b_0(\omega - c) + c\alpha$ and $L_3 = a_1 + \beta\theta + (\omega - c)\alpha + b_1c$.

3.4.2 Centralized decision-making in the classical model

If the manufacturer and retailer aim to maximize the overall supply chain's profit, the dual-channel supply chain is regarded as a system. The second derivative conditions of the profit function π_{sc} are $\frac{\partial^2 \pi_{sc}}{\partial {p_0}^2} = -2b_0 < 0$ and $\frac{\partial^2 \pi_{sc}}{\partial {p_1}^2} = -2b_1 < 0$. Hence, the optimal prices of the classical model exist. From the first derivative condition $\frac{\partial \pi_{sc}}{\partial p_j} = 0, j = 0, 1$, we have:

$$\begin{bmatrix} -2b_0 & 2\alpha \\ 2\alpha & -2b_1 \end{bmatrix} \begin{bmatrix} \overline{P}{}_0^* \overline{P}{}_1^* \end{bmatrix} = \begin{bmatrix} -(L_1 - L_2) \\ -(L_3 - \omega\alpha) \end{bmatrix}. \quad (30)$$

The determinant:

$$|\overline{D}| = \begin{vmatrix} -2b_0 & 2\alpha \\ 2\alpha & -2b_1 \end{vmatrix} = 4b_0b_1 - 4\alpha^2 > 0.$$

By Cramer's rule, the optimal prices are:

$$\overline{p}_0^* = \frac{1}{|\overline{D}|}\left[2b_1(L_1 - L_2) + 2\alpha(L_3 - \omega\alpha)\right], \quad (31)$$

$$\overline{p}_1^* = \frac{1}{|\overline{D}|}\left[2\alpha(L_1 - L_2) + 2b_0(L_3 - \omega\alpha)\right], \quad (32)$$

where $L_1 = a_0 + \beta\theta + b_0\omega$, $L_2 = b_0(\omega - c) + c_\alpha$ and $L_3 = a_1 + \beta\theta + (\omega - c)\alpha + b_1$c.

4. Quantum game model under carbon tax constraints and investment cooperation

Assuming that the quantum structure of the model is as shown in Figure 3, the retailer T and manufacturer M start the quantum game from the initial state $|vac\rangle_0 \otimes |vac\rangle_1$. The quantum entangled state:

$$|\psi\rangle = \hat{\mathcal{J}}(\gamma)\left(|vac\rangle_0 \otimes |vac\rangle_1\right) \quad (33)$$

can be transformed from the initial state through a unitary operator:

$$\hat{\mathcal{J}}(\gamma) = \exp\left[i\gamma\left(\hat{X}_0\hat{P}_1 + \hat{X}_1\hat{P}_0\right)\right], \quad (34)$$

where γ is called the quantum entanglement degree, $\hat{X}_j = \left(\hat{a}_j^+ + \hat{a}_j\right)/\sqrt{2}$, $\hat{P}_j = i\left(\hat{a}_j^+ - \hat{a}_j\right)/\sqrt{2}$, $\hat{a}_j^+$ and $\hat{a}_j$ are the creation operator and annihilation operator, respectively, $j = 0, 1$.

The action strategies of the retailer T and manufacturer M are determined by unitary operators $\hat{D}_0$ and $\hat{D}_1$. The set of strategies is:

$$S_j = \left\{\hat{D}_j(x_j) = \exp\left(x_j\left(\hat{a}_j^+ - \hat{a}_j\right)/\sqrt{2}\right) \;\middle|\; x_j \in (0, +\infty)\right\}, \quad j = 0, 1. \quad (35)$$

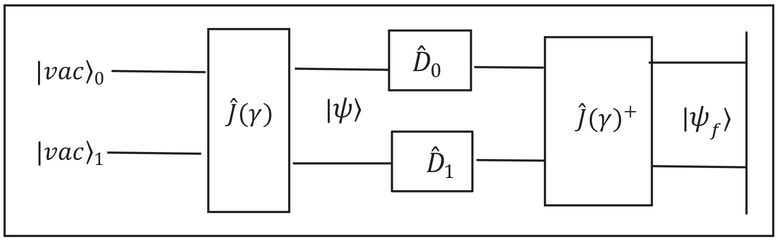
The unitary operator $\hat{j}^+$ can measure the state, the final state is expressed as:

$$|\psi_f\rangle = \hat{\mathcal{J}}(\gamma)^+\left(\hat{D}_0 \otimes \hat{D}_1\right)\hat{\mathcal{J}}(\gamma)\left(|vac\rangle_0 \otimes |vac\rangle_1\right). \quad (36)$$

4.1 Quantum game model of cooperative emission reduction

The relationships between the quantum strategies and the prices are:

Figure 3 Quantum structure of the model



$$p_0(x_0, x_1) = x_0 \cosh\gamma + x_1 \sinh\gamma, \tag{37}$$

$$p_1(x_0, x_1) = x_1 \cosh\gamma + x_0 \sinh\gamma, \tag{38}$$

where $\sinh\gamma = (e^{\gamma} - e^{-\gamma})/2$ and $\cosh\gamma = (e^{\gamma} + e^{-\gamma})/2$. Bring equations (37) and (38) into equations (20) and (21), the demand functions Q_0 and Q_1 can be converted as follows:

$$Q_0 = (a_0 + \beta\theta) + (-b_0 \cosh\gamma + \alpha \sinh\gamma)x_0 + (-b_0 \sinh\gamma + \alpha \cosh\gamma)x_1, \tag{39}$$

$$Q_1 = (a_1 + \beta\theta) + (-b_1 \sinh\gamma + \alpha \cosh\gamma)x_0 + (-b_1 \cosh\gamma + \alpha \sinh\gamma)x_1. \tag{40}$$

4.2 Decentralized decision-making in the quantum model

When the partners in the dual-channel supply chain cooperate to reduce emissions, if the manufacturer and retailer aim to maximize their profit respectively for the quantum game, then the optimal prices p_0^{d*} and p_1^{d*} can be obtained as the following theorem.

Theorem 1. Under decentralized decision-making, the optimal prices p_0^{d*} and p_1^{d*} of the retailer and manufacturer are:

$$p_0^{d*} = \frac{1}{|A|}\Big\{\cosh^2\gamma\left[2b_1(L_1 - mnL_2) + \alpha(1+mn)L_3\right] - \cosh\gamma\sinh\gamma[\alpha(L_1 + L_2 + 2\omega b_1)] + \sinh^2\gamma\left[mn(2b_1L_2 - \alpha L_3) + \omega\alpha^2\right]\Big\}, \tag{41}$$

$$p_1^{d*} = \frac{1}{|A|}\Big\{\cosh^2\gamma[\alpha(L_1 - mnL_2) + 2b_0L_3] - \cosh\gamma\sinh\gamma\left[2b_0L_2 + \alpha L_3 + \omega\alpha^2\right] + \sinh^2\gamma[\alpha(1+mn)L_2]\Big\}, \tag{42}$$

where the determinant:

$$|A| = 4b_0b_1\cosh^2\gamma - 2\alpha(b_0 + b_1)\cosh\gamma\sinh\gamma - (1+nm)\alpha^2, \tag{43}$$

and $L_1 = a_0 + \beta\theta + b_0\omega$, $L_2 = b_0(\omega - c) + c\alpha$, $L_3 = a_1 + \beta\theta + b_1c + \alpha(\omega - c)$.

