Numerical investigation of fracture in double-edge notched FGM plates under tension load

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Abstract

Purpose – The purpose of this paper is to introduce a numerical investigation used to calculate the *J*-integral of the main crack behavior emanating from a semicircular notch and double semicircular notch and its interaction with another crack which may occur in various positions in (TiB/Ti) functionally graded material (FGM) plate subjected to tensile mechanical load.

Design/methodology/approach – For this purpose the variations of the material properties are applied at the integration points and at the nodes by implementing a subroutine USDFLD in the ABAQUS software. The variation of the *J*-integral according to the position, the length and the angle of rotation of cracks is demonstrated. The variation of the *J*-integral according to the position, the length and the angle of rotation of cracks is examined; also the effect of different parameters for double notch FGM plate is investigated as well as the effect of band of FGM within the ceramic plate to reduce *J*-integral.

Findings – According to the numerical analysis, all parameters above played an important role in determining the *J*-integral.

Originality/value – The present study consists in investigating the simulation used to calculate the *J*-integral of the main crack behavior emanating from a semicircular notch and double semicircular notch and its interaction with another crack which may occur in various positions in (TiB/Ti) FGM plate under Mode I. The *J*-integral is determined for various load applied. The cracked plate is joined by bonding an FGM layer to TiB plate on its double side. The determination of the gain on *J*-integral by using FGM layer is highlighted. The calculation of *J*-integral of FGM's involves the direction of the radius of the notch in order to reduce the *J*-integral.

Keywords Functionally graded material (FGM), Finite element method (FEM), Crack, Notch, *J*-integral, Mode I, Interaction

Paper type Research paper

1. Introduction

Composite materials in which the composition or microstructure or both are locally varied so that a certain variation of the local material properties is achieved are defined as functionally graded materials (FGMs). FGMs possess non-homogeneous macrostructure with continuously varying mechanical and/or thermal properties in one or more than one direction. The fatigue life of these components is normally estimated without accounting for the effect of defects/discontinuities. Fatigue life is significantly affected by presence of voids and micro-defects near the tip of a major crack and further enhances the effective SIF at the tip of the major crack. The severity of failure is more when a structure is subjected to mixed mode loading as compared to Mode-I loading. Moreover, the crack growth under mixed mode loading may not be in a self-similar direction adherence issues like crack growth direction also become important. In view of this, the fatigue analysis of FGM under mixed mode loading becomes quite important. Cotterell and Rice (1980) employed the perturbation method and obtained the

