Uncertainty, agency costs and investment behavior in the Euro area and in the USA

Johannes Strobel
Department of Real Estate, University of Regensburg, Regensburg, Germany
Kevin D. Salyer
Department of Economics, University of California, Davis, California, USA, and
Gabriel S. Lee
Department of Real Estate, University of Regensburg, Regensburg, Germany

Abstract

Purpose – The purpose of this paper is to analyze the credit channel effects on investment behavior for the US and the Euro area.

Design/methodology/approach – This paper uses the dynamic stochastic general equilibrium model and calibrates a version of the Carlstrom and Fuerst's (1997) agency cost model of business cycles with time-varying uncertainty in the technology shocks that affect capital production. To highlight the differences between the US and European financial sectors, the paper focuses on two key components of the lending channel: the risk premium associated with bank loans and the bankruptcy rates.

Findings – This paper shows that the effects of minor differences in the credit market translate into large, persistent and asymmetric fluctuations in real and financial variables and depend on the type of shocks. The results imply that the Euro areas supply elasticities for capital are less elastic than that of the USA following a technology shock. Finally, the authors find that the adverse impact of uncertainty shocks is heterogeneous across countries and amplified by the steady-state bankruptcy rate and risk premium.

Originality/value – This paper quantifies the effects of uncertainty shocks when there is a credit channel due to asymmetric information between lenders and borrowers for the Euro area countries, and then compares the results to that of the USA. This paper shows that financial accelerator mechanism could potentially play a significant role in business cycles in the Euro area. This result directly lends one to conclude the following: the credit channel that affects the financial sector does indeed matter for macroeconomic behavior, and that policy makers should be attentive in smoothing out uncertainties if the economic policies are to lower the business and financial cycle volatilities.

Keywords Agency costs, Investment behaviour, Credit channel, EU area

Paper type Research paper

1. Introduction

In recent years, a number of theoretical models that highlight the role of the financial accelerator in propagating and amplifying macroeconomic shocks have further cast doubts on aggregate technology shocks in the standard real business cycle (RBC) model as the driving force in business activities[1]. Financial accelerator literature addresses the question

"can credit constraints and (or) asymmetric information between borrows and lenders propagate and amplify business cycles?" Although the theoretical contributions have improved our understanding of the propagation mechanism, the lack of empirical support has led many to question the relevance of financial accelerator type models – from shortly after they were developed up until today[2].

In this paper, we continue with this debate by posing the following question:

RQ1. How do differences in the credit channel affect investment behavior in the US and the Euro area?

To analyze this question, we calibrate a version of the Carlstrom and Fuerst's (1997) agency cost model of business cycles with time-varying uncertainty in the technology shocks that affect capital production as in Dorofeenko *et al.* (2008) for the US and European economies. We follow the work of Dorofeenko *et al.* (2008) and model time-varying uncertainty as a mean-preserving spread in the distribution of the technology shocks affecting capital production and explore how changes in uncertainty affect equilibrium characteristics and economic performance. This setting is useful for three reasons: first, the impact of uncertainty on investment via the lending channel is fairly transparent so that economic intuition is enhanced. Second, Justiniano and Primiceri (2008) identify the equilibrium condition of investment as the major source of changes in US macroeconomic variables' volatility. Third, Ludvigson *et al.* (2016) identify that uncertainty about financial markets is a likely source of business cycle fluctuations. We examine the impact of uncertainty that come about this channel and find them to be quantitatively substantial.

We compare the US and the Euro area for our analysis as Agresti and Mojon (2001) and Cecchetti (1999) show that these two economies exhibit similar business cycle patterns but are quite different in financial structures. Figure 1 shows the autocorrelation functions (ACF) for output growth for the USA and some of the Euro area countries (including the aggregate EMU11). These ACFs clearly show that the business cycle patterns between the two monetary unions are similar. But to highlight the differences in the US and European financial sectors, we focus on two key components of the lending channel: the risk premium associated with bank loans and bankruptcy rates. We take Austria, Ireland and Spain as the representative European member states for our calibration analysis as these three countries represent three different legal systems and are known to have either low bankruptcy rate (e.g. Spain) or high risk premium (e.g. Ireland) (see Table I)[3].

Our main results can be summarized as follows. In contrast to an aggregate technology shock which affects investment demand, an increase in uncertainty will cause an increase in the price of capital and a fall in investment activity. Our empirical results then indicate that the differences in financial structures quantitatively affect the cyclical behavior in the two areas: the magnitude of the credit channel effects is amplified by the differences in the financial structures. We further demonstrate that the effects of minor differences in the credit market may translate into large, persistent and asymmetric fluctuations in both real (output, consumption and investment) and financial variables (price of capital, bankruptcy rate and risk premium).

More precisely, for the technology shock, real variables' response is very similar across countries, but there is an asymmetric response in financial variables: the effects imply that the Euro area's supply elasticities for capital are less elastic than the USA. Furthermore, we examine two types of uncertainty shock: a standard unexpected shock following Dorofeenko *et al.* (2008) as well as a hump-shaped shock, in order to capture the richer dynamics displayed by uncertainty (Strobel, 2017). For the standard shock, we find that a higher steady-state bankruptcy rate amplifies the adverse impact on both real and credit channel variables. Output decreases by 3.5 percent, in the USA and 3 percent in Ireland and Austria; the impact in Spain is much less severe and about one-tenth of the other countries' impact.
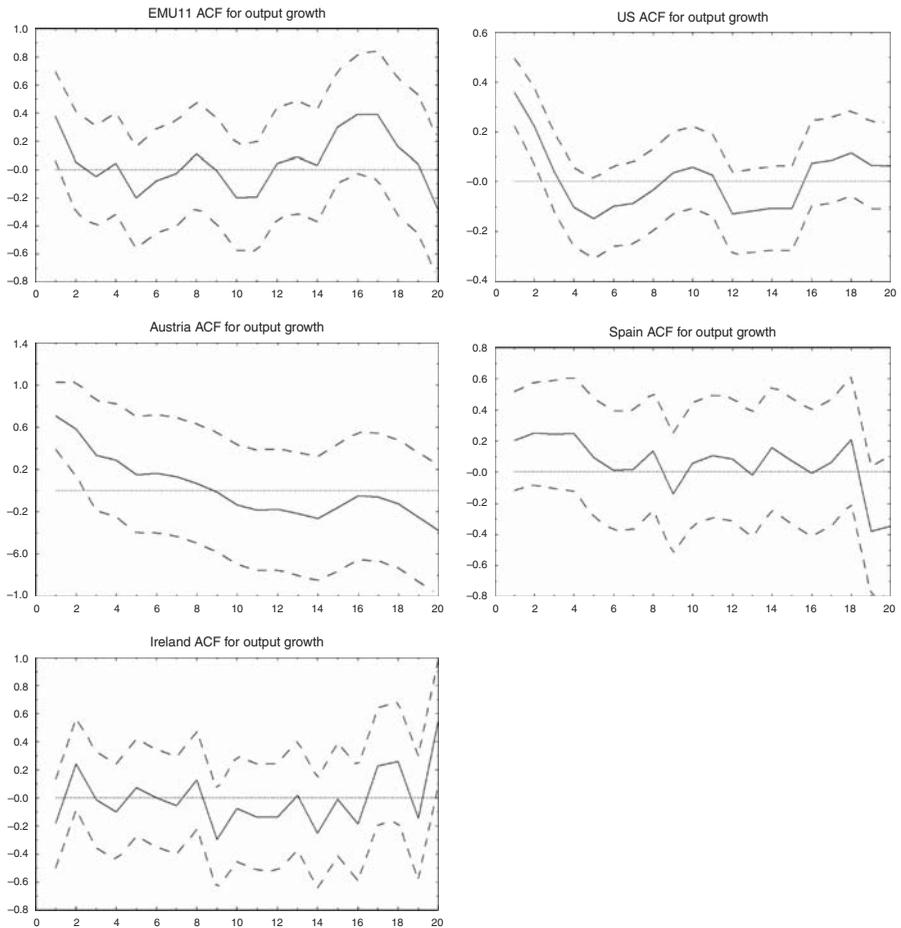
Figure 1.
Autocorrelation
functions for the
USA and selected
EMU countries
output growth

Country	Bankruptcy rate	Risk premium
Austria (German Civil Law)	0.332	3.76
Ireland (English Common Law)	0.685	8.85
Spain (French Civil Law)	0.005	1.99
USA (English Common Law)	0.974	1.87

Notes: The bankruptcy rates for the EU countries are calculated as an average percentage of bankruptcies to number of firms for the period between 1990 and 1999. Risk premia are the differences between lending and deposit rates. For the US numbers, see Carlstrom and Fuerst (1997)

Table I.
Financial sector
information on Euro
area countries
and USA

For the dynamic uncertainty shock, we find that the risk premium and the bankruptcy rate play important roles in influencing the price of capital, which, in turn, affect investment and output. As households save precautionarily because they anticipate deteriorating investment opportunities, a sluggish recovery ensues. We conclude that the heterogeneity of the Euro area countries' response depends on the shock and that the financial accelerator mechanism can potentially play a significant role in business cycles.