Proof of Theorem 1. Refer to Appendix 1.

Note: (1) If the quantum entanglement degree $\gamma = 0$, the determinant:

$$|A| = 4b_0b_1 - (1+nm)\alpha^2 = |D|,$$

and the optimal prices:

$$p_0^{d*} = \frac{1}{|D|}\left[2b_1(L_1 - mnL_2) + \alpha(1+mn)L_3\right] = p_0^*,$$
$$p_1^{d*} = \frac{1}{|D|}[\alpha(L_1 - mnL_2) + 2b_0L_3] = p_1^*.$$

In other words, when the quantum entanglement degree $\gamma = 0$, the quantum game model is consistent with the classical game model.
(2) When the quantum entanglement tends to infinity, the optimal prices are converged as follows:

$$p_0^{d*} = \frac{(2b_1 - \alpha)L_1 + \alpha(L_3 - L_2 - 2\omega b_1 + \omega\alpha)}{4b_0b_1 - 2\alpha(b_0 + b_1)},$$
$$p_1^{d*} = \frac{\alpha(L_1 + L_2 + L_3 + \omega\alpha)}{4b_0b_1 - 2\alpha(b_0 + b_1)}.$$

(3) If the retailer does not invest in the carbon emission reduction project, in this case, $m = 0$ and $n = 0$, the determinant becomes:

$$|A| = 4b_0b_1\cosh^2\gamma - 2\alpha(b_0 + b_1)\cosh\gamma\sinh\gamma - \alpha^2,$$

and the optimal prices become:

$$p_0^{d*} = \frac{1}{|A|}\{\cosh^2\gamma[2b_1L_1 + \alpha L_3] - \cosh\gamma\sinh\gamma[\alpha(L_1 + L_2 + 2\omega b_1)] + \sinh^2\gamma[\omega\alpha^2]\},$$

$$p_1^{d*} = \frac{1}{|A|}\{\cosh^2\gamma[\alpha L_1 + 2b_0L_3] - \cosh\gamma\sinh\gamma\left[2b_0L_2 + \alpha L_3 + \omega\alpha^2\right] + \sinh^2\gamma[\alpha L_2].$$

Corollary 1. Under decentralized decision-making, the consumers' sensitivity β and the total annual carbon emission reduction θ affect the retailer and manufacturer's selling prices. In fact:

(1) $\frac{\partial p_j^{d*}}{\partial\theta} > 0$, $j = 0, 1$. *Both the retailer and manufacturer's selling prices are increasing functions of θ. When the total annual carbon emission reduction increases, the retailer and manufacturer's selling prices increase.*

(2) $\frac{\partial p_j^{d*}}{\partial\beta} > 0$, $j = 0, 1$. *Both the retailer and manufacturer's selling prices are increasing functions of β. When the consumers' sensitivity β to carbon emission reduction increases, the retailer and manufacturer's selling prices increase.*

Proof. By using $\sinh\gamma = (e^{\gamma} - e^{-\gamma})/2$ and $\cosh\gamma = (e^{\gamma} + e^{-\gamma})/2$, we have:

$$\frac{\partial p_0^{d*}}{\partial\theta} = \frac{\beta}{4|A|}\left[2b_1e^{2\gamma} + (2b_1 + 2\alpha)e^{-2\gamma} + (4b_1 + 2\alpha + mn\alpha)\right] > 0,$$

$$\frac{\partial p_1^{d*}}{\partial\theta} = \frac{\beta}{4|A|}\left[(2b_0e^{2\gamma} + (2b_0 + 2\alpha)e^{-2\gamma} + 4b_0 + 2\alpha\right] > 0,$$

$$\frac{\partial p_0^{d*}}{\partial\beta} = \frac{\theta}{4|A|}\left[2b_1e^{2\gamma} + (2b_1 + 2\alpha)e^{-2\gamma} + (4b_1 + 2\alpha + mn\alpha)\right] > 0,$$

$$\frac{\partial p_1^{d*}}{\partial\beta} = \frac{\theta}{4|A|}\left[(2b_0e^{2\gamma} + (2b_0 + 2\alpha)e^{-2\gamma} + 4b_0 + 2\alpha\right] > 0.$$

This completes the proof.

4.3 Centralized decision-making in the quantum model

If the manufacturer and retailer aim to maximize the overall supply chain's profit, the dual-channel supply chain is regarded as a system. In this case, there is no quantum phenomenon. Then, the optimal prices p_0^{c*} and p_1^{c*} can be obtained as the following theorem.

Theorem 2. Under centralized decision-making, the optimal prices p_0^{c*} and p_1^{c*} of the retailer and manufacturer are:

$$p_0^{c*} = \frac{2(b_1V_0 + \alpha V_1)}{|\overline{A}|} = \frac{2\left[b_1(L_1 - L_2) + \alpha(L_3 - \alpha\omega)\right]}{|\overline{A}|} \quad (44)$$

$$p_1^{c*} = \frac{2(\alpha V_0 + b_0V_1)}{|\overline{A}|} = \frac{2\left[\alpha(L_1 - L_2) + b_0(L_3 - \alpha\omega)\right]}{|\overline{A}|} \quad (45)$$

where the determinant:

$$|\overline{A}| = 4\left(b_0b_1 - \alpha^2\right), \quad (46)$$

and $V_0 = a_0 + \beta\theta + b_0c - \alpha c = L_1 - L_2$, $V_1 = a_1 + \beta\theta + b_1c - \alpha c = L_3 - \alpha\omega$.

Proof of Theorem 2. Refer to Appendix 2.

Note: Under centralized decision-making, the members of the dual-channel supply chain make decisions together, there is no competition between them, so the phenomenon of quantum entanglement does not exist. Hence, the optimal solutions of the quantum game model are consistent with the classical game. That is, $p_0^{c*} = \overline{p}_0^*$ and $p_1^{c*} = \overline{p}_1^*$.

Corollary 2. Under centralized decision-making, the consumers' sensitivity β and the total annual carbon emission reduction θ affect the retailer and manufacturer's selling prices. In fact:

(1) $\frac{\partial p_j^{c*}}{\partial\theta} > 0, j = 0, 1$. *Both the retailer and manufacturer's selling prices are increasing functions of θ. When the total annual carbon emission reduction increases, the retailer and manufacturer's selling prices increase.*

(2) $\frac{\partial p_j^{c*}}{\partial\beta} > 0, j = 0, 1$. *Both the retailer and manufacturer's selling prices are increasing functions of β. When the consumers' sensitivity β to carbon emission reduction increases, the retailer and manufacturer's selling prices increase.*

Proof. By a direct calculation, we have:

$$\frac{\partial p_0^{c*}}{\partial\theta} = \frac{2\beta}{|\overline{A}|}[b_1 + \alpha] > 0, \frac{\partial p_1^{c*}}{\partial\theta} = \frac{2\beta}{|\overline{A}|}[b_0 + \alpha] > 0,$$
$$\frac{\partial p_0^{c*}}{\partial\beta} = \frac{2\theta}{|\overline{A}|}[b_1 + \alpha] > 0, \frac{\partial p_1^{c*}}{\partial\beta} = \frac{2\theta}{|\overline{A}|}[b_0 + \alpha] > 0.$$

This completes the proof.