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exact solutions for stress intensity factors of a curved crack in an isotropic plane under remote traction. The plane elasticity problem for a non-homogeneous medium containing a crack was examined by Delale and Erdogan (1983). In this work, the integral equation for a crack problem was obtained. The results were shown that the effect of the Poisson's ratio and consequently that of the thickness constraint on the stress intensity factors are rather negligible and the results are highly affected by the FG parameter. Konda and Erdogan (1994) considered the mixed mode plane strain problem for an arbitrarily orientated crack in a non-homogeneous medium. Analytical expressions for mixed-mode stress intensity factors have been obtained by Gu and Asaro (1997) for cases where the crack tip was oriented perpendicular to the material gradient. Stress intensity factors in an inhomogeneous medium contain collinear cracks under Mode I plane strain or plane stress loading conditions was analyzed by Ozturk and Erdogan (2001). Huang and Kardomateas (2001) studied the Modes I and II stress intensity factors in an anisotropic strip with a crack. Kim and Paulino (2002) conducted finite element studies to evaluate stress intensity factor of an orthotropic FGMs. They investigated the effects of boundary conditions, crack-tip mesh discretization and materials properties on fracture behavior of the medium. In another work, Kim and Paulino (2003) presented Mode I and mixed mode crack problems in an orthotropic FGMs based on the concept of the *J*-integral, with the application of finite element method. In this study, both exponential and linear variations of the material properties were considered, also, stress intensity factors and energy release rate for pure Mode I and mixed mode loading conditions were calculated. A new interaction energy integral method for the computation of mixed-mode stress intensity factors for arbitrarily oriented cracks in FGMs was considered by Dolbow and Gosz (2002). Rao and Rahman (2003) solved crack problems for isotropic FGMs under mixed mode loading conditions. They used Galerkin-based meshless method for calculating stress intensity factors. Chen (2004) obtained mixed-mode stress intensity factors for multiple curved cracks in a homogeneous plane by using the distributed dislocation method. A numerical procedure based on the concept of the *J*-integral, for computation of the mixed-mode stress intensity factors for curved cracks, was obtained by Chang and Wu (2007). The dynamic behavior of a finite crack in FGMs subjected to normally incident elastic time harmonic waves was investigated by Xia and Ma (2007). The effects of the shear stress wave velocity, the geometry of the crack and frequency of the incident wave on DSIFs were investigated. Dag *et al.* (2007) studied the mixed mode fracture problem for an orthotropic FGM under mechanical and thermal loading conditions. Fotuhi and Fariborz (2008) employed distributed dislocation technique to analyze a strip having multiple cracks. The results were used to evaluate Modes I and II stress intensity factors. Dag and Ilhan (2008) analyzed the mixed-mode stress intensity factors in a functionally graded orthotropic material coating bonded coat substrate by using of analytical and computational methods. The effects of material non-homogeneity and orthotropic constants, the thickness of bond coat, as well as crack periodicity on the mixed mode SIFs and the energy release rate, were studied. It has been shown that the orthotropic and non-homogeneity parameters significantly influence the fracture behavior of an orthotropic FGM coating. Cracking characteristics of a moving screw dislocation near an interfacial crack in two dissimilar orthotropic media has been discussed by Xie and Liu (2008). Asymptotic analysis coupled with Westergaard stress function approach in order to obtain stress fields for a crack oriented along one of the principal axes of inhomogeneous orthotropic medium under Mode I loading condition examined by Chalivendra (2008). The results shown that stiffness ratio has significant influence subjected to non-homogeneity parameter on the spatial distribution of the stress field around the crack tip. Multiple circular arc cracks in an infinite linear elastic media by using numerical approach presented by Yan (2010). Monfared and Ayatollahi (2016) employed distributed dislocation technique to compute mixed-mode stress intensity factors in a non-homogeneous orthotropic plane.

The investigation of multiple crack interactions in fracture mechanics is important to predict the safety and reliability of structures. This paper introduces a numerical investigation used to calculate the *J*-integral of the main crack behavior emanating from a semicircular notch and double semicircular notch and its interaction with another crack which may occur in various positions in (TiB/Ti) FGM plate subjected to tensile mechanical load. Young's modulus of the FGM plate varies along the specimen width (notch radius direction r-FGM) with exponential-law (E-FGM) function. Furthermore, the Poisson's ratio is taken as a constant in normal direction. For this purpose the variations of the material properties are applied at the integration points and at the nodes by implementing a subroutine USDFLD in the ABAQUS software. The variation of the *J*-integral according to the position, the length and the angle of rotation of cracks is demonstrated. The variation of the *J*-integral according to the position, the length and the angle of rotation of cracks is examined; also the effect of different parameters for double notch FGM plate is investigated as well as the effect of band of FGM within the ceramic plate to reduce *J*-integral. According to the numerical analysis, all parameters above played an important role in determining the *J*-integral.

2. Finite element analysis

This section details the finite-element formulation of enriched crack tip elements for Mode I fracture analyses of two-dimensional cracks in FGMs. Similar to many other studies in the literature, the form of material property gradient functions is selected to be exponential. For the FGM domain containing an inclined embedded crack, the modulus of elasticity varies according to:

$$E(x) = E_1 e^{\beta r}, \tag{1}$$

$$\beta = \frac{1}{h} \ln\left(\frac{E_1}{E_2}\right), \tag{2}$$

where β is a material constant and r is the coordinate by which the material property changes along the notch radius (r-FGM) (Bouchikhi *et al.*, 2019; Bouida *et al.*, 2018). E is the Young modulus; E_1 the Young modulus at $r = R$, E_1 and E_2 the Young modulus for ceramic and metal, respectively. Table I indicates the scale of length over variations of the properties for validation of the FE model.