2. Model
We employ the agency cost business cycle model of Carlstrom and Fuerst (1997) to address
the financial intermediaries' role in the propagation of productivity shocks and extend their
analysis by introducing time-varying uncertainty following Dorofeenko *et al.* (2008).
Since, for the most part, the model is identical to that in Dorofeenko *et al.* (2008),
the exposition of the model will be brief with primary focus on the lending channel.
A full presentation of the model is given in Appendix 2.

Carlstrom and Fuerst (1997) include capital-producing entrepreneurs, who default if they
are not productive enough, into a RBC model. In this framework, households and final goods
producing firms are identical and perfectly competitive. Households save by investing in a
risk-neutral financial intermediary that extends loans to entrepreneurs. Entrepreneurs are
heterogeneous and produce capital using an idiosyncratic and stochastic technology with
constant volatility. Dorofeenko *et al.* (2008) introduce stochastic shocks to the volatility
(uncertainty shocks) of entrepreneurs' technology (the aggregate production technology is
also subject to technology shocks as is standard).

The conversion of investment to capital is not one-to-one here because heterogeneous
entrepreneurs produce capital using idiosyncratic and stochastic technology. If a capital-
producing firm realizes a low technology shock, it declares bankruptcy and the financial
intermediary takes over production after paying monitoring costs.

2.1 Optimal financial contract
For expository purposes as well as to explain our approach in addressing the effect of risk,
we briefly introduce the contract set up and leave the complete contract model to the
Appendix 2. In deriving the optimal contract, both entrepreneurs and lenders take the price
of capital, q_t, and net worth, n_t, as given.

As described above, the entrepreneur has access to a stochastic technology that
transforms i_t units of consumption into $\omega_t i_t$ units of capital. In the work of Carlstrom and
Fuerst (1997), the technology shock ω_t is assumed to be distributed as i.i.d. with $E(\omega_t) = 1$.
While we maintain the assumption of constant mean, we follow the work of Dorofeenko *et al.*
(2008) and assume that the standard deviation is varying with time. Specifically, we assume
that the standard deviation of the capital production technology shock is governed by the
following AR(1) process:

$$log(\sigma_{\omega,t+1}) = (1-\rho_{\sigma_\omega})log(\overline{\sigma}_\omega) + \rho_{\sigma_\omega}log(\sigma_{\omega,t}) + \varphi_\sigma u_{t+1}, \tag{1}$$

where $\rho_{\sigma_\omega} \in (0, 1)$ and $u_t \sim^{\text{i.i.d.}} N(0, 1)$. The unconditional mean of the standard deviation is
given by $\overline{\sigma}_w$. This structure is such that innovations are unexpected such that uncertainty
jumps to the peak and then converges back to the long-run mean. Recent empirical evidence in
the work of Strobel *et al.* (2016), however, suggests that uncertainty shocks are more persistent
and display a hump-shaped time path. We follow model this time path as:

$$log(\sigma_{\omega,t+1}) = (1-\rho_{\sigma_\omega})log(\overline{\sigma}_\omega) + \rho_{\sigma_\omega}log(\sigma_{\omega,t}) + x_{t+1}$$

$$x_{t+1} = \rho_x x_t + \varphi_x \varepsilon_{x,t+1}, \tag{2}$$

where x_{t+1} induces a hump-shape and $\varepsilon_{x,t+1} \sim^{\text{i.i.d.}} N(0, 1)$. As shown in the work of Strobel
et al. (2016), this approach to modeling uncertainty is not *ad hoc* but based on the time-series
evidence of Ludvigson *et al.* (2016) and Jurado *et al.* (2015).

The realization of ω_t is privately observed by entrepreneurs – banks can observe the
realization at a cost of μi_t units of consumption. The entrepreneur enters period t with one
unit of labor endowment and z_t units of capital. Labor is supplied inelastically while capital
is rented to firms; hence income in the period is $w_t + r_t z_t$. This income along with remaining

capital determines net worth (denoted as n_t and denominated in units of consumption) at time t:

$$n_t = w_t + z_t(r_t + q_t(1-\delta)). \tag{3}$$

With a positive net worth, the entrepreneur borrows $(i_t - n_t)$ consumption goods and agrees to pay back $(1+r^k)(i_t - n_t)$ capital goods to the lender, where r^k is the interest rate on loans. Thus, the entrepreneur defaults on the loan if his realization of output is less than the re-payment, i.e.:

$$\omega_t < \frac{(1+r^k)(i_t-n_t)}{i_t} \equiv \overline{\omega}_t. \tag{4}$$

The optimal borrowing contract is given by the pair $(i_t, \overline{\omega}_t)$ that maximizes entrepreneur's return subject to the lender's willingness to participate (all rents go to the entrepreneur). Denoting the c.d.f. and p.d.f. of ω_t as $\Phi(\omega_t; \sigma_{\omega,t})$ and $\phi(\omega_t; \sigma_{\omega,t})$, respectively, the contract is determined by the solution to[4]:

$$\max_{\{i,\overline{\omega}\}} qi_t f(\overline{\omega}_t; \sigma_{\omega,t}) \text{ subject to } qi_t g(\overline{\omega}_t; \sigma_{\omega,t}) \geqslant (i-n),$$

where

$$f(\overline{\omega}_t; \sigma_{\omega,t}) = \int_{\overline{\omega}_t}^{\infty} \omega\phi(\omega; \sigma_{\omega,t})d\omega - \left[1-\Phi(\overline{\omega}_t; \sigma_{\omega,t})\right]\overline{\omega}_t,$$

which can be interpreted as the fraction of the expected net capital output received by the entrepreneur:

$$g(\overline{\omega}_t; \sigma_{\omega,t}) = \int_{-\infty}^{\overline{\omega}_t} \omega\phi(\omega; \sigma_{\omega,t})d\omega + \left[1-\Phi(\overline{\omega}_t; \sigma_{\omega,t})\right]\overline{\omega}_t - \Phi(\overline{\omega}_t; \sigma_{\omega,t})\mu,$$

which represents the lender's fraction of expected capital output, $\Phi(\overline{\omega}_t; \sigma_{\omega,t})$ is the bankruptcy rate. Also note that $f(\overline{\omega}_t; \sigma_{\omega,t}) + g(\overline{\omega}_t; \sigma_{\omega,t}) = 1 - \Phi(\overline{\omega}_t; \sigma_{\omega,t})\mu$: the right-hand side is the average amount of capital that is produced. This is split between entrepreneurs and lenders while monitoring costs reduce net capital production.

The necessary conditions for the optimal contract problem are:

$$\frac{\partial(.)}{\partial\overline{\omega}} : qif'(\overline{\omega}) = -\lambda qi\frac{\partial g(\overline{\omega}_t; \sigma_{\omega,t})}{\partial\overline{\omega}},$$

where λ_t is the shadow price of capital. Using the definitions of $f(\overline{\omega}_t; \sigma_{\omega,t})$ and $g(\overline{\omega}_t; \sigma_{\omega,t})$, this can be rewritten as:

$$1 - \frac{1}{\lambda_t} = \frac{\phi(\overline{\omega}_t; \sigma_{\omega,t})}{1-\Phi(\overline{\omega}_t; \sigma_{\omega,t})}\mu. \tag{5}$$

As shown by Equation (5), the shadow price of capital is an increasing function of the relevant Inverse Mill's ratio (interpreted as the conditional probability of bankruptcy) and the agency costs. If the product of these terms equals 0, then the shadow price equals the cost of capital production, i.e. $\lambda_t = 1$.

The second necessary condition is:

$$\frac{\partial(.)}{\partial i_t} : qf(\overline{\omega}_t; \sigma_{\omega,t}) = -\lambda_t[1 - qg(\overline{\omega}_t; \sigma_{\omega,t})].$$

Solving for q using the first-order conditions, we have:

$$q^{-1} = \left[(f(\overline{\omega}_t; \sigma_{\omega,t}) + g(\overline{\omega}_t; \sigma_{\omega,t})) + \frac{\phi(\overline{\omega}_t; \sigma_{\omega,t})\mu f(\overline{\omega}_t; \sigma_{\omega,t})}{\frac{\partial f(\overline{\omega}_t; \sigma_{\omega,t})}{\partial \overline{\omega}}} \right]$$

$$= \left[1 - \Phi(\overline{\omega}_t; \sigma_{\omega,t})\mu + \frac{\phi(\overline{\omega}_t; \sigma_{\omega,t})\mu f(\overline{\omega}_t; \sigma_{\omega,t})}{\frac{\partial f(\overline{\omega}_t; \sigma_{\omega,t})}{\partial \overline{\omega}}} \right]$$

$$\equiv [1 - D(\overline{\omega}_t, \sigma_{\omega,t})] = F(\overline{\omega}_t, \sigma_{\omega,t}), \tag{6}$$

where $D(\overline{\omega}_t; \sigma_{\omega,t})$ can be thought of as the total default costs.