Corollary 3. Under centralized decision-making, if:

$$\beta\theta > \frac{(2b_0b_1 + \alpha^2)c}{b_0 + b_1 + 2\alpha}, \quad (47)$$

the dual-channel supply chain system profit π_{SC} is an increasing function of β. That is, $\frac{\partial\ \pi_{SC}}{\partial\beta} > 0, j = 0, 1$.

Proof. By a direct calculation, we have:

$$\frac{\partial\ \pi_{SC}}{\partial\beta} = \frac{2\theta}{|\overline{A}|^2}\left[(b_0 + b_1 + 2\alpha)\beta\theta - (2b_0b_1 + \alpha^2)c\right.$$
$$\left. + 3c\alpha^2\left(4b_0b_1 - \alpha^2\right) + U\right],$$

where $U = 2(2b_0b_1 + a^2)\ (a_0(b_1 + a) + a_1(b_0 + a))$. If $\beta\theta > \frac{(2b_0b_1 + \alpha^2)c}{b_0 + b_1 + 2\alpha}$, we can obtain $\frac{\partial\ \pi_{SC}}{\partial\beta} > 0$.

4.4 Management significance

Whether decentralized or centralized decision-making is adopted, the following facts $\frac{\partial p_j^{d*}}{\partial\theta} > 0$ and $\frac{\partial p_j^{c*}}{\partial\theta} > 0$, $j = 0, 1$, in Corollary 1 and Corollary 2 tell us that the increase of the total amount of annual emission reduction θ is probably to increase the cost, and then the increased cost of emission reduction will eventually be passed on to consumers, resulting both manufacturer and retailer's selling prices have increased. Therefore, for the sustainable development of the earth, carbon emission reduction is imperative, and consumers need to accept the rise in product prices caused by carbon emission reduction. How to develop low-cost carbon reduction technologies and equipment to reduce product prices and attract consumers has become a problem that manufacturers must face.

With consumers' attention to carbon emission reduction, consumers' sensitivity β increases. The facts $\frac{\partial p_j^{d*}}{\partial\beta} > 0$ and $\frac{\partial p_j^{c*}}{\partial\beta} > 0$, $j = 0, 1$, in Corollary 1 and Corollary 2 show that consumers' acceptance of the rise in sales prices of the manufacturer and retailer increases with consumers' attention to carbon emission reduction.

If the consumer's sensitivity β greater than a certain threshold, the supply chain system's profit increases with the consumer's sensitivity β increases. Publicity and strengthening education on carbon emission reduction for consumers can improve the consumers' sensitivity to carbon emission reduction by improving consumers' attention, then encourage consumers to actively choose carbon emission reduction products and finally increase the dual-channel supply chain's profit.

To sum up, to cope with the carbon tax and make a contribution to environmental protection, business managers should invest in carbon emission reduction. But for the competitiveness of products, they should develop low-cost carbon reduction technologies. It is necessary to strengthen the environmental protection education of consumers, improve their sensitivity to carbon emission reduction and make consumers more likely to accept carbon emission reduction products.

5. Numerical analysis

This section studies the quantum model in the previous by numerical analysis and sensitivity analysis and is divided into decentralized and centralized decision-making to discuss. The parameters of numerical analysis are shown in Table 1. The numerical analysis is a simulation, but all the values of variables are reasonable and completely based on the theoretical model. For example, the direct price elasticity coefficient for the retail channel b_0 and the direct price elasticity coefficient for the online channel b_1 are all greater than 1, so $b_0 > 1$ and $b_1 > 1$. Because consumers in retail channels are more vulnerable to price fluctuations and switch to online channels to purchase, therefore, the direct price elasticity coefficient b_0 of retail channels is greater than the direct price elasticity coefficient b_1 of online channels. So, we assume that $b_0 = 1.6$ and $b_1 = 1.5$. The channel cross-elasticity coefficient α, carbon tax rate λ, investment ratio m and proportion of the profit as the reward n are all less than 1, so we choose $\alpha = 0.5$, $\lambda = 0.2$, $m = 0.3$ and $n = 0.05$.

Table 1 Value of model variables in numerical analysis

Symbol	Description	Value
a_0	Potential market size to Q_0	1,500
a_1	Potential market size to Q_1	1,000
b_0	Direct price elasticity coefficient for the retail channel	1.6
b_1	Direct price elasticity coefficient for the online channel	1.5
α	Channel cross-elasticity coefficient	0.5
c	Product cost	200
ω	Wholesale price	220
k	Difficulty degree coefficient of reduction in carbon emissions	0.8
m	Investment ratio	0.3
n	Proportion of the profit as the reward	0.05
λ	Carbon tax rate	0.2
C_e	Original carbon emission during product-making process	5,000

5.1 Decentralized decision-making analysis

We discuss the impact of the quantum entanglement degree γ, the customer's sensitivity β and the total amount of emission reduction θ on selling prices and profits, respectively.

5.1.1 Influence of quantum entanglement

When the partners in the dual-channel supply chain cooperate to reduce emission, choose the consumer's sensitivity $\beta = 0.6$ and the total amount of emission reduction $\theta = 1{,}000$. Assuming that the quantum entanglement γ increases, the influence of quantum entanglement on selling prices and profits under decentralized decision-making is shown in Table 2 and Figure 4.

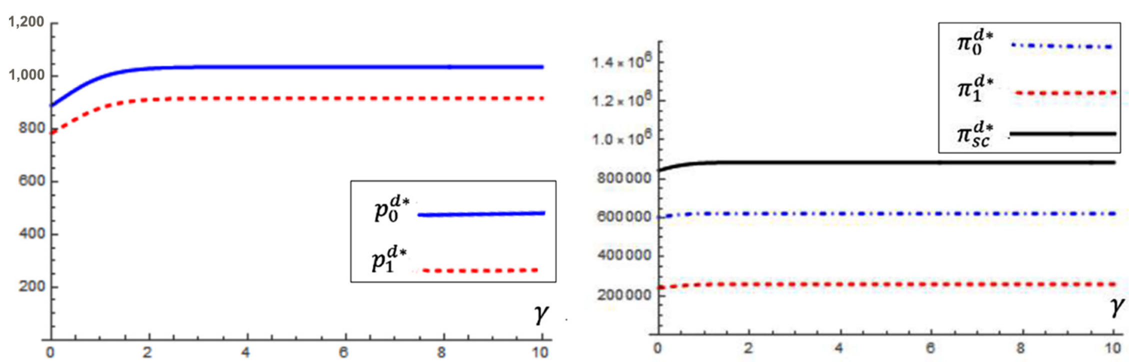
The retailer and manufacturer will adjust their product price strategy because of another's price strategy when making product sales price, so as to counter it to benefit market competition. This status of adjusting the competitive strategy according to the competitor's strategy forms a phenomenon of quantum entanglement. The significance of the quantum entanglement degree is to adjust the adjustment range or speed of the competitive strategy according to the competitor's strategy. The increase in quantum entanglement degree means that the reaction speed to competitors' strategy increases or the range of price change is increased.