As shown in Figure 1, a cracked rectangular FGM plate (Ti-TiB) with a double semicircular notch at side under uniform loading ($\sigma_0 = 100$ Mpa) is numerically simulated in the ABAQUS commercial FEM code Version 6.9.126 (ABAQUS Finite Element Program, 2008).

The geometrical characteristics of the FGM plate are the width $W = 200$ mm; the length $H = 2\ W = 400$ mm. To analyze the fracture behavior, a crack of length (c) is supposed to be initiated at the notch root with radius (R).

We used 9,316 elements Quadratic CPS6M: a six-node modified quadratic plane stress triangle Figure 2(a). In total, 65,546 nodes in bulk plate model, we used CPS8R element: an eight-node biquadratic plane stress quadrilateral Figure 2(b), and quad-dominant sweep mesh near the crack tips as shown in Figure 3.

Table I.
Material properties of Ti and TiB

Materials	Young's modulus (GPa)	Poisson's ratio
Ti	110	0.34
TiB	375	0.14

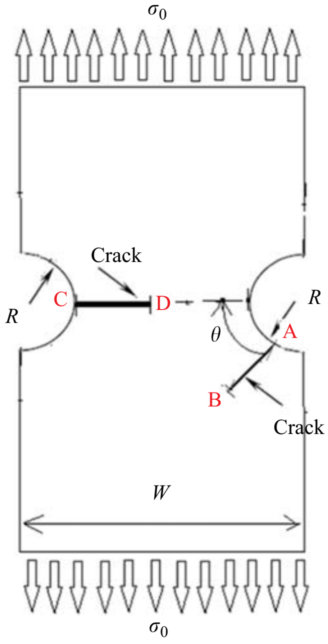
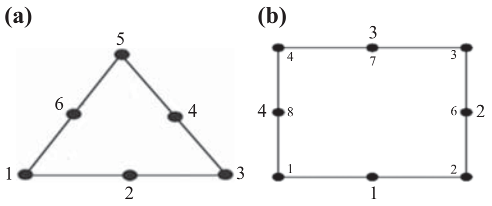
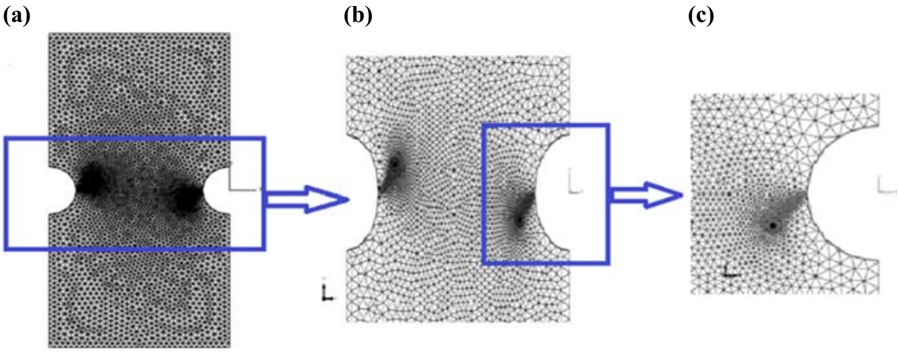


Figure 1.
Geometrical model of FGM plate with fixed and oblique crack at notches



Notes: (a) CPS6M; (b) CPS8R

Figure 2.
Gaussian quadrature



Notes: (a) Geometry and complete finite element mesh; (b) mesh details of two crack tips; (c) zoom of the right crack tip

Figure 3.
Meshing model of FGM plate with two cracks emanating from notches

3. Results and discussions

In this study the numerical calculation is based on FE using ABAQUS software. The direction of variation of the Young's modulus of the FGM is according to r-FGM (Bouchikhi *et al.*, 2019; Bouida *et al.*, 2018).

3.1 Interaction between two cracks with the same orientation and different size crack (c/c_0) on the J-integral

For two-crack interactions, the *J*-integral of each crack is evaluated through varying the crack length ratio for the same angle inclination. First, the ratio of the second crack length to the first crack length (c/c_0) ranges from 0.2 to 1.5 (Figure 1).

Figures 4 and 5 describe the evolution of *J*-integral in different ratios of crack lengths size (c/c_0) emanating from notch at angle θ. It is the fact that the *J*-integral exhibit an asymptotic behavior as the crack length increases whatever the position of the crack in the FGM plate.