It is straightforward to show that Equation (6) defines an implicit function $\overline{\omega}(q, \sigma_{\omega,t})$ that is increasing in q. Also note that, in equilibrium, the price of capital, q, differs from unity due to the presence of the credit market frictions (Note that: $\partial f(\overline{\omega}_t; \sigma_{\omega,t})/\partial \overline{\omega} = \Phi(\overline{\omega}_t; \sigma_{\omega,t}) - 1 < 0$).

The incentive compatibility constraint implies:

$$i_t = \frac{1}{(1 - qg(\overline{\omega}_t; \sigma_{\omega,t}))} n. \tag{7}$$

Equation (7) implies that investment is linear in net worth and defines a function that represents the amount of consumption goods placed in to the capital technology: $i(q, n, \sigma_{\omega,t})$. The fact that the function is linear implies that the aggregate investment function is well defined.

The effect of an increase in uncertainty on investment in this model can be understood by first turning to Equation (6). Under the assumption that the price of capital is unchanged, this implies that the costs of default, represented in the function $D(\overline{\omega}_t, \sigma_{\omega,t})$, must also be unchanged. With a mean-preserving spread in the distribution for ω_t, this implies that $\overline{\omega}_t$, and in turn $g(\overline{\omega}_t; \sigma_{\omega,t})$, will fall. The effect of an uncertainty shock is summarized graphically, and contrasted with an aggregate technology shock, in Figure 2 (taken from Dorofeenko *et al.*, 2008).

3. Equilibrium characteristics
3.1 Steady-state analysis
While our focus is primarily on the cyclical behavior of the economy, we briefly examine the steady-state properties of the economies. For this analysis, we use, to a large extent, the parameters employed in Carlstrom and Fuerst's (1997) analysis for the USA and Casares (2001) for the Euro area countries. Specifically, the parameter values used are shown in Table II.

Agents discount factor β, the depreciation rate δ and capital's share α are fairly standard in the RBC analysis. The remaining parameter, μ, represents the monitoring costs associated with bankruptcy. This value, as noted by Carlstrom and Fuerst (1997), is relatively prudent given estimates of bankruptcy costs (which range from 20 percent (Altman, 1984) to 36 percent (Alderson and Betker, 1995) of firm assets).

The remaining parameters $(\overline{\sigma}, \overline{\omega}, \gamma)$ determine the steady-state bankruptcy rate $\Phi(\overline{\omega}, \overline{\sigma})$ (which we denote as *br* and is expressed in percentage terms) and the risk

Figure 2.
The partial
equilibrium impact of
an uncertainty shock
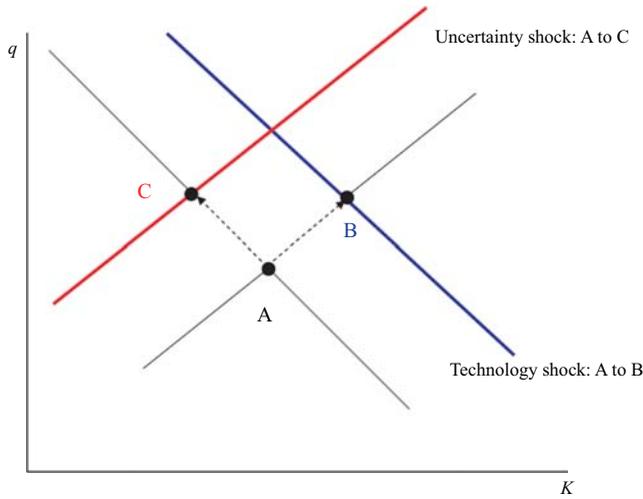
Source: Dorofeenko *et al.* (2008)

	β	α	δ	μ
USA	0.99	0.36	0.02	0.25
Euro area	0.995	0.36	0.025	0.25

Table II.
Parameter values

premium (denoted *rp*) associated with bank loans (also, recall that γ is calibrated so that the rate of return to internal funds is equal to $1/\gamma$)[5]. While Carlstrom and Fuerst found it useful to use the observed bankruptcy rate to determine $\bar{\sigma}$, for our analysis we treat *rp* and *br* as exogenous and examine the steady-state behavior of the economy under different scenarios. In particular, to examine the role of uncertainty on the steady-state behavior of the economy, we hold the bankruptcy rate constant to that studied in the work of Carlstrom and Fuerst (1997) and vary *rp* for each country. That is, once the values of *rp* and *br* are specified, the values of $\bar{\sigma}, \bar{\omega}$, and γ are determined endogenously. Table III reports the steady-state analysis for four economies, where the values of γ are reported strictly for comparison. The main message from Table III is that the decrease in the bankruptcy rate contributes broadly to a decrease in the cut-off points for the changes in the distribution of the lending channel ($\bar{\omega}$) and an increasing uncertainty ($\bar{\sigma}$), although this relation is non-linear. For example, while the risk premia in the USA and Spain are not very different from each other, $\bar{\omega}$: mover is much lower while $\bar{\sigma}$ is much higher. For Ireland, the combination of a low bankruptcy rate and relatively high risk premium, compared to the USA, leads to high degrees of steady-state uncertainty, and a relatively

Table III.
Calibration key credit
channel variables of
the four economies in
the quarterly
frequency

Economy	$\bar{\omega}$	$\bar{\sigma}$	*br* (%)	*rp* (%)	γ
USA (C&F)	0.606	0.205	0.974	1.87	0.9471
Ireland	0.085	0.852	0.685	8.84	0.9337
Austria	0.095	0.760	0.332	3.76	0.9653
Spain	0.002	1.315	0.005	1.99	0.9845

lower lending cut-off point. Finally, the values of $\overline{\omega}$ and $\overline{\sigma}$ in Austria and Ireland are not as different as one might expect when comparing the bankruptcy rate and the risk premia. On the other hand, γ is quite different for these two countries.

The effects of the different calibration on the steady-state values are seen in Table IV (all values in Table IV are percentage changes relative to the US (Carlstrom and Fuerst) economy). Table IV indicates that the bankruptcy rate plays the most important role in determining the steady-state level of both real and credit channel variables. The very low bankruptcy rate of Spain implies that the level of quantities – investment, capital stock, output and consumption – is highest compared to the other countries. Analogously, the lower bankruptcy rates of the European countries also imply higher quantities. The relatively higher values of investment and the capital stock in the context of a higher steady-state price of capital also suggest that the bankruptcy rate plays a more important than the risk premium.

3.2 Cyclical behavior

As described in detail in Appendix 2, Equations (A16)-(A23) determine the equilibrium properties of the economy. To analyze the cyclical properties of the economy, we linearize (i.e. take a first-order Taylor series expansion) of these equations around the steady-state values and express all terms as percentage deviations from steady-state values. We then examine the impact of a shock to aggregate technology and to the second moment of entrepreneurs' distribution of productivity.

3.2.1 Technology shocks. The behavior of these four economies is analyzed by examining the impulse response functions of several key variables – output, aggregate consumption and investment – to a 1 percent innovation in θ_t with a persistency of 0.95. The impulse response functions are presented in Figure 3.

Following an aggregate productivity shock, as expected, aggregate output, consumption and investment all increase. The magnitude of increase across different economies is quite similar, especially for consumption and output. These effects are shown in Figure 3. As shown in the work of Carlstrom and Fuerst (1997), a technology shock increases output and the demand for capital. The resulting increase in the price of capital implies greater lending activity and, hence, an increase in the bankruptcy rate (and risk premia) as shown in Figure 4. Our focus, as was in Carlstrom and Fuerst (1997), is on the effects of an innovation to the aggregate technology shock and, because of the assumed persistence in this shock, is driven by the change in the first moment of the aggregate production shock. What is different in our results in compare to Carlstrom and Fuerst (1997) is the magnitude of the impulse response functions for bankruptcy rate, risk premium and price of capital across different economies. As the cut-off point decreases ($\overline{\omega}$), the response of investment increases (see Figure 4) and the response of the price of capital decreases. This is a direct evidence that the Euro area's supply elasticities for capital are less elastic than that of the USA following a technology shock.

