Guest editorial: Networking theories for understanding and guiding lesson study

Introduction

As an effective teacher collaborative professional learning approach, lesson study (LS) has been adapted in different settings and cultures around the world (e.g. Huang *et al.*, 2019). Although the effects of LS on students' learning, teachers' learning, the building of professional learning communities, and connecting theory and practice have been widely documented (e.g. da Ponte, 2017; Huang and Shimizu, 2016; Willems and Van den Bossche, 2019), the exploration of what theories could be used as frameworks for researching LS and/or as intervention instruments for strengthening LS is still an emerging field (e.g. Borko and Potari, 2020; Clivaz, 2018; Huang *et al.*, 2019; Quaresma *et al.*, 2018). For example, in *Theory and Practice of Lesson Study in Mathematics* (Huang *et al.*, 2019), 16 different theoretical perspectives were explicitly utilized to guide the design of LS and/or frame studies on LS. Researchers argued that the diversity of theories is "the *indicator* for the dynamic character of the field and also an *outcome* of the dynamics of the theory" (Bikner-Ahsbahs and Prediger, 2014, p. 5). Yet, it was emphasized that "the diversity of theoretical approaches can only become fruitful if connections between them are actively established" (Bikner-Ahsbahs and Prediger, 2014, p. 8). In mathematics education, there has been systematic exploration into the potential and challenges of networking theories to deepen the understanding of complex education phenomena (e.g. Bikner-Ahsbahs and Prediger, 2014; Prediger *et al.*, 2008a, b). In the case of LS, some researchers have explored how multiple theories might be networked to guide LS and/or study LS (e.g. Clivaz and Ni Shuilleabhain, 2019; Huang *et al.*, 2016). In this special issue, we focus on LS in mathematics as a case to explore the use of networking theories in LS.

Theories and networking theories

There are various interpretations of the meaning of theory (Silver and Herbst, 2007; Radford, 2008). We subscribe to the view that "from its role in research practices, theories can be understood as guiding research practices and at the same time being influenced by or being the aim of research practices" (Bikner-Ahsbahs and Prediger, 2014, p. 7). According to Radford (2008), a "theory can be seen as a way of producing understandings and ways of action based on:

(1) A system, P, of basic principles, which includes implicit views and explicit statements that delineate the frontier of what will be the universe of discourse and the adopted research perspective.

(2) A methodology, M, which includes techniques of data collection and data interpretation as supported by P.

(3) A set, Q, of paradigmatic research questions (templates or schemas that generate specific questions as new interpretations arise or as the principles are deepened, expanded or modified)." (Radford, 2008, p. 320)

Various theories can be classified into three levels according to Kieran *et al.* (2015).

(1) Local theoretical frameworks: frames that specify a particular feature of teachers' knowledge or activity (e.g. teacher noticing, teacher design).

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(2) Intermediate theoretical frameworks: frames that do not specify a particular feature, but rather a general aspect of teachers' knowledge or activity (e.g. knowledge for teaching, professional growth).

(3) Grand theoretical frameworks: well-established frames for research inside and outside mathematics education that have been developed in a broader context of human activity (e.g. ATD, CHAT).