It can be noted that the *J*-integral at crack tip A increases with the crack length. An opposite behavior is observed for the *J*-integral at crack tip D. When two-crack lengths are almost similar (e.g. $c/c_0 = 1.5$), the magnitude of *J*-integral is similar. The result also shows that the interaction vanishes with higher angle $\theta = 80°$, at this point the value of *J* is maximum for crack A and inverse at angle $\theta = 20°$ .The size ratio affects the value of *J* proportionally at crack A, and inversely at crack D. On the other hand, the difference in the *J*-integral becomes increasingly important with decreasing crack length and increasing in crack inclination.

Thus, the maximum reduction of the *J*-integral is obtained when the cracks are parallel to the applied load direction. It can be said that the importance of *J*-integral reduction is strongly dependent on crack size. The larger the crack length, the relatively more important the reduction is. This is explained by the fact that the small cracks are requested in the stress field generated by the semicircular notch. The reduction in the *J*-integral influences directly the kinetics of the crack.

3.2 Effect of notch radius and material gradient β on J-integral

The maximum stress is localized in the notch root which allows to characterize the notch by a *J*-integral that depends only of the notch geometry. Figure 6 presents a function of the

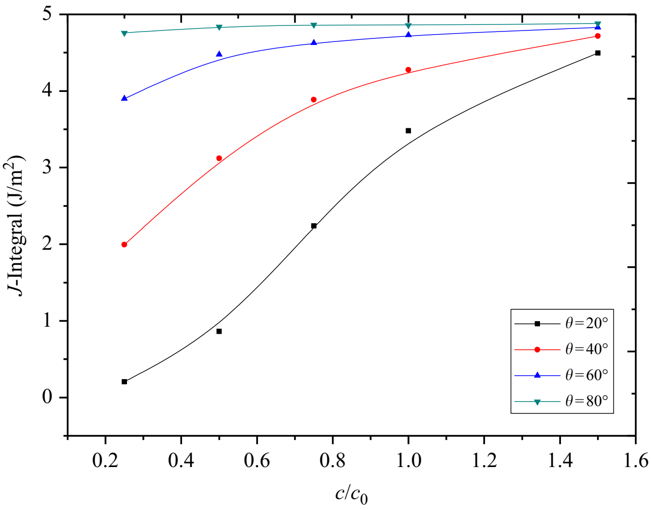


Figure 4.
Variation of the *J*-integral at crack tip A vs c/c_0 crack length ratio for different changing orientation θ

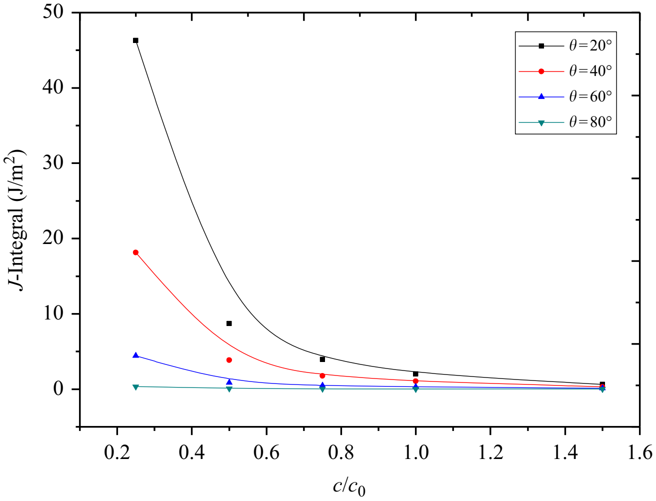


Figure 5.
Variation of the J-integral at crack tip D vs c/c_0 crack length ratio for different changing orientation θ