3.2.2 Unanticipated uncertainty shocks. We now turn to the impact of an unanticipated risk shock on real variables in Figure 5. We match the innovation in uncertainty relative to

Variable	Austria	Ireland	Spain
c	3.27	1.45	4.25
k	31.53	25.19	35.12
y	10.37	8.42	11.44
q	0.93	4.17	−0.79
i	31.32	25.10	34.79
br	−65.91	−29.67	−99.49

Table IV.
Steady-state effects of greater uncertainty (comparison to the US economy, in percent)

Figure 3.
Response of output, aggregate consumption and investment to a 1 percent productivity shock
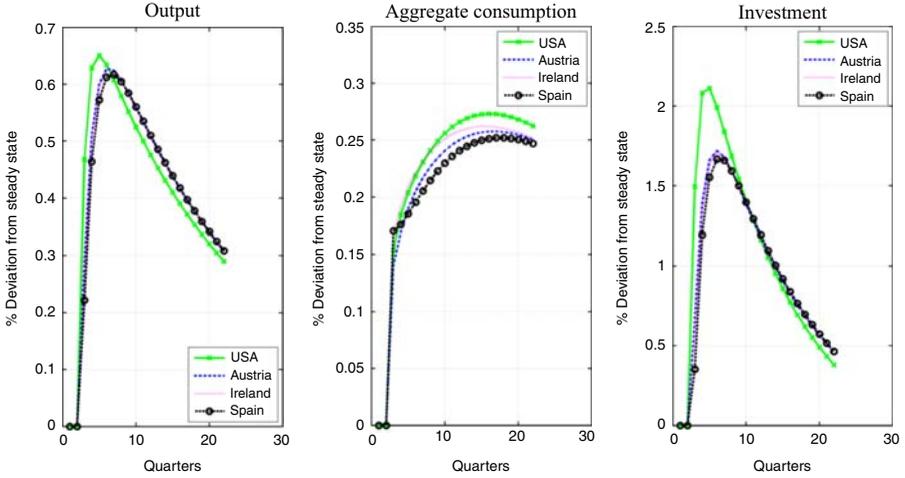
Figure 4.
Response of the price of capital, the bankruptcy rate and the risk premium to a 1 percent productivity shock
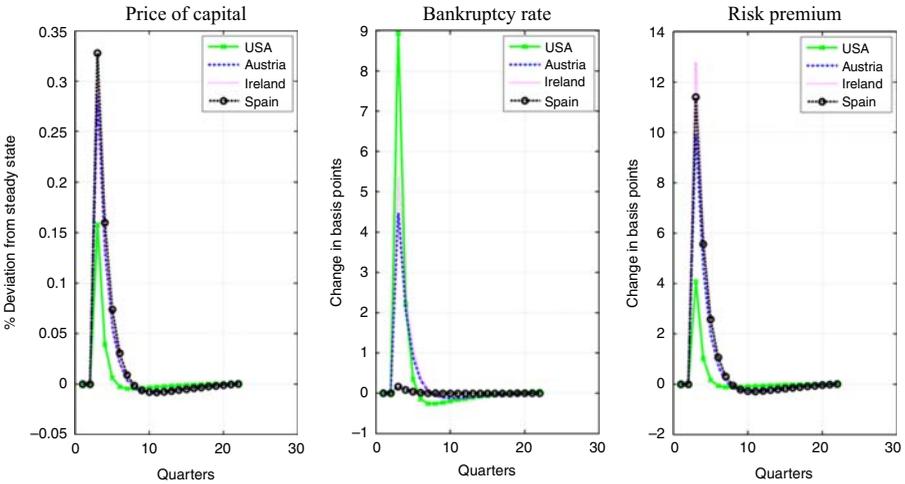
the steady state based on the uncertainty proxy of Jurado *et al.* (2015). More precisely, we average the increase of their macro uncertainty measure (relative to the long-run mean) at each one of the three shocks' peaks indicated by Jurado *et al.* (2015) and calculate the increase relative to the long-run mean. This results in an innovation of 48 percent relative to the steady state, i.e. we set $\varphi_\sigma = 0.48$[6]. Following the work of Dorofeenko *et al.* (2008), the persistency of the AR(1) process of uncertainty, ρ_{σ_ω}, is 0.9.

As shown in Figure 5, risk shocks induce adverse effects for all the countries. As expected from the partial equilibrium analysis, there is a drop in investment and output; in response to the drop in investment households increase consumption, which strongly contributes to the countercyclical increase in aggregate consumption. The extent of the drop in investment correlates with the bankruptcy rate: the higher the steady-state bankruptcy rate, the stronger the adverse impact. Surprisingly, as shown in Figure 6, the bankruptcy rate responds highly asymmetrically, increasing in the USA and decreasing in the Euro area countries. The reason

Output

Aggregate consumption
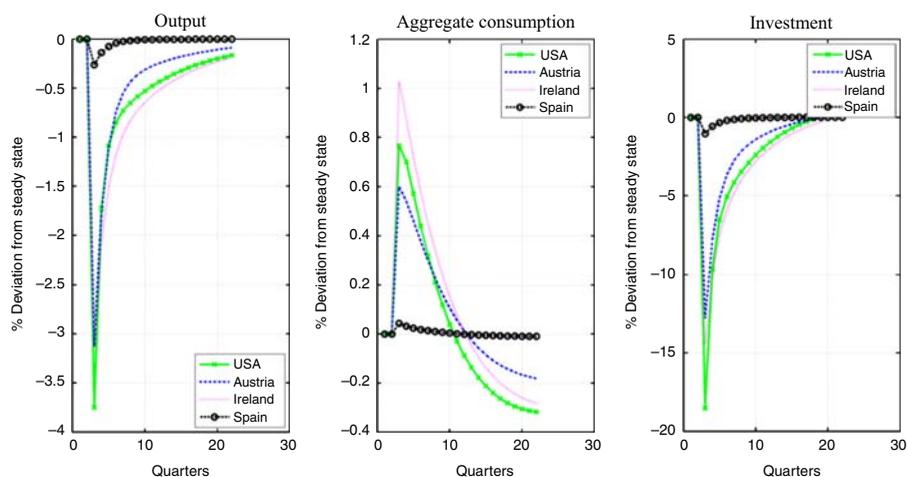
Investment

Figure 5.
Response of
output, aggregate
consumption and
investment to a
48 percent
unanticipated
risk shock

Price of capital

Bankruptcy rate
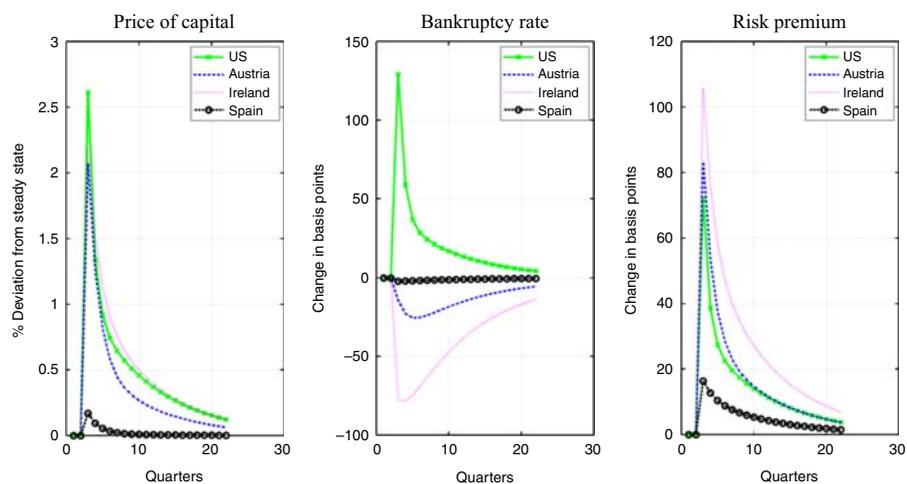
Risk premium

Figure 6.
Response of the
price of capital, the
bankruptcy rate
and the risk premium
to a 48 percent
unanticipated
risk shock

is that, as analyzed in partial equilibrium, a risk shock leads to a drop in both investment and the default threshold ($\overline{\omega}$). With the steady-state values of $\overline{\omega}$ in the Euro area already relatively low compared to the USA, a further decrease in $\overline{\omega}$ dominates the increase $\sigma_{\omega,t}$ such that $\Phi(\overline{\omega}_t; \sigma_{\omega,t})$ decreases. The price of capital in the USA then responds most strongly, as do investment and output. Regarding the risk premium, the risk shock acts as an amplification: the higher the steady-state value, the larger the response following a risk shock. In conclusion, we find the bankruptcy rate plays the key role in amplifying unanticipated risk shocks.

3.2.3 Persistent uncertainty shocks. The empirical evidence for uncertainty shocks in the USA, as depicted in Ludvigson *et al.* (2016) and Jurado *et al.* (2015), suggests that the dynamics in uncertainty are richer than implied by a simple autoregressive process. As described in Strobel (2017), financial uncertainty peaks, on average, for the six shocks indicated by Ludvigson *et al.* (2016) after rising for 24 months and after increasing by 48.42 percent. We analyze the impact of this dynamic shock in the monthly frequency and adjust the calibration of the parameters accordingly, as shown in Table V.

Figure 7 shows the hump-shaped time path of uncertainty following a shock to ε_x, with $\varphi_x = 0.048$, for different values of the persistence parameter. The horizontal axis measures time in monthly periods, while the vertical axis shows the percentage deviation from the steady state. If $\rho_x = 0$, there is a jump in uncertainty, as analyzed previously. The larger ρ_x, the more pronounced the hump in $\sigma_{\omega,t}$ and the longer uncertainty rises before it peaks. For our analysis, we set $\rho_x = 0.96$, such that uncertainty peaks after rising for 25 months and to match the increase relative to the steady state.

Figures 8 and 9 show the impulse response of real and key credit channel variables following a dynamic uncertainty shock. For Spain, there is essentially no response for any of the variables. As before, this result is due to the very low bankruptcy rate and the associated very low default threshold value, which decreases further following a dynamic uncertainty shock and precludes a quantitatively relevant effect for most variables.