The diversity of theories can be seen as a problem or as a resource. In line with the point of view of Prediger *et al.* (2008b), we consider "that the variety of different theoretical approaches and perspectives in mathematics education research is a rich resource upon which the scientific community should build more consequently" (p. 163). To do so, Prediger *et al.* (2008a) described networking strategies and methods as lying on a spectrum of intentions in relation to the use of different theories. At one extreme, the privilege is given to just one theory while *ignoring* others; at the other extreme, the aim is to *unify* some theories into a global grand theory for research and practice in mathematics education. They describe three networking strategies for bringing together two or more theories in terms of different intentions and conditions along this spectrum: understanding others and making understandable, contrasting and comparing; combining and coordinating; synthesizing and integrating locally. First, all attempts to connect theories must start with the hard work of *understanding the theoretical work of others* and, reciprocally, with *making your own theoretical stance understandable*. Second, the most used pair of networking theories is *comparing and contrasting* theoretical approaches primarily used for a better understanding of typical characteristics of the theoretical approaches. Whereas comparing refers to similarities and differences, contrasting is more focused on stressing differences. Through contrasting, the specificity of theories and their possible connections can be made more visible: the strong similarities are for linking and the strong differences are for highlighting the individual strengths. Third, *coordinating and combining strategies of networking* are mainly used to deepen the understanding of an empirical phenomenon or a piece of data. The networking strategies of combining and coordinating are typical for conceptual frameworks for using different analytical tools in the context of a practical problem or the analysis of a concrete empirical phenomenon. Coordinating is used when a conceptual framework is built by carefully fitting together elements from different theories. It is necessary to carefully analyze the compatibility of the core elements of the different theories and how the empirical components are complementary. Combining is used when theoretical approaches are only juxtaposed, which does not necessitate the complementarity or even the complete coherence of the theoretical approaches. Fourth, whereas the strategies of combining and coordinating aim at a deeper insight into an empirical phenomenon, the strategies of *synthesizing and integrating locally* are focused on the development of theories by putting together a small number of theoretical approaches into a new framework. The notion of synthesizing is used when two (or more) equally stable theories are connected in such a way that a new theory evolves. Often, there are only some concepts or aspects of one theory integrated into an already more elaborated dominant theory which is emphasized as integrating locally.

Beyond identifying these networking strategies, Bikner-Ahsbahs and Prediger (2014, p. 10) also indicated that networking theories could be on three levels: (1) empirical, to gain deep and complex insights into empirical conceptualized phenomena; (2) theoretical, to give impetus to the development of further theory such as sharpening theoretical principles or constructs, extending theoretical approaches, building new concepts, posing new questions; and (3) methodological, to offer insights transferrable from concrete cases to networking in principle.

Goals of this special issue and major contributions

Building on the literature on networking theories in mathematics education and studies on LS, the editors of this special issue believe that networking theories could advance the theorizing of LS across the curriculum. Our goal is twofold: (1) to strengthen the design of LS, and (2) to develop theories for framing research of LS and deepening understanding of why and how LS works for what purposes.

Eight papers are included in this special issue. Two focus on networking theories at the theoretical level and six at the empirical level. Thirteen different theoretical frameworks are discussed and both coordinating/combining and integrating locally networking strategies are used to network at least two of the above thirteen theoretical frameworks. Major themes explored in these papers are briefly summarized below.

Networking theories at the theoretical level

Two papers aim at networking theories at the theoretical level. Based on a two-year study of a large-scale, LS-based intervention for improving post-16 learners' learning, Wake (this issue) explores how the two grand theories of Cultural Historical Activity Theory (Engeström, 2001) and Community of Practice (Wenger, 1998) could be coordinated to guide the design of a project intended to improve teaching and learning in mathematics. The theoretical analysis suggests how the careful design of LS can facilitate both individual and collective learning. The "Disturbances" in activity systems need to be purposefully designed to support collective learning, while deliberate architectures should be designed to support individual learning. Together, both individual and collective learning could be appropriately supported through the careful design of LS.

In Clivaz *et al.*'s (this issue) paper, the authors present a conceptual framework constructed by combining two intermediate theoretical perspectives of Mathematical Knowledge for Teaching Problem-solving (Chapman, 2015) and Lesson Study Dialogue Analysis (Hennessy, 2020). The paper also provides an illustrative example to describe the methodology of using the framework to analyze how dialogic interactions contribute to construction of teachers' mathematical problem-solving knowledge with an illustrative example.