From Table 2 and Figure 4 it can be seen that, with the increase of the quantum entanglement degree, the retailer

Table 2 Influence of quantum entanglement

Quantum entanglement γ	0	0.1	1	5	10
Retailer price p_0^{d*}	890.13	902.47	997.00	1,037.52	1,037.54
Online price p_1^{d*}	785.02	795.92	881.17	918.69	918.71
Retailer's profit π_0^{d*}	603,823	607,295	622,772	623,450	623,449
Manufacturer's profit π_1^{d*}	240,170	243,228	258,995	261,295	261,295
Overall system profit π_{sc}^{d*}	843,993	850,523	881,768	884,744	884,774

Figure 4 Selling prices and profits vary with the quantum entanglement



price, online price and the profits of the manufacturer, retailer and overall supply chain show an upward trend. When the quantum entanglement degree is 0, the solution of the quantum model is consistent with the classical model. In this case, the retailer and manufacturer's selling prices are the lowest and the profits of the retailer, manufacturer and overall supply chain system are the lowest. Therefore, the solution of the quantum game model is better than that of the classical game model.

5.1.2 Influence of the consumer's sensitivity to carbon emission reduction

Choose the quantum entanglement degree $\gamma = 1$ and the total amount of carbon emission reduction θ = 1,000. Choose the quantum entanglement degree $\gamma = 1$ and the total amount of carbon emission reduction θ = 1,000. With the consumer's sensitivity β increases, the influence of consumer's sensitivity β on selling prices and profits under decentralized decision-making is shown in Table 3 and Figure 4.

The retailer price, online price and the profits of the manufacturer, retailer and overall supply chain show an upward trend. Figure 5 verifies the result of Corollary 1 that both the retailer and manufacturer's selling prices are increasing functions of β. When the customers' sensitivity β to carbon emission reduction is rise that represents that customers' degree of pay attention to and of acceptance for carbon emission reduction products is rise. Consumers willingly accept high-degree emission reduction products with a higher price for supporting human sustainable development. Therefore, the manufacturer and retailer can set higher selling prices for high-degree emission reduction products and obtain more profits.

5.1.3 Influence of the total amount of emission reduction

Choose the quantum entanglement degree $\gamma = 1$ and consumer's sensitivity $\beta = 0.6$ to carbon emission reduction carbon. The influence of the total emission reduction is shown in Table 4 and Figure 6.

With the increase in the total carbon emission reduction, the retailer price, online price and profit of the retailer show an upward trend, but the profits of manufacture and overall supply chain system show a trend of rising first and then falling. Figure 6 verifies the result of Corollary 1 that the selling price of the retailer and manufacturer is an increasing function of θ, respectively.

When the supply chain members cooperate to reduce emissions, the investment in purchasing equipment or developing technology to reduce carbon emissions is borne by

Table 3 Influence of the consumer's sensitivity to carbon emission reduction

Consumer's sensitivity β	0.2	0.4	0.6	0.8	1
Retailer price p_0^{d*}	821.90	909.45	997.00	1,084.56	1,172.11
Online price p_1^{d*}	695.03	788.10	881.17	974.25	1,067.32
Retailer's profit π_0^{d*}	325,305	464,573	622,772	799,903	995,964
Manufacturer's profit π_1^{d*}	10,798.5	125,347	258,995	411,473	583,590
Overall system profit π_{sc}^{d*}	336,103	589,920	881,768	1,211,646	1,579,554

Figure 5 Selling prices and profits vary with the consumer's sensitivity

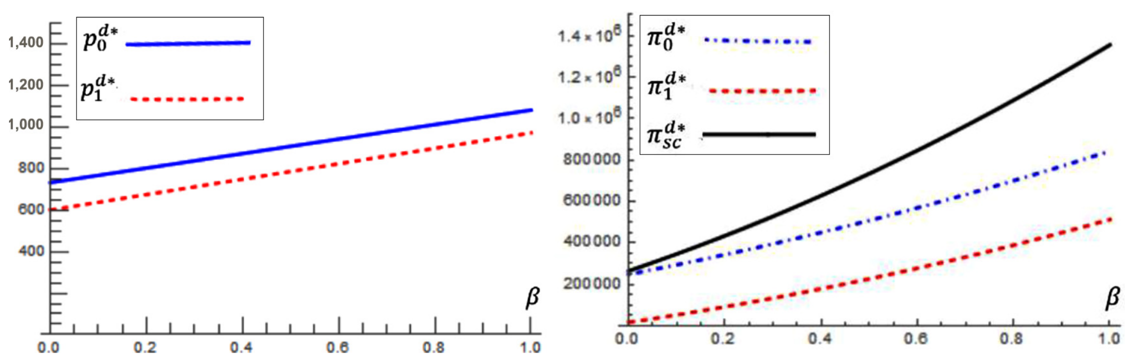
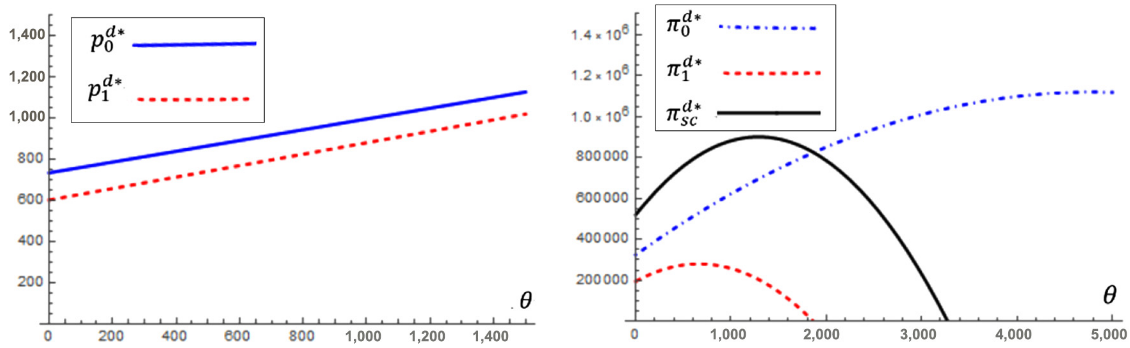


Table 4 Influence of the total emission reduction

Carbon emission reduction θ	200	400	600	800	1,000
Retailer price p_0^{d*}	786.88	839.41	891.94	944.47	997.00
Online price p_1^{d*}	657.80	713.64	769.49	825.33	881.17
Retailer's profit π_0^{d*}	390,096	452,442	512,004	568,780	622,772
Manufacturer's profit π_1^{d*}	238,969	267,262	280,031	277,275	258,995
Overall system profit π_{sc}^{d*}	629,065	719,704	792,034	846,056	881,768

Figure 6 Selling prices and profits vary with the total emission reduction



the retailer and manufacturer. Therefore, it is bound to lead to an increase in product cost and product prices. The increase in the total carbon emission reduction θ represents an increase in the cost of capital invested in the purchase of equipment or the developing technology, resulting in the rise of product prices. The retailer, manufacturer and overall supply chain system can benefit from higher selling prices. However, when the total emission reduction increases to a certain threshold, the increase of the total emission reduction will cause the high capital cost of investing in the purchase of equipment or the development of technology, form a major burden, reduce the manufacturer's profit and affect the profit of overall supply chain. The retailer bears part of the cost of emission reduction, which affects its profit, but it also receives part of the profit of the manufacturer, so its profit does not reduce.