Comparing the remaining countries, the initial drop in output is the greatest for the Euro area countries. Subsequently, however, the rebound and drop in output are greater for the USA – although the slump in output that ensues is similar to that in Ireland. These movements are best understood by considering investment, the price of capital and the bankruptcy rate. Because the model's agents anticipate the time path of uncertainty after a shock, they save as a precaution as investment opportunities further deteriorate in future periods. This increase is greatest in the USA because the increase in the bankruptcy rate is greatest in the USA. Conversely, the slump in investment (and output) that sets in after about ten months is also the greatest in the USA – but very similar in Ireland. While the initial drop and rebound in investment in Ireland and Austria is similar, the subsequent drop is considerably larger in Ireland because of the large increase in the price of capital. This increase, in turn, is driven by the large increase in the risk premium and despite the large drop in the bankruptcy rate. In other words, the high risk premium in Ireland prevents households from further increasing investment. For the USA, the evolution of the price of capital similar to the Irish, but mainly driven by the elevated

Economy	$\overline{\omega}$	$\overline{\sigma}$	br (%)	rp (%)	γ	β	δ
USA	0.5883	0.188	0.974	1.87	0.9778	0.9966	0.02/3
Ireland	0.083	0.773	0.685	8.85	0.9720	0.9983	0.025/3
Austria	0.094	0.694	0.332	3.76	0.9858	0.9983	0.025/3
Spain	0.0025	1.315	0.005	1.99	0.9942	0.9983	0.025/3

Table V.
Calibration key credit channel variables of the four economies in the monthly frequency
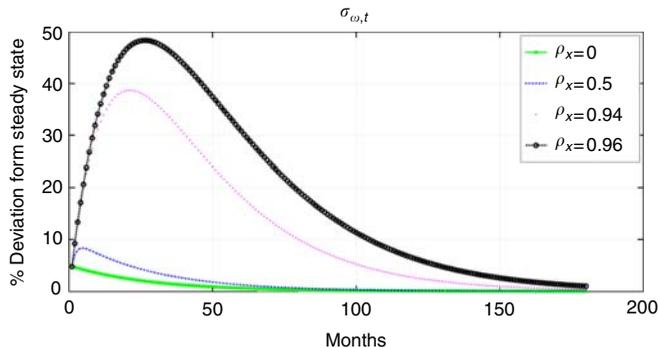
Figure 7.
Response of uncertainty, σ_ω, for different values of the persistence parameter ρ_x
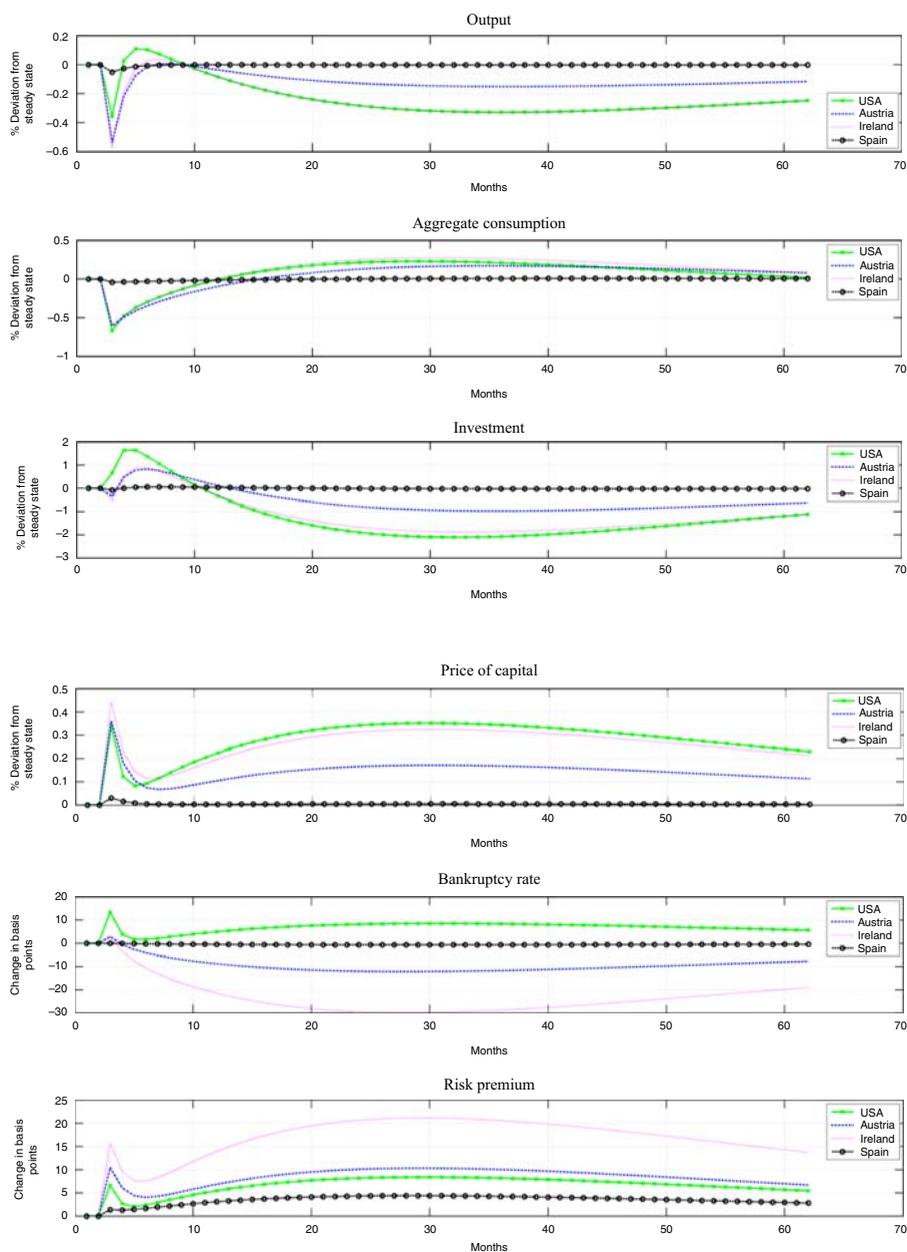
bankruptcy rate rather than the risk premium. Overall, the dynamic uncertainty shocks induce a long slump in investment and output after inducing a precautionary increase in investment. We find that risk premium and bankruptcy rate play an important role in influencing the price of capital relatively, which, in turn, reduces investment and output.

4. Conclusion

Theoretical works on the credit channel effect on aggregate economic variables in the last ten years have seen a proliferation of macroeconomic models. The common element in this literature is that lending activity is characterized by asymmetric information between borrowers and lenders. As a consequence, interest rates may not move to clear lending markets (as in models with moral hazard and adverse selection elements) or firms' net worth may play a critical role as collateral in influencing lending activity (as in models with agency costs). While debate on the empirical support for these models continues, there is little doubt that, as a whole, they have improved our understanding of financial intermediation and broadened the scope of how monetary policy, through the impact of interest rates on firms' net worth, can influence macroeconomic performance. Our attempt in this paper is to show empirically that the credit channel effect matters and that the effect propagates and amplifies business cycles. Our result is in direct contrast to the recent findings by Angeloni *et al.* (2003) who state that the interest rate channel alone could explain most of the monetary policies in the Euro Area. Our and Angeloni *et al.*'s results differ due mainly due to the nature of methodology: we calibrate a dynamic stochastic general equilibrium whereas Angeloni *et al.* (2003) estimate reduced form equations.

Our primary findings fall into two broad categories. First, aggregate technology shock could propagate and amplify various aggregate macroeconomic variables in an environment where there is a financial intermediation, i.e. where there is a credit channel effect. The bankruptcy rate, in particular, plays a major role for this amplification. Second, when compared to various economies that differ only in two financial dimensions (bankruptcy rate and risk premium), we find that the magnitude of shocks to aggregate technology may be quantitatively large. We demonstrate that the effects of minor differences in the credit market translate into large, persistent and asymmetric fluctuations in investment, price of capital, bankruptcy rate and risk premium. The effects imply that the Euro area's supply elasticities for capital are less elastic than the USA following a technology shock. The importance of the bankruptcy rate extends to the risk shocks. For jumps in risk, the bankruptcy rate seems to dominate; the larger the bankruptcy rate, the more adverse the consequences, especially in terms of real variables. The results are a bit more nuanced for the dynamic uncertainty shocks, where we find that both the bankruptcy and the risk premium may strongly influence the price of capital, investment (demand) and finally output. We conclude that the financial accelerator mechanism could potentially play a significant role in business cycles in the Euro area. This result directly lends one to conclude the following: the credit channel that affects the financial sector does indeed matter for macroeconomic behavior.

Notes

1. Financial accelerator models are usually classified into two categories: agency costs models and credit constraint models. Some prominent contributions in agency costs literature are: Williamson (1987), Bernanke and Gertler (1989, 1990), Bernanke *et al.* (1999) and Carlstrom and Fuerst (1997). For constraint models, see Scheinkman and Weiss (1986), Kiyotaki and Moore (1997), Kiyotaki (1998), Cooley and Quadrini (2001) and Kocherlakota (2000). Walsh (2003) presents an overview, both theoretical and empirical, of the literature.