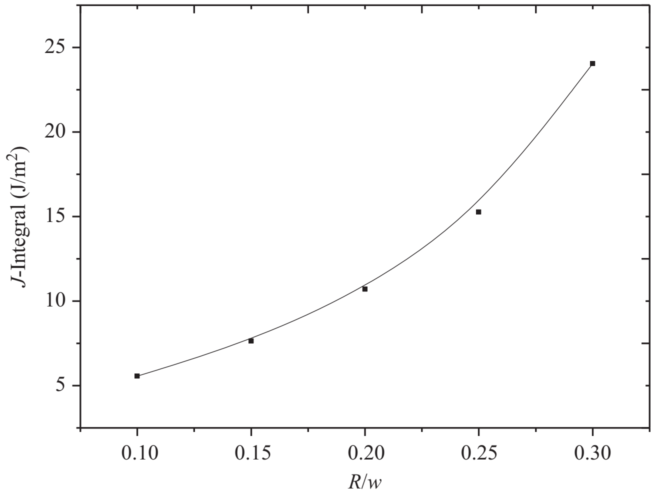


Figure 6.
Variation of the J-integral vs normalized R/w ratio with ($c/w = 0.1$, $c_0/w = 0.1$)

normalized radius (R/w) ratio of the lateral semicircular notches. It can be observed also that the J-integral becomes larger by increasing the control radius. When the notches' normalized, radius lies between 0.1 and 0.3, and the J-integral lies between 5 and 24 J/m^2 for circular notches and for semicircular notches, respectively.

The Young's modulus may vary exponentially through the FGM plate width. In this case, for example, the Young's modulus varies from 70 to 140 GPa. The Poisson's ratio assumed to be constant and equal to 0.3. The variation of the J-integral vs values of β is plotted in Figure 7.

It should be noted again that $\beta = 0$ may represent the homogenous material.

For high material gradient (the exponent β) the J-integral is negligible, it may be concluded that the material gradient (the exponent β) has larger effect on the J-integral (Figure 7).

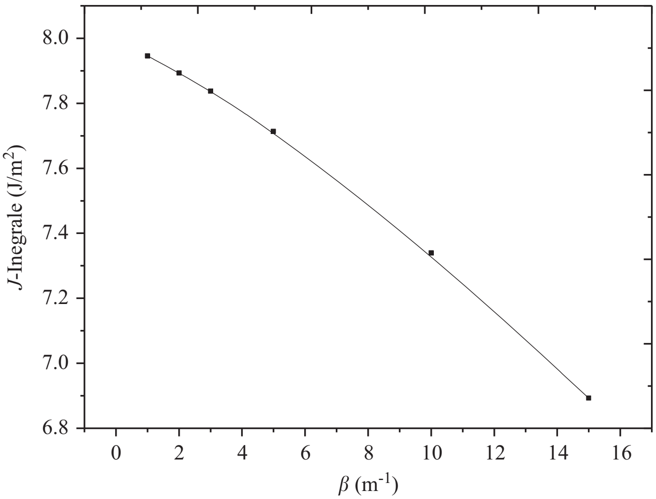


Figure 7.
Variation of J-integral vs material gradient β ($c/w = 0.1$, $c_0/w = 0.1$)

3.3 Behavior of a fixed and oblique cracks emanating from double notched FGM plate under tension

Effect of interaction of the orientation between two inter notch-cracks with equal lengths emanating from each semicircular notch root in double notched plate is studied in this part; in Figure 1, one considers the geometrical model of the FGM plate, to ensure the opening of cracks, we let the plate be under constant unidirectional loadings σ_0 at the far-field. The influence of the orientation of the second crack AB is highlighted while maintaining the angle of the first crack CD constant (Crack CD is fixed, whereas the orientation of crack AB is changed).

Figure 8 illustrates the variation of the J-integral at crack tip B and D in Mode I as a function of the variation of the inclination angle of the crack AB at semicircular notch root. One shows, respectively, the effect of the main crack AB rotation located at notch with the angle for various values between 60° and −60°. Because of symmetry, the J-integral possesses the same curve for the positive and negative angles of inclination; this shows that it is independent