5.2 Centralized decision-making analysis

Under centralized decision-making, the manufacturer and retailer make decisions with the goal to maximize the supply chain system's profit. In this case, the supply chain system is regarded as a whole to make decisions, there is no competition, so there is no quantum entanglement.

5.2.1 Influence of the customer's sensitivity to carbon emission reduction

We use the parameters in Table 1, and choose the total amount of carbon emission reduction $\theta = 1{,}000$.

With the consumer's sensitivity β increases, the influence of consumer's sensitivity β on selling prices and the overall supply chain's profit under centralized decision-making is shown in Table 5 and Figure 7. With the consumer's sensitivity β increases, the retailer price, online price and the overall supply chain's profit show an upward trend. Figure 7 verifies the result of Corollary 2 that the selling price of the retailer and manufacturer is an increasing function of β, respectively.

When the customers' sensitivity β to carbon emission reduction is on the rise, consumers are willing to accept high-degree emission reduction products at a higher price, then the dual-channel supply chain system can set higher selling prices for high-degree emission reduction products and obtain more profit.

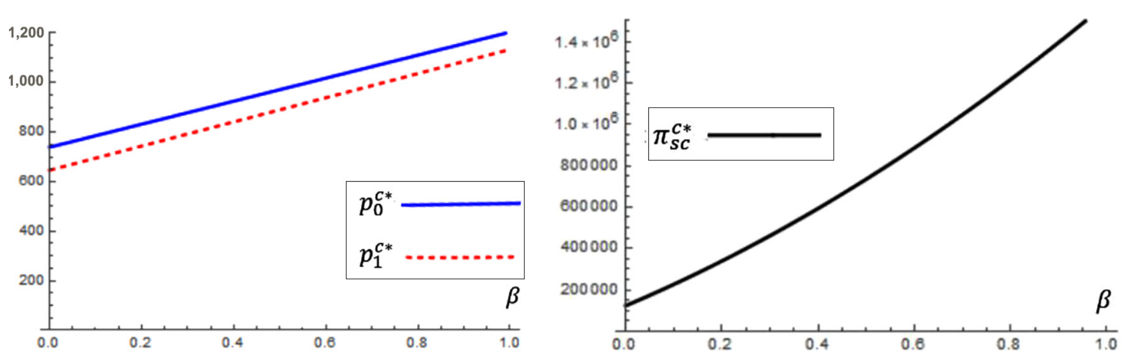
5.2.2 Influence of the total emission reduction

Choose the consumer's sensitivity $\beta = 0.6$. Assuming the total emission reduction θ increases, the influence of the total

Table 5 Influence of β under centralized decision-making

Consumer's sensitivity β	0.2	0.4	0.6	0.8	1
Retailer price p_0^{c*}	832.558	925.58	1,018.60	1,111.63	1,204.65
Online price p_1^{c*}	744.186	841.86	939.54	1,037.21	1,134.88
Overall system profit π_{sc}^{c*}	339,386	593,805	886,363	121,706	158,590

Figure 7 Selling prices and profit vary with β under centralized decision-making



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emission reduction on prices and profit under centralized decision-making is shown in Table 6 and Figure 8.

With the increase of the total emission reduction θ, the retailer price, online price and the overall supply chain's profit show an upward trend. Figure 8 verifies the result of Corollary 2 that the selling price of the retailer and manufacturer is an increasing function of θ, respectively.

When the dual-channel supply chain system carries out carbon emission reduction, the investment in purchasing equipment or developing carbon emission reduction technology is borne by the overall supply chain system. The increased cost is passed on to consumers, then resulting in the product's selling price increase. When the total amount of carbon emission reduction θ is less than a certain threshold, the dual-channel supply chain system can obtain higher profit from higher selling prices. However, when the total emission reduction θ is greater than a certain threshold, the investment of purchase equipment or developing technology to reduce carbon emission will cause a heavy financial burden and affect the overall supply chain's profit. And when the total amount of annual emission reduction is too large, the profit of that year will be negative.

5.2.3 Customer's attention and the manufacturer's emission reduction

We use the parameters in Table 1 and assume that the consumer's sensitivity β to emission reduction and the total emission reduction θ increase, and the influence on profit of the overall supply chain system under centralized decision-making is shown in Figure 9. Figure 9 verifies Corollary 3, when $\beta\theta > \frac{(2b_0b_1+\alpha^2)c}{b_0+b_1+2\alpha}$ holds, the overall supply chain's profit is an increasing function of customers' sensitivity β.

From Figure 9 it can be seen that, when the manufacturer carries out emission reduction, the overall supply chain system transfers the cost of carbon reduction to consumers and obtains high profit from the high selling prices. The more carbon emission reduction, the higher profit. However, if the customer's sensitivity β to carbon emission reduction is not high enough, the more difficult it is for customers to accept the high prices of carbon reduction products. As a result, the product of the manufacturer that strives to achieve carbon emission reduction will not be accepted by consumers because of the high price due to the high total emission reduction, finally resulting in a reduced profit.

5.2.4 Comparison of decentralized and centralized decision-making

The profit comparison between the two decision-making is shown in Figure 10. After comparing the profit between decentralized and centralized decision-making, we can find that the centralized decision-making's overall profit is higher than the decentralized decision-making's overall profit. This is because the manufacturer and retailer aim to maximize each profit under decentralized decision-making, but the overall supply chain's profit cannot be maximized because of mutual competition. Under centralized decision-making, the retailer and manufacturer aim at maximizing the overall supply chain's profit. Therefore, for the overall supply chain's profit, centralized decision-making is more economical and efficient than decentralized decision-making.

6. Conclusions and suggestions

6.1 Conclusions

This paper studies a green product optimal strategy problem of the dual-channel supply chain under carbon tax constraints and emission reduction investment cooperation from the quantum game perspective. The classical game model and quantum game model of the dual-channel supply chain are established, and the optimal solutions of the two models are obtained under decentralized and centralized decision-making, respectively. Comparing the quantum game model with the classical game model, we find the following facts: when the quantum entanglement degree is 0, the solutions of the quantum model are consistent with the classical model and when the quantum

Table 6 Influence of θ under centralized decision-making

Carbon emission reduction θ	200	400	600	800	1,000
Retailer price p_0^{c*}	795.35	851.16	906.98	962.79	1,018.60
Online price p_1^{c*}	705.12	763.72	822.33	880.93	939.54
Supply chain's profit π_{sc}^{c*}	632,138	723,099	795,790	850,211	886,363