2. See, for example, Fisher (1999), Kocherlakota (2000), Cole and Ohanian (2000), Cooper and Ejarque (2000), Cordoba and Ripoll (2003), Campagne *et al.* (2015), Bachmann and Bayer (2013), Chugh (2016) and Dmitriev and Hoddenbagh (2015) for a negative stance on the role that financial

sector plays in the actual economy. In contrast, Christiano *et al.* (2014) and Strobel *et al.* (2016) find that the financial accelerator plays an important role for the business cycle.

3. We include Austria as a case where both the bankruptcy rate and risk premium lie between the two extremes of Ireland and Spain. We also analyze all EMU11 countries but they are not reported here. The complete results are available upon request.

4. The notation $\Phi(\omega; \sigma_{\omega,t})$ is used to denote that the distribution function is time-varying as determined by the realization of the random variable, $\sigma_{\omega,\ t}$. For expositional purposes, we suppress the time notation on the price of capital and net worth since these are treated as parameters in this section.

5. The fraction of entrepreneurs in the economy, η, is not a critical parameter for the behavior of the economy. As Carlstrom and Fuerst note, it is simply a normalization. Aggregate consumption in the model is indeed a weighted average of household and entrepreneurial consumption but the weights are determined by the steady-state level of per capita consumption for these groups. This is endogenously determined, but not by η. This is demonstrated at the end of Appendix 2.

6. Using the same procedure, Ludvigson *et al.* (2016) with their financial uncertainty measure gives 45 percent relative to the steady state. Meinen and Röhe (2017) provide uncertainty measures for several European countries. To maintain comparability, however, we examine an uncertainty shock of the same magnitude.

7. Note that we denote aggregate variables with upper case while lower case represents per capita values. Prices are also lower case.

8. As in Carlstrom and Fuerst, we assume that the entrepreneur's labor share is small, in particular, $\alpha_{H^e} = 0.0001$. The inclusion of entrepreneurs' labor into the aggregate production function serves as a technical device so that entrepreneurs' net worth is always positive, even when insolvent.

9. As noted above, we require in a steady state: $1 = \gamma q_t f(\overline{\omega}_t)/(1 - q_t g(\overline{\omega}_t))$.

10. A more thorough presentation of the equilibrium conditions is provided in Appendix 2.

References

Agresti, A.-M. and Mojon, B. (2001), "Some stylized facts on the Euro area business cycles", Working Paper No. 95, European Central Bank, Frankfurt.

Alderson, M.J. and Betker, B.L. (1995), "Liquidation costs and capital structure", *Journal of Financial Economics*, Vol. 39 No. 1, pp. 45-69.

Altman, E. (1984), "A further investigation of the bankruptcy cost question", *Journal of Finance*, Vol. 39 No. 4, pp. 1067-1089.

Angeloni, I., Kashyap, A., Mojon, B. and Terlizzese, D. (2003), "The output composition puzzle: a difference in the monetary transmission mechanism in the Euro area and The United States", *Journal of Money, Credit and Banking*, Vol. 35 No. 6, pp. 1265-1306.

Bachmann, R. and Bayer, A. (2013), " 'Wait-and-see' business cycles?", *Journal of Monetary Economics*, Vol. 60 No. 6, pp. 704-719.

Bernanke, B. and Gertler, M. (1989), "Agency costs, net worth, and business fluctuations", *American Economic Review*, Vol. 79 No. 1, pp. 14-31.

Bernanke, B. and Gertler, M. (1990), "Financial fragility and economic performance", *Quarterly Journal of Economics*, Vol. 105 No. 1, pp. 87-114.

Bernanke, B., Gertler, M. and Gilchrist, S. (1999), "The financial accelerator in a quantitative business cycle framework", in Taylor, J.B. and Woodford, M. (Eds), *Handbook of Macroeconomics*, Vol. 1, Elsevier, B.V, Amsterdam, pp. 1341-1393.

Campagne, B., Alhenc-Gelas, V. and Bernard, J. (2015), "No evidence of financial accelerator in France", Insee Documents de travail No. G2015/07, Malakoff Cedex.

Carlstrom, C. and Fuerst, T. (1997), "Agency costs, net worth, and business fluctuations: a computable general equilibrium analysis", *American Economic Review*, Vol. 87, pp. 893-910.

Casares, M. (2001), "Business cycle and monetary policy analysis in a structural sticky price model of the Euro area", Working Paper No. 49, European Central Bank, Frankfurt

Cecchetti, S. (1999), "Legal structure, financial structure, and the monetary policy transmission mechanism", *FRBNY Economic Policy Review*, July, pp. 9-28.

Christiano, L.J., Motto, R. and Rostagno, M. (2014), "Risk shocks", *American Economic Review*, Vol. 104 No. 1, pp. 27-65.

Chugh, S. (2016), "Firm risk and leverage-based business cycles", *Review of Economic Dynamics*, Vol. 20, April, pp. 111-131.

Claessens, S. and Klapper, L.F. (2002), "Bankruptcy around the world - explanations of its relative use", *World Bank Policy Research*, Working Paper WPS2865.

Cole, H.L. and Ohanian, L.E. (2000), "Re-examining the contribution of money and banking shocks to the US Great Depression", in Bernanke, B. and Rogoff, K. (Eds), *NBER Macroeconomics Annual*, Vol. 15, MIT Press, Cambridge, MA; and London, pp. 183-227.

Cooley, T. and Quadrini, V. (2001), "Financial markets and firm dynamics", *American Economic Review*, Vol. 91 No. 5, pp. 1286-1310.

Cooper, R. and Ejarque, J. (2000), "Financial intermediation and aggregate fluctuations: a quantitative analysis", *Macroeconomic Dynamics*, Vol. 4 No. 4, pp. 423-447.

Cordoba, J.C. and Ripoll, M. (2003), "Credit cycles redux", *International Economic Review*, Vol. 45 No. 4, pp. 1011-1046.

Dmitriev, M. and Hoddenbagh, J. (2015), *The Financial Accelerator and the Optimal Lending Contract*, Mimeo, available at: https://ssrn.com/abstract=2706599 (accessed September 1, 2015).

Dorofeenko, V., Lee, G. and Salyer, K. (2008), "Time varying uncertainty and the credit channel", *Bulleting of Economic Research*, Vol. 60 No. 4, pp. 375-403.

Fisher, J. (1999), "Credit market imperfections and the heterogeneous response of firms to monetary shocks", *Journal of Money, Credit and Banking*, Vol. 31 No. 2, pp. 187-211.

Jurado, K., Ludvigson, S. and Ng, S. (2015), "Measuring uncertainty", *American Economic Review*, Vol. 105 No. 3, pp. 1177-1216.

Justiniano, A. and Primiceri, G. (2008), "The time-varying volatility of macroeconomic fluctuations", *American Economic Review*, Vol. 98 No. 3, pp. 604-641.

Kiyotaki, N. (1998), "Credit and business cycles", *The Japanese Economic Review*, Vol. 49 No. 1, pp. 19-36.

Kiyotaki, N. and Moore, J. (1997), "Credit cycles", *Journal of Political Economy*, Vol. 105 No. 2, pp. 211-248.

Kocherlakota, N. (2000), "Creating business cycles through credit constraints", *Federal Reserve Bank of Minneapolis Quarterly Review*, Vol. 24 No. 3, pp. 2-10.

Ludvigson, S., Ma, S. and Ng, S. (2016), *Uncertainty and Business Cycles: Exogenous Impulse or Endogenous Response?*, Mimeo, available at: www.sydneyludvigson.com/s/lmn1.pdf (accessed May 25, 2018).

Meinen, P. and Röhe, O. (2017), "On measuring aggregate uncertainty and its impact on investment: cross-country evidence from the Euro area", *European Economic Review*, Vol. 92, pp. 161-179.

Scheinkman, J. and Weiss, L. (1986), "Borrowing constraints and aggregate economic activity", *Econometrica*, Vol. 54 No. 1, pp. 23-46.

Strobel, J. (2017), "Persistent uncertainty shocks", available at: https://drive.google.com/open?id=1m0 aAO81qp4sS7NIZLb3S_YUwXUq-HJvS (accessed May 28, 2018).

Strobel, J., Salyer, K.D., Lee, G. and Dorofeenko, V. (2016), "On modeling risk shocks", mimeo, Regensburg, available at: www.dropbox.com/s/w0dvayvcorqdi0z/mixture_model_Nov_1% 202016%20.pdf?dl=0 (accessed May 18, 2016).

Walsh, C. (2003), *Monetary Theory and Policy*, 2nd ed., The MIT Press, Cambridge, MA.

Williamson, S. (1987), "Financial intermediation, business failures, and real business cycles", *Journal of Political Economy*, Vol. 95 No. 6, pp. 1196-1216.

Further reading

House, C. (2006), "Adverse selection and the financial accelerator", *Journal of Monetary Economics*, Vol. 53 No. 6, pp. 1117-1134.

King, R. and Rebelo, S. (2000), "Resuscitating real business cycles", in Taylor, J.B. and Woodford, M. (Eds), *Handbook of Macroeconomics*, 1st ed., Elsevier B.V. Amsterdam, Vol. 1, pp. 927-1007.