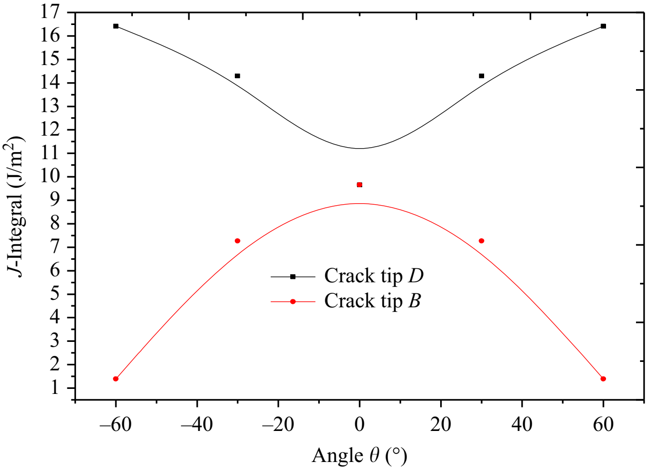


Figure 8.
Variation of J-integral at crack tips vs angle inclination θ oblique crack at crack tip B and D

of the sense of orientation of the existing crack AB. In the case of the negative angles, the *J*-integral takes positive values and the curve is asymmetric with regard to the *y*-axis.

The results show also that the crack the rotation of the crack AB has relatively great effect on *J*-integral when the crack angle is relatively small. For this case that the crack inclined angle is smaller, the traction on the crack is larger and the stresses near the crack are larger. So the crack AB has greater effect on the crack CD in the case.

Figure 8 shows the effect of right cracks on crack tip D. In this case, the *J*-integral for both crack tips are identical as $\theta = 0$ because of the symmetry. At this angle, the value of *J*-integral is maximum. The angle of crack tip B is inversely influenced at crack tip D. The *J*-integral of crack tip D is minimum when θ increases.

3.4 Behavior of two oblique cracks with same angle inclination emanating from double notched FGM plate under tension

In this part, the interaction of two cracks emanating from notches in a plate is investigated. As Figure 2 indicates, we present the geometrical model of the double notched plate in the presence of two cracks located at each notch emanating from the notch root. The plate considered is subjected to the same boundary conditions.

Figure 2(a) shows the FGM plate geometry, Figure 2(a) shows the complete mesh configuration, Figure 2(b) shows mesh detail of two cracks and Figure 2(c) shows a zoom of the left crack tip region showing mesh.

For two-crack interactions, the *J*-integral of each crack is evaluated through varying the crack angle inclination which is the same of the two cracks for the same length ratio.

Figure 9 shows the variation of the *J*-integral as a function of the variation of the inclination angle of the two cracks. The results are, respectively, obtained for seven positions of the two cracks emanating from the semicircular notch, which have the same angle rotation between $\theta = -60°$and 60°. The angle is positive following the application of the principle of local symmetry. It is clear that the *J*-integral curves are symmetric with regard to the ligament of the plate. The curves take the importance when the angle θ decreases. One notices that function *J*-integral (θ) passes by a maximum corresponding to the angle $\theta = 0$ of the two existing cracks (i.e. the preexisting cracks is perpendicularly oriented with respect the applied load).

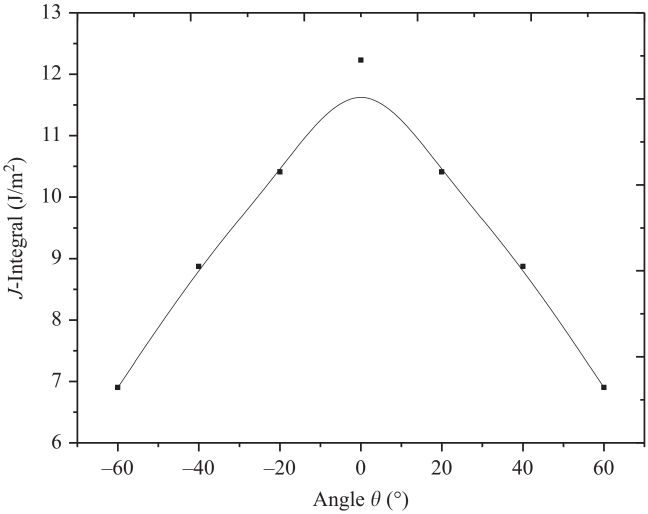


Figure 9.
Variation of *J*-integral vs angle inclination θ of two cracks

3.5 New techniques to reduce the J-integral with FGM materials

3.5.1 Effect of FGMs layers jointed on ceramic plate (TiB) to reduce the J-integral. The real structures are of complex geometrical forms containing numerous zones of stress concentrations. These sites are characterized by weak sections due to the presence of notches which are the main causes of cracks initiation. The knowledge of the distribution of the stress field in the neighborhood of a notch is of an extreme importance for the analysis of the variation of the *J*-integral.