Figure 8 Centralized decision-making's selling prices and profit varying with θ

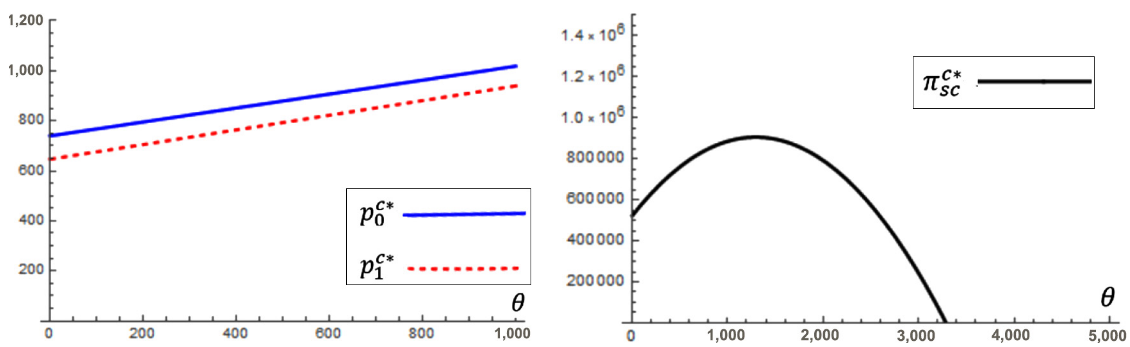

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Figure 9 Overall supply chain's profit varies with θ and β

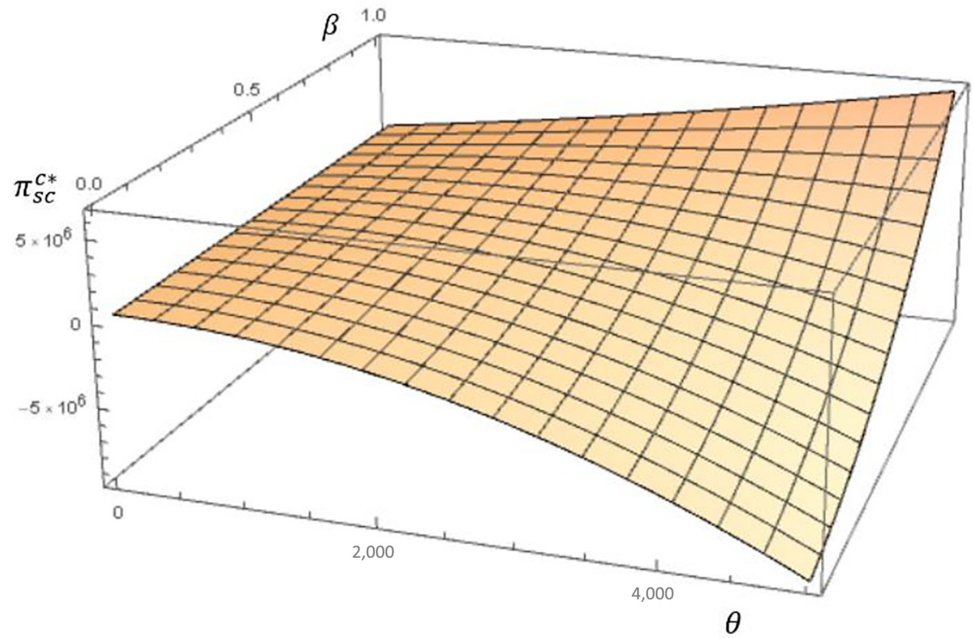
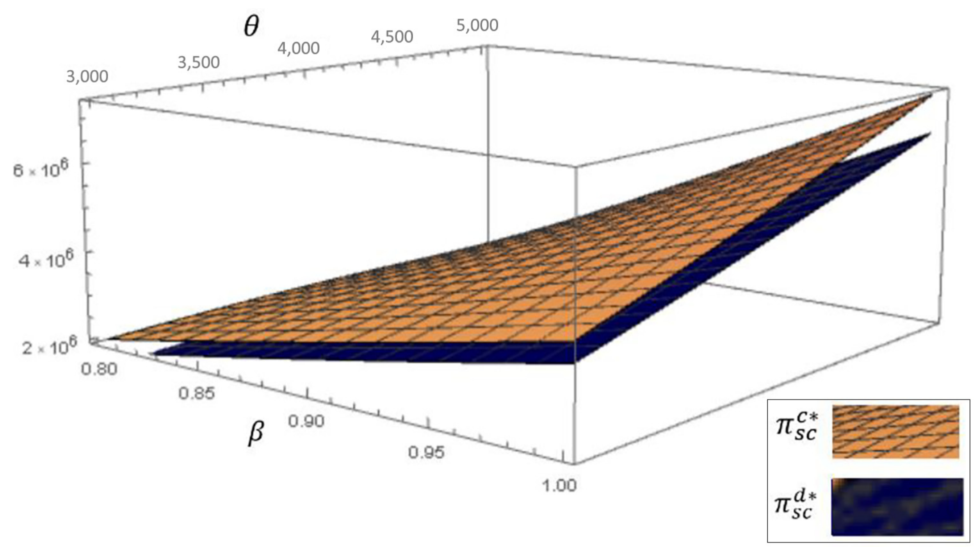


Figure 10 Profit comparison between two decision-making



entanglement degree is greater than 0, the solutions of the quantum game model are higher than the solutions of the classical game model, including the product sales price of the manufacturer and retailer and the profit of the manufacturer, retailer and the overall supply chain system. Therefore, the method of the quantum game model is better than that of the classical game model.

Through theoretical research and numerical analysis results, we have the following conclusions. For the overall supply chain system's profit, the profit obtained by centralized decision-making is higher than that of decentralized. In other words, centralized decision-making is more economical and efficient than decentralized decision-making in pursuing the maximum overall supply chain profit.

Whether decentralized decision-making or centralized decision-making is adopted, when the dual-channel supply chain members cooperate to reduce emissions under carbon tax constraints, the increased cost to purchase equipment or develop technology will eventually be transferred to consumers, resulting the manufacturer and retailer's selling prices increase. If the consumer's sensitivity to the manufacturer reducing carbon emissions is not enough, it will lead to low consumer recognition of the green products and will be difficult to accept high product prices, the profits of the overall system and every member in the dual-channel supply chain will reduce. Therefore, it is important to publicize and strengthen carbon emission reduction education for consumers, which can improve consumers' attention and sensitivity to manufacturers reducing carbon emissions, promote consumers to choose low-carbon products and increase the profits of the overall system and every member in the dual-channel supply chain.

The increase in the total emission reduction represents an increase in the cost of purchasing equipment or developing technology to reduce carbon emissions, resulting in an increase in product prices. The overall system and every member in the dual-channel supply chain can obtain higher profits from higher product prices. When the total amount of emission reduction increases too fast, the investment cooperation capital cost of purchasing equipment or developing technology to reduce carbon emission will be too high, forming a major burden. Too high product prices will lead consumers to reduce purchases, reduce the manufacturer's profit and affect the overall supply chain's profit. Although the retailer bears the part cost of emission reduction, which affects the retailer's profit, it also gets part of the manufacturer's profit, so the profit will not be reduced. How to develop low-cost carbon reduction technologies and equipment to reduce product prices and attract consumers is an important problem that the manufacturer must face.

6.2 Business suggestions

From the theoretical derivation and numerical simulation results of this paper, it can be found that when decentralized decision-making is adopted, the manufacturer and retailer in the dual-channel supply chain will consider their own interests and then there will be sales price competition. Because of mutual competition, the profits of the manufacturer and retailer as well as the profits of the dual-channel supply chain system cannot be maximized. However, under centralized decision-making, the goal of the manufacturer and retailer is to maximize the profit of the dual-channel supply chain system. And then the result is that not only the profit of the dual-channel supply chain system is maximized but also the profits of the manufacturer and retailer are higher than when they adopt decentralized decision-making. Therefore, when conducting business cooperation, managers of enterprises should try their best to get rid of selfishness and take the overall interests as the starting point to maximize the benefits of business cooperation.