Olver, F.W.J. (1997), *Asymptotics and Special Functions*, A.K. Peters, Ltd, Wellesley, MA.

Appendix 1

Data source for Table I:

- Bankruptcy rates for the EU nations: Claessens and Klapper (2002), Table II. For the US bankruptcy rate, see Carlstrom and Fuerst (1997).

- Risk premium: lending minus deposit rates. Source: European Central Bank. National Retail Interest Rates. 1995:4-2002:8

(1) Austria

- Lending rate; N4 short-term loans to enterprises. "Loans to enterprises" .

- Deposit rate; N8 time deposits. "Saving deposits with maturity up to 12 months."

(2) Ireland

- Lending rate; N4 short-term loans to enterprises. "Overdrafts and term loans up to 1 year —AA rate/lending to firms."

- Deposit rate; N92 savings accounts. "Clearing banks demand deposits IEP 25,000 to IEP 100,000 – enterprises."

(3) Spain

- Lending rate; N4 short-term loans to enterprises. "Variable rate; monthly reviewable."

- Deposit rate; N8 Time deposits. "Deposits with maturity over 1 up to 2 years."

(4) USA (Source: Carlstrom and Fuerst (1997))

- Risk premium: the average spread between the prime rate and the three-month commercial paper rate for the period April 1971-June 1996.

Data Source for ACFs:

All the European GDP per capita series are from the Datastream from 1960 to 2000. These are seasonally adjusted and are expressed in current US dollars. The Datastream source codes are as follows:

- Austria: OEGDPH; Ireland: IRGDPH; Spain: ESGDPH; EMU11: EMGDPCR

- USA: GDP per capita is calculated using the quarterly data for total GDP, population over 16 and CPI from 1948:1 to 2002:1. Source: Federal Reserve Bank Data Bank.

Appendix 2

Model description

This exposition closely follows the work of Dorofeenko *et al.* (2008).

Households
The representative household is infinitely lived and has expected utility over consumption c_t and leisure $1 - l_t$ with functional form given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t [ln(c_t) + v(1 - l_t)], \tag{A1}$$

where E_0 denotes the conditional expectation operator on time zero information, $\beta \in (0,1)$, $\nu > 0$, and l_t is time t labor. The household supplies labor, l_t, and rents its accumulated capital stock, k_t, to firms at the market clearing real wage, w_t, and rental rate r_t, respectively, thus earning a total income of $w_t l_t + r_t k_t$. The household then purchases consumption good from firms at price of 1 (i.e. consumption is the numeraire), and purchases new capital, i_t, at a price of q_t. Consequently, the household's budget constraint is:

$$w_t l_t + r_t k_t \geqslant c_t + q_t i_t. \tag{A2}$$

The law of motion for households' capital stock is standard:

$$k_{t+1} = (1 - \delta)k_t + i_t, \tag{A3}$$

where $\delta \in (0, 1)$ is the depreciation rate on capital.

The necessary conditions associated with the maximization problem include the standard labor–leisure condition and the intertemporal efficiency condition associated with investment. Given the functional form for preferences, these are:

$$vc_t = w_t, \tag{A4}$$

$$\frac{q_t}{c_t} = \beta E_t \left(\frac{q_{t+1}(1 - \delta) + r_{t+1}}{c_{t+1}} \right). \tag{A5}$$

Firms
The economy's output is produced by firms using Cobb-Douglas technology[7]:

$$Y_t = \theta_t K_t^{\alpha_K} H_t^{\alpha_H} \left(H_t^e \right)^{\alpha_{H^e}}, \tag{A6}$$

where Y_t represents the aggregate output; θ_t denotes the aggregate technology shock; K_t denotes the aggregate capital stock; H_t denotes the aggregate household labor supply; H_t^e denotes the aggregate supply of entrepreneurial labor, and $\alpha_K + \alpha_H + \alpha_{H^e} = 1$[8].

The profit maximizing representative firm's first-order conditions are given by the factor market's condition that wage and rental rates are equal to their respective marginal productivities:

$$w_t = \alpha_H \frac{Y_t}{H_t}, \tag{A7}$$

$$r_t = \alpha_K \frac{Y_t}{K_t}, \tag{A8}$$

$$w_t^e = \alpha_{H^e} \frac{Y_t}{H_t^e}, \tag{A9}$$

where w_t^e denotes the wage rate for entrepreneurial labor.

Entrepreneurs
A risk-neutral representative entrepreneur's course of action is as follows. To finance his project at period t, he/she borrows resources from the capital mutual fund (CMF) according to an optimal financial contract. The entire borrowed resources, along with his total net worth at period t, are then invested into his/her capital creation project. If the representative entrepreneur is solvent after observing his own technology shock, he/she then makes his/her consumption decision; otherwise, he/she declares bankruptcy and production is monitored (at a cost) by the CMF.

Entrepreneur's consumption choice
To rule out self-financing by the entrepreneur (i.e. which would eliminate the presence of agency costs), it is assumed that the risk-neutral entrepreneur discounts the future at a higher rate than the household:

$$E_0 \sum_{t=0}^{\infty} (\beta\gamma)^t c_t^e, \tag{A10}$$

where c_t^e denotes entrepreneur's consumption at date t, and $\gamma \in (0,1)$. γ is then chosen so that it offsets the steady-state internal rate of return to entrepreneurs' investment.

At the end of the period, the entrepreneur finances consumption out of the returns from the investment project implying that the law of motion for the entrepreneur's capital stock is:

$$z_{t+1} = n_t \left\{ \frac{f(\overline{\omega}; \sigma_{\omega,t})}{1 - q_t g(\overline{\omega}; \sigma_{\omega,t})} \right\} - \frac{c_t^e}{q_t}. \tag{A11}$$

Note that the expected return to internal fund is $q_t f(\overline{\omega}; \sigma_{\omega,t}) i_t / n_t$; that is, the net worth of size n_t is leveraged into a project of size i_t, entrepreneurs keep the share of the capital produced and capital is priced at q_t consumption goods. Since these are intra-period loans, the opportunity cost is 1[9].

Consequently, the representative entrepreneur maximizes his expected utility function in Equation (A10) over consumption and capital subject to the law of motion for capital, Equation (A11), and the definition of net worth given in Equation (3). The resulting Euler equation is as follows:

$$q_t = \beta\gamma E_t \left\{ (q_{t+1}(1-\delta) + r_{t+1}) \left(\frac{q_{t+1} f(\overline{\omega}; \sigma_{\omega,t})}{(1 - q_{t+1} g(\overline{\omega}; \sigma_{\omega,t}))} \right) \right\}.$$

Financial intermediaries
The CMFs act as risk-neutral financial intermediaries who earn no profit and produce neither consumption nor capital goods. There is a clear role for the CMF in this economy since, through pooling, all aggregate uncertainty of capital production can be eliminated. The CMF receives capital from three sources: entrepreneurs sell undepreciated capital in advance of the loan, after the loan, the CMF receives the newly created capital through loan re-payment and through monitoring of insolvent firms, and, finally, those entrepreneur's that are still solvent, sell some of their capital to the CMF to finance current period consumption. This capital is then sold at the price of q_t units of consumption to households for their investment plans.

Equilibrium
Equilibrium in the economy is represented by market clearing in four markets: the labor markets for households and entrepreneurs and the goods markets for consumption and capital. Letting (H_t, H_t^e) denote the aggregate labor supply of households and entrepreneurs, respectively, we have:

$$H_t = (1-\eta)l_t, \tag{A12}$$

where l_t denotes the labor supply of households and η denotes the fraction of entrepreneurs in the economy:

$$H_t^e = \eta. \tag{A13}$$

Goods market equilibrium is represented by:

$$C_t + I_t = Y_t, \tag{A14}$$

where $C_t = (1-\eta)c_t + \eta c_t^e$ and $I_t = \eta i_t$ (note that upper case variables denotes aggregate quantities while lower case denote per capita quantities).