This section has been made to determine the performance of the (Ti-TiB) FGM materials to reduce the *J*-integral calculated for the crack tip emanating from the semicircular notch root in Mode I, when the crack is propagated from the notch root.

Two-crack interactions in a double notched FGM plate are investigated with remote tension (ð^0). The geometry of a notched plate with two cracks is shown in Figure 10. For two cracks of AB and CD, the lengths of two cracks are c and c_0.

In order to analyze the effect of an FGM layer jointed on ceramic plate (TiB) for reducing the integral J on the repair of the cracks emanating from the notch subjected to unidirectional tensile load was studied (Figure 10(c)), we consider the geometric model of a TiB ceramic plate containing two FGM layers (R_1–FGM, (Ti-TiB)) around a circular notch of radius R, the layer is of width w_0 ($w_0 = R_1 – R$) as shown in Figure 10(b).

For a better illustration of the beneficial effect of the FGM materials to reduce the *J*-integral, it is plotted in Figure 11, the variation of the asymptotic *J*-integral according to the FGM layer w_0 defined as $w_0 = R_1 – R$ for a double notched jointed to TiB plate for various load was applied.

On the one hand, the *J*-integral is affected by the FGM layer jointed. The increase in the width leads to the increase in the *J*-integral. The *J*-integral is stabilized when FGM layer jointed width exceeds $w_0/w = 0.4$. This effect is due to the fact that the increase in the size of the FGM layer jointed width involves an increase in the stress field at the crack tip, which leads to a saturation of the stresses transfer. This saturation involves a stabilization of the increase of the *J*-integral according to the crack size, an optimization of the FGM layer width is recommended. On the other hand, the *J*-integral increases proportionally with the increment in the applied loads.

The use of an FGM layer of size $w_0/w = 0.2$ results in a significant decrease of the *J*-integral with a gain of 69 percent. The maximum value of J is almost at the level of $w_0/w = 0.4$ with gain of 52 percent.

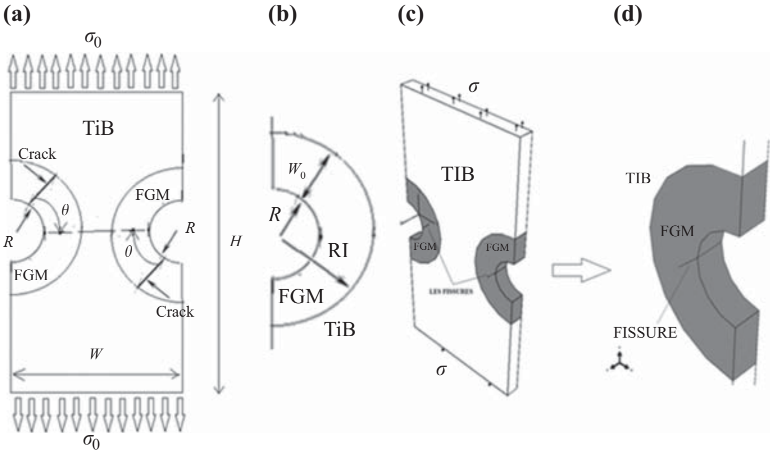


Figure 10. Geometric model of a ceramic plate (TiB) jointed with two FGMs layers at notches

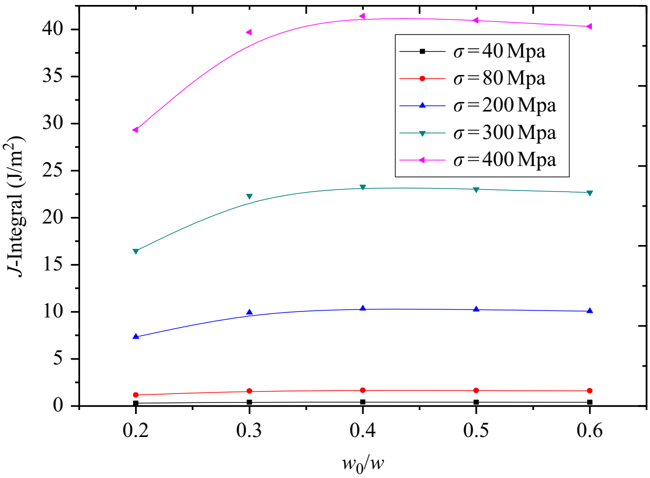


Figure 11. Variation of the J-integral vs (w_0/w) ratio (coming $c/w = c_0/w = 0.1$, $\theta = 45°$)