Enterprises' investment in carbon emission reduction not only reflects their conscience but also fulfills their social responsibilities and should be more publicity promoted given to people. Strengthening the publicity of enterprises on carbon emission reduction investment and results can not only establish a good image of enterprises but also attract consumers' attention, improve their sensitivity to carbon emission reduction and awaken their environmental awareness. When consumers' awareness of environmental protection is awakened, they may start to pursue a low-carbon lifestyle, which will be beneficial to the sustainability of the environment and the improvement of the earth's ecosystem. Based on the theoretical derivation and numerical analysis simulation results of this paper, it can be found that the increased sensitivity of consumers to carbon emission reduction will promote consumers to choose low-carbon products, help to increase the sales quantity of carbon emission reduction products and thus increase the profits of each member of the dual-channel supply chain and the entire system. Therefore, a manager of an enterprise should widely publicize the results of the enterprise's carbon emission reduction, establish a positive image of the enterprise and educate consumers to awaken their environmental awareness, so as to improve the sales quantity and profit of the enterprise's carbon emission reduction products. In addition, if customers are more sensitive to carbon emission reduction, they will not only be more receptive to carbon emission reduction products but also in turn urge enterprises to reduce carbon emissions, which will be more beneficial to the earth's ecology and environmental protection.

Business managers should attach importance to developing new technologies to reduce carbon emissions. Simply increasing the amount of money to purchase carbon emission reduction equipment will lead to a substantial increase in the price of carbon emission reduction products, which will eventually rise to a high price that many consumers cannot accept. Therefore, a successful business manager must plan as soon as possible to develop low-cost carbon reduction technologies and equipment.

To improve the sales and profitability of industry suppliers, we need to understand the weaknesses of each part of the supply chain to reduce different cost components and then implement better industry marketing strategies. According to the research in this paper, we find that the manufacturer and retailer in the dual-channel supply chain can not only reduce the risk of capital contraction of the manufacturer and improve the income of the retailer through carbon emission reduction investment cooperation. More importantly, we can publicize the supply chain's investment in carbon emission reduction and technology development through advertising, establish a good image of the enterprise and build an environmental protection brand with sustainability and leading carbon reduction technology. Then, we can actively win the support or rewards of the government departments through the established good image. The operation of the customer group is not limited to the general customers with green environmental awareness; we can improve customers' sensitivity to carbon emission reduction through environmental education and publicity, awaken their environmental awareness and actively cultivate them to become a broader customer base.

Finally, we put forward the reflection of the research in this paper. Although the quantum game is a very useful tool, it requires more mathematical knowledge reserves and a lot of mathematical calculation processes, which is not conducive to generalizing the research results to general business managers. Therefore, the follow-up research direction should be simplified mathematical tools or related theories.

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Appendix 1. The Proof of Theorem 1

Proof. Substitute equations (37)–(40) into equations (22) and (23), we can get the profit functions of the retailer and manufacturer, respectively. According to the first derivative condition of the profit functions $\frac{\partial \pi_j}{\partial x_j} = 0, j = 0, 1$, the following are obtained:

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} -(L_1 - nmL_2)cosh\gamma - (mnL_3 - \omega\alpha)\sinh\gamma \\ L_2 sinh\gamma - L_3 cosh\gamma \end{bmatrix},$$

where:

$A_{11} = -2[b_0 \cosh^2 \gamma - \alpha(1+nm)\cosh\gamma \sinh\gamma + nmb_0 \sinh^2 \gamma]$;

$A_{12} = a(1+nm)\cosh^2 \gamma - 2(b_0 + nmb_1)\cos \gamma \sinh \gamma + a(1+nm) \sinh^2\gamma$;

$A_{21} = \alpha\cosh^2 \gamma - 2b_1 \cosh\gamma \sinh\gamma + \alpha \sinh 2\gamma$;

$A_{22} = -2b_1 \cosh^2 \gamma + 2\alpha \cosh \gamma \sinh \gamma$;

$L_1 = a_0 + \beta\theta + b_0\omega$;

$L_2 = b_0 (\omega - c) + c\alpha$; and

$L_3 = a1 + \beta\theta + b_1 c + \alpha(\omega - c)$.

The second derivative condition of the profit functions are:

$$\frac{\partial^2 \pi_0}{\partial {x_0}^2} = -2\left[b_0 \cosh^2\gamma - \alpha(1+nm)\cosh\gamma\sinh\gamma + nmb_0 \sinh^2\gamma\right],$$
$$\frac{\partial^2 \pi_1}{\partial {x_1}^2} = -2b_1\cosh^2\gamma + 2\alpha\cosh\gamma\sinh\gamma.$$

By using $\sinh \gamma = (e^\gamma - e^{-\gamma})/2$, $\cosh \gamma = (e^\gamma + e^{-\gamma})/2$ and $b_j > 1 > n, m, \alpha, j = 0, 1$, we can get:

$$\frac{\partial^2 \pi_0}{\partial {x_0}^2} = -\frac{1}{2}\left\{e^{2\gamma}\left[(1+nm)(b_0 - \alpha)\right] + e^{-2\gamma}\left[(1-nm)b_0 + (1+nm)\alpha\right] + 2(1-nm)b_0\right\} < 0$$

$$\frac{\partial^2 \pi_1}{\partial {x_1}^2} = -\frac{1}{2}\left\{e^{2\gamma}(b_1+\alpha) + e^{-2\gamma}(b_1 - \alpha) + 2b_1\right\} < 0.$$

Hence, this model has the optimal strategies. From the first derivative condition, by using $\cosh^2 \gamma - \sinh^2 \gamma = 1$, $\sinh \gamma = (e^\gamma - e^{-\gamma})/2$, $\cosh \gamma = (e^\gamma + e^{-\gamma})/2$ and $b_j > 1 > n, m, \alpha, j = 0, 1$, the determinant:

$$|A| = \begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix} = 4b_0b_1\cosh^2\gamma - 2\alpha(b_0+b_1)\cosh\gamma\sinh\gamma - (1+nm)\alpha^2,$$
$$= \frac{1}{4}\left\{[4b_0b_1 - 2\alpha(b_0+b_1)](e^{2\gamma}+e^{-2\gamma}) + 2b_0b_1 - (1+nm)\alpha^2\right\} > 0.$$