The law of motion of aggregate capital is given by:

$$K_{t+1} = (1-\delta)K_t + I_t\left[1 - \Phi(\overline{\omega}_t; \sigma_{\omega,t})\mu\right]. \tag{A15}$$

A competitive equilibrium is defined by the decision rules for (aggregate capital, entrepreneurs capital, household labor, entrepreneur's labor, the price of capital, entrepreneur's net worth, investment, the cut-off productivity level, household consumption and entrepreneur's consumption) given by the vector: $\{K_{t+1}, Z_{t+1}, H_t, H_t^e, q_t, n_t, i_t, \overline{\omega}_t, c_t, c_t^e\}$ where these decision rules are stationary functions of $\{K_t, Z_t, \theta_t, \sigma_{\omega,t}\}$ and satisfy the following equations[10]:

$$vc_t = \alpha_H \frac{Y_t}{H_t}, \tag{A16}$$

$$\frac{q_t}{c_t} = \beta E_t\left\{\frac{1}{c_{t+1}}\left(q_{t+1}(1-\delta) + \alpha_K\frac{Y_{t+1}}{K_{t+1}}\right)\right\}, \tag{A17}$$

$$q_t = \left\{1 - \Phi(\overline{\omega}; \sigma_{\omega,t})\mu + \frac{\phi(\overline{\omega}; \sigma_{\omega,t})\mu f(\overline{\omega}; \sigma_{\omega,t})}{f'(\overline{\omega}_t)}\right\}^{-1}, \tag{A18}$$

$$i_t = \frac{1}{\left(1 - q_t g(\overline{\omega}; \sigma_{\omega,t})\right)}n_t, \tag{A19}$$

$$q_t = \beta\gamma E_t\left\{\left(q_{t+1}(1-\delta) + \alpha_K\frac{Y_{t+1}}{K_{t+1}}\right)\left(\frac{q_{t+1}f(\overline{\omega}; \sigma_{\omega,t})}{\left(1 - q_{t+1}g(\overline{\omega}; \sigma_{\omega,t})\right)}\right)\right\}, \tag{A20}$$

$$n_t = \alpha_{H^e}\frac{Y_t}{H_t^e} + Z_t\left(q_t(1-\delta) + \alpha_K\frac{Y_t}{K_t}\right), \tag{A21}$$

$$Z_{t+1} = \eta n_t\left\{\frac{f(\overline{\omega}; \sigma_{\omega,t})}{1 - q_t g(\overline{\omega}; \sigma_{\omega,t})}\right\} - \eta\frac{c_t^e}{q_t}, \tag{A22}$$

$$log(\theta_{t+1}) = \rho log(\theta_t) + \varphi_\theta\xi_{t+1} \text{ where } \xi_t \overset{i.i.d.}{\sim} N(0,1). \tag{A23}$$

The first equation represents the labor-leisure choice for households while the second equation is the necessary condition associated with household's savings decision. The third and fourth equations are

from the optimal lending contract while the fifth equation is the necessary condition associated with entrepreneur's savings decision. The sixth equation is the determination of net worth while the seventh gives the evolution of entrepreneur's capital (the evolution of aggregate capital is given in Equation (A15). The final two equations represent the laws of motion for the aggregate technology and uncertainty shock, respectively.

Steady-state conditions in the Carlstrom and Fuerst agency cost model

We first present the equilibrium conditions and express these in scaled (by the fraction of entrepreneurs in the economy) terms. Then the equations are analyzed for steady-state implications. As in the text, upper case variables denote aggregate wide while lower case represent household variables. Preferences and technology are:

$$U(\tilde{c}, 1-l) = ln\tilde{c} + v(1-l),$$

$$Y = \theta K^{\alpha}[(1-\eta)l]^{1-\alpha-\phi}\eta^{\phi},$$

where η denotes the fraction of entrepreneurs in the economy and θ is the technology shock. Note that aggregate household labor is $L = (1-\eta)l$ while entrepreneurs inelastically supply one unit of labor. We assume that the share of entrepreneur's labor is approximately 0 so that the production function is simply:

$$Y = \theta K^{\alpha}[(1-\eta)l]^{1-\alpha}.$$

This assumption implies that entrepreneurs receive no wage income see Equation (A2) in C&F.

There are nine equilibrium conditions:

(1) The resource constraint:

$$(1-\eta)\tilde{c}_t + \eta c_t^e + \eta i_t = Y_t = \theta_t K_t^{\alpha}[(1-\eta)l_t]^{1-\alpha}. \tag{A24}$$

Let $c = (1-\eta)\tilde{c}/\eta$, $h = (1-\eta)/\eta l$, and $k_t = K_t/\eta$, then Equation (A24) can be written as:

$$c_t + c_t^e + i_t = \theta_t k_t^{\alpha} h_t^{1-\alpha}. \tag{A25}$$

(2) Household's intratemporal efficiency condition:

$$\tilde{c}_t = \frac{(1-\alpha)}{v} K_t^{\alpha}[(1-\eta)l_t]^{-\alpha}.$$

Defining $v_0 = \eta/1-\eta v$, this can be expressed as:

$$v_0 c_t = (1-\alpha)k_t^{\alpha} h_t^{-\alpha}. \tag{A26}$$

(3) Law of motion of aggregate capital stock:

$$K_{t+1} = (1-\delta)K_t + \eta i_t \left[1 - \Phi(\overline{\omega}; \sigma_{\omega,t})\mu\right],$$

dividing by η yields the scaled version:

$$k_{t+1} = (1-\delta)k_t + i_t \left[1 - \Phi(\overline{\omega}; \sigma_{\omega,t})\mu\right]. \tag{A27}$$

(4) Household's intertemporal efficiency condition:

$$q_t\frac{1}{\tilde{c}_t} = \beta E_t \left\{ \frac{1}{\tilde{c}_{t+1}} \left[q_{t+1}(1-\delta) + \theta_{t+1}\alpha K_{t+1}^{\alpha-1}[(1-\eta)l_{t+1}]^{1-\alpha}\right] \right\},$$

dividing both sides by $1-\eta/\eta$ and scaling the inputs by η yields:

$$q_t \frac{1}{c_t} = \beta E_t \left\{ \frac{1}{c_{t+1}} \left[q_{t+1}(1-\delta) + \theta_{t+1} \alpha k_{t+1}^{\alpha-1} h_{t+1}^{1-\alpha} \right] \right\}, \tag{A28}$$

the conditions from the financial contract are already in scaled form.
(5) Contract efficiency condition:

$$q_t = \frac{1}{1 - \Phi(\overline{\omega}; \sigma_{\omega,t}) \mu + \phi(\overline{\omega}; \sigma_{\omega,t}) \mu \frac{f(\overline{\omega}; \sigma_{\omega,t})}{f'(\overline{\omega}_t)}}. \tag{A29}$$

(6) Contract incentive compatibility constraint:

$$\frac{i_t}{n_t} = \frac{1}{1 - q_t g(\overline{\omega}; \sigma_{\omega,t})}, \tag{A30}$$

where n_t is entrepreneur's net worth.
(7) Determination of net worth

$$\eta n_t = Z_t \left[q_t(1-\delta) + \theta_t K_t^{\alpha-1} [(1-\eta)l_t]^{1-\alpha} \right],$$

or, in scaled terms:

$$n_t = z_t \left[q_t(1-\delta) + \theta_t k_t^{\alpha-1} h_t^{1-\alpha} \right], \tag{A31}$$

Note that z_t denotes (scaled) entrepreneur's capital.
(8) Law of motion of entrepreneur's capital:

$$Z_{t+1} = \eta n_t \left\{ \frac{f(\overline{\omega}; \sigma_{\omega,t})}{1 - q_t g(\overline{\omega}; \sigma_{\omega,t})} \right\} - \eta \frac{c_t^e}{q_t},$$

or, dividing by η, it gives:

$$z_{t+1} = n_t \left\{ \frac{f(\overline{\omega}; \sigma_{\omega,t})}{1 - q_t g(\overline{\omega}; \sigma_{\omega,t})} \right\} - \frac{c_t^e}{q_t}. \tag{A32}$$

(9) Entrepreneur's intertemporal efficiency condition:

$$q_t = \gamma \beta E_t \left\{ \left[q_{t+1}(1-\delta) + \theta_{t+1} \alpha K_{t+1}^{\alpha-1} [(1-\eta)l_{t+1}]^{1-\alpha} \right] \left(\frac{q_{t+1} f(\overline{\omega}; \sigma_{\omega,t})}{1 - q_{t+1} g(\overline{\omega}; \sigma_{\omega,t})} \right) \right\},$$

or, in scaled terms:

$$q_t = \gamma \beta E_t \left\{ \left[q_{t+1}(1-\delta) + \theta_{t+1} \alpha k_{t+1}^{\alpha-1} h_{t+1}^{1-\alpha} \right] \left(\frac{q_{t+1} f(\overline{\omega}; \sigma_{\omega,t})}{1 - q_{t+1} g(\overline{\omega}; \sigma_{\omega,t})} \right) \right\}. \tag{A33}$$

The role of η in aggregate consumption
The parameter η does not play a role in the characteristics of equilibrium and, in particular, the behavior of aggregate consumption. This can be seen by first defining aggregate consumption:

$$(1-\eta)\tilde{c}_t + \eta c_t^e = C_t^A,$$

dividing by η and using the earlier definitions:

$$c_t + c_t^e = c_t^A. \tag{A34}$$

Since the policy rules for household and entrepreneurial consumption are defined as the percentage deviations from steady state, aggregate consumption will be similarly defined (and note that since $c_t^A = 1/\eta C_t^A$, percentage deviations of aggregate consumption and scaled aggregate consumption are identical). Using an asterisk to denote percentage deviations from steady-state, we have:

$$\frac{\hat{c}}{\hat{c}+\hat{c}^e}c_t^* + \frac{\hat{c}^e}{\hat{c}+\hat{c}^e}c_t^{e*} = c_t^{A*}. \tag{A35}$$

It is this equation that is used to analyze the cyclical properties of aggregate consumption.

Corresponding author
Johannes Strobel can be contacted at: johannes.strobel@ur.de