3.5.2 Effect of the radius of notched FGM layer for reduce J-integral. The influence of the geometry plate is very important, in order to determine the effect of the notch radius on the performance of the bonded TiB plate repair by FGM layer jointed in Mode I. The orientation of notched crack is $\theta = 45^0$ for FGM layer width of $w_0/w = 0.125$ maintained constant.

Figure 12 presents the variation of the J-integral as a function of the notch radii for various subject loads. It is noted that the J-integral increased such that the important factor (R/w) in FGMs layers $w_0/w = 0.125$, the difference of the J-integral becomes larger and more important with the increase of the notch radius and important load.

4. Conclusion

The obtained results allow us to deduce the following conclusions:

- A normal crack to load leads to a highest J-integral at the crack tip. This energy decreases gradually as the orientation angle decreases.

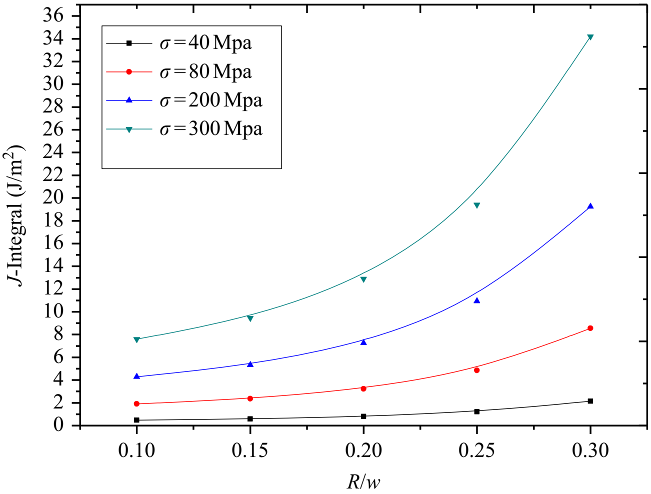


Figure 12. Variation of the J-integral vs R/w ($w_0/w = 0.125$, $c/w = c_0/w = 0.1$, $\theta = 45°$)

- The *J*-integral at the crack tip depends on both the crack length and its position.
- The increase of the crack length causes an increase in the *J*-integral. The crack propagation leads to an increase of fracture energy.
- In case of two cracks emanating from notch in an FGM plate under tension, the interaction and their effect on the *J*-integral is important when value of θ is maximum.
- Effect of the orientation with different crack sizes is remarkable; the maximum of *J*-integral is obtained when the angle increases corresponding obtained when the crack is parallel to loading and higher size, and the minimum value is obtained for the crack perpendicular to loading.
- For the case of two cracks emanating from double notched FGM plate under tension, the *J*-integral for both crack tips are identical as $\theta = 0$ due to symmetry. At this angle, the *J*-integral value is maximum. The angle of crack B is inversely influenced at crack A. The *J*-integral of crack A is minimum when θ increases.
- The effect of the loading and the notch radius ratio ($\sigma 0$, R/w) on the *J*-integral was highlighted; the interaction grows for high value of the loading and notch radius.
- The good choice of the FGMs layers at the notches in ceramic plate (TiB) and notch radius must be optimized; because it is the best means for decreasing the *J*-integral considerably and improves the fracture strength.
- The decrease of the *J*-integral becomes more and more important with the importance of the semicircular FGM layer radius around the notch. The use of an FGM layer of size $w_0/w = 0.2$ results in a significant decrease of the *J*-integral with a gain of 69 percent. The maximum of *J* is almost at the level of $w_0/w = 0.4$ with gain of 52 percent.

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