By Cramer's rule, the optimal strategies are:

$$x_0 = \frac{1}{|A|}\left\{\cosh^3\gamma\left[2b_1(L_1 - mnL_2) + \alpha(1+mn)L_3\right] - \cosh^2\gamma\sinh\gamma\left[\alpha(2L_1 + (1-mn)L_2 + 2\omega b_1) + 2b_0L_3\right] + \cosh\gamma\sinh^2\gamma\left[2(b_0 + mnb_1)L_2 + \alpha(1-mn)L_3 + 2\omega\alpha^2\right] - \sinh^3\gamma\left[\alpha(1+mn)L_2\right]\right\},$$

$$x_1 = \frac{1}{|A|}\Big\{\cosh^3\gamma[\alpha(L_1 - mnL_2 - \omega\alpha) + 2b_0L_3] - \cosh^2\gamma\sinh\gamma[\alpha((2+mn)L_3 + \omega\alpha) + 2(b_1L_1 + (b_0 - mnb_1)L_2)] + \cosh\gamma\sinh^2\gamma[\alpha(L_1 + (2+mn)L_2 + 2\omega b_1)] - \sinh^3\gamma[mn(\alpha L_3 - 2b_1L_2)]\Big\}.$$

Substitute the optimal strategies x_0 and x_1 into equations (37) and (38), then the optimal prices p_0^{d*} and p_1^{d*} can be obtained as follows:

$$p_0^{d*} = \frac{1}{|A|}\Big\{\cosh^2\gamma[2b_1(L_1 - mnL_2) + \alpha(1+mn)L_3] - \cosh\gamma\sinh\gamma[\alpha(L_1 + L_2 + 2\omega b_1)] + \sinh^2\gamma\left[mn(2b_1L_2 - \alpha L_3) + \omega\alpha^2\right]\Big\},$$

$$p_1^{d*} = \frac{1}{|A|}\Big\{\cosh^2\gamma[\alpha(L_1 - mnL_2) + 2b_0L_3] - \cosh\gamma\sinh\gamma\left[2b_0L_2 + \alpha L_3 + \omega\alpha^2\right] + \sinh^2\gamma[\alpha(1+mn)L_2].$$

Appendix 2. The Proof of Theorem 2

Proof. Substitute equations (37)–(40) into equation (24), then we can get the overall supply chain's profit function. The second-order partial derivatives of x_0 and x_1 for the profit function π_{sc} are:

$$\frac{\partial^2\pi_{sc}}{\partial {x_0}^2} = -2\left[b_0\cosh^2\gamma - 2\alpha\cosh\gamma\sinh\gamma + b_1\sinh^2\gamma\right],$$
$$\frac{\partial^2\pi_{sc}}{\partial x_0\partial x_1} = \frac{\partial^2\pi_{sc}}{\partial x_1\partial x_0} = 2\left[\alpha\cosh^2\gamma - (b_0 + b_1)\cosh\gamma\sinh\gamma + \alpha\sinh^2\gamma\right],$$
$$\frac{\partial^2\pi_{sc}}{\partial {x_1}^2} = -2\left[b_1\cosh^2\gamma - 2\alpha\cosh\gamma\sinh\gamma + b_0\sinh^2\gamma\right].$$

The determinant of the Hessian matrix:

$$|H(\pi_{sc})| = 4\left(b_0b_1 - \alpha^2\right)\left(\cosh^2\gamma - \sinh^2\gamma\right)^2 = 4\left(b_0b_1 - \alpha^2\right) > 0.$$

By using $\sinh\gamma = (e^\gamma - e^{-\gamma})/2$ and $\cosh\gamma = (e^\gamma + e^{-\gamma})/2$, we can get:

$$\frac{\partial^2\pi_{sc}}{\partial {x_0}^2} = -\frac{1}{2}\left[e^{2\gamma}(b_0 + b_1 - 2\alpha) + e^{-2\gamma}(b_0 + b_1 + 2\alpha) + 2(b_0 - b_1)\right],$$

$$\frac{\partial^2\pi_{sc}}{\partial {x_1}^2} = -\frac{1}{2}\left[e^{2\gamma}(b_0 + b_1 - 2\alpha) + e^{-2\gamma}(b_0 + b_1 + 2\alpha) + 2(b_1 - b_0)\right].$$

Because $\gamma = 0$, we have $\frac{\partial^2\pi_{sc}}{\partial {x_0}^2} = -2b_0 < 0$ and $\frac{\partial^2\pi_{sc}}{\partial {x_1}^2} = -2b_1 < 0$. Hence, the Hessian matrix is negative definite. Then the optimal prices p_0^{c*} and p_1^{c*} exist.

According to the first derivative condition of the profit functions $\frac{\partial\pi_{sc}}{\partial x_j} = 0, j = 0, 1$, the following are obtained:

$$\begin{bmatrix}\bar{A}_{11} & \bar{A}_{12}\\ \bar{A}_{21} & \bar{A}_{22}\end{bmatrix}\begin{bmatrix}x_0\\ x_1\end{bmatrix} = \begin{bmatrix}-V_0\cosh\gamma - V_1\sinh\gamma\\ -V_0\sinh\gamma - V_1\cosh\gamma\end{bmatrix},$$

where:

$$\bar{A}_{11} = -2\left[b_0\cosh^2\gamma - 2\alpha\cosh\gamma\sinh\gamma + b_1\sinh^2\gamma\right];$$

$$\bar{A}_{12} = \bar{A}_{21} = 2\left[\alpha\cosh^2\gamma - (b_0 + b_1)\cosh\gamma\sinh\gamma + \alpha\sinh^2\gamma\right];$$

$$\bar{A}_{22} = -2\left[b_1\cosh^2\gamma - 2\alpha\cosh\gamma\sinh\gamma + b_0\sinh^2\gamma\right];$$

$V_0 = a_0 + \beta\theta + b_0c - \alpha c = L_1 - L_2$; and
$V_1 = a_1 + \beta\theta + b_1c - \alpha c = L_3 - \alpha\omega$.
Because the determinant:

$$|\bar{A}| = \begin{vmatrix}\bar{A}_{11} & \bar{A}_{12}\\ \bar{A}_{21} & \bar{A}_{22}\end{vmatrix} = 4\left(b_0b_1 - \alpha^2\right)\left(\cosh^2\gamma - \sinh^2\gamma\right)^2 = 4\left(b_0b_1 - \alpha^2\right) > 0,$$

by Cramer's rule, the optimal strategies are:

$$x_0 = \frac{-2[(\sinh\gamma\,\alpha - \cosh\gamma\,b_1)V_0 + (-\cosh\gamma\,\alpha + \sinh\gamma\,b_0)V_1]}{|\bar{A}|} = \frac{2b_1V_0 + \alpha V_1}{|\bar{A}|},$$

$$x_1 = \frac{-2[(-\cosh\gamma\,\alpha + \sinh\gamma\,b_1)V_0 + (\sinh\gamma\,\alpha - \cosh\gamma\,b_0)V_1]}{|\bar{A}|} = \frac{\alpha V_0 + 2b_0V_1}{|\bar{A}|}.$$

Substitute the optimal strategies x_0 and x_1 into equations (37) and (38), then the optimal prices p_0^{c*} and p_1^{c*} can be obtained as follows:

$$p_0^{c*} = \frac{2(b_1V_0 + \alpha V_1)}{|\bar{A}|} = \frac{2\left[b_1(L_1 - L_2) + \alpha(L_3 - \alpha\omega)\right]}{|\bar{A}|},$$

$$p_1^{c*} = \frac{2(\alpha V_0 + b_0V_1)}{|\bar{A}|} = \frac{2\left[\alpha(L_1 - L_2) + b_0(L_3 - \alpha\omega)\right]}{|\bar{A}|}.$$

This completes the proof.

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