Networking theories at the empirical level

Six papers demonstrate the application of networking theories at the empirical level. Two of them explore the effectiveness of theory-informed LS regarding teachers' professional learning. Adler *et al.* (this issue) report a case study about a theory-informed LS in Malawi. The Mathematics Teaching Framework (MTF), rooted in Vygotskian sociocultural perspective (Vygotsky, 1978), is coordinated with two intermediate theories: (1) Variation Theory (Marton, 2015) and its wide application in research on examples in the field; and (2) Socio-linguistic Theory (Planas, 2018) and selected tools to investigate explanatory communication. The MTF is utilized as the guiding principle for planning, teaching and reflecting on the LS. By focusing on explanatory communication in MTF, this study documented how teachers' lexicalization, a key component of language responsive teaching, evolved as a function of the MTF through the theory-informed LS.

Jessen *et al.* (this issue) present two cases on theory-informed LS from a 3-year European project, "Teacher inquiry in mathematics education," through university and school partnership in Denmark and the Netherlands. Two intermediate theoretical frameworks of Realistic Mathematics Education (Van den Heuvel-Panhuizen and Drijvers, 2020) and the Theory of Didactical Situations (Brousseau, 1997) *are combined* as the theoretical framework for guiding the project, and the LS process in particular. The case studies showed that to a certain extent, the participating teachers learned the vocabulary to describe situations and variables in their lesson designs and to focus on aspects that are crucial to students' learning according to the combined framework.

The other four papers focus on exploring teachers' professional learning through networking theories. Skott (this issue) explores what and how new mathematics teachers learn when participating in a LS-based induction program. One sociocultural theoretical framework, the Pattern of Participation (Skott, 2017), focuses on changes in how a teacher participates in social interaction over time. Another cognitive perspective, The Knowledge Quartet focuses on the forms of knowledge that are important for teaching a subject effectively (Turner and Rowland, 2011). These two intermediate frameworks are combined to examine teacher learning through LS. The study yields a more complete insight into what the teacher learned and provides complementary views on what and how the teacher learned.

In da Ponte *et al.*'s (this issue) study, the Interconnected Model of Teacher Professional Growth (IMTPG) (Clarke and Hollingsworth, 2002) was modified to examine teachers' collective learning with the coordination of the framework of Mathematics Knowledge for Teaching (Ball *et al.*, 2008). The coordination of the two intermediate frameworks helps reveal what teachers learned about specific knowledge needed for teaching (such as knowledge of content, of task design and of students) and how the collective learning takes place through the interactions among different domains.

Huang and Huang (this issue) explored teachers' collective learning through LS as document development based on networking two intermediate theoretical frameworks, IMTPG and Documentational Approach to Didactics (DAD) (Gueudet and Trouche, 2009). The DAD attributes teachers' learning as document system development through the interaction between teachers and resources. The DAD is locally integrated with IMTPG to investigate the dialectical process by examining the dynamic between external domains and other domains in the IMTPG model. A case study shows that the teachers' documents evolved from adopting the traditional teaching materials to adapting both traditional ones and e-resources with careful consideration of student learning through the LS process. Meanwhile, the DAD theory is enriched by illustrating the dialectical process of instrumentation and instrumentalization.

In Qi *et al.*'s (this issue) study, the intermediate IMTPG model is locally integrated with an enriched Community of Practice (CoP) framework (Wenger, 1998) to examine teachers' collective learning through LS. The study demonstrates that within a context of project-based learning LS, the teachers collectively developed their knowledge about the understanding of the authenticity of problems and the assessment of students' project activity. Moreover, this study reveals the teacher collective activity and how it triggers teachers' collective learning through the dual process of enactment and reflection.

Final remarks

This special issue demonstrates the richness and fruitfulness of using networking theories in LS. First, at least two of the 13 explored theories, which include two grand theoretical perspectives and 11 intermediate theoretical perspectives, could be networked through coordinating/combining (six papers) and locally integrating (two papers) to serve as a networked theoretical perspective for LS. Second, networking theories could not only strengthen the design of LS, which aims to promote teachers' professional learning, but also advance the research of LS through which more comprehensive frames are used to deepen understanding of teachers' learning in LS at both theoretical and empirical levels. It is networking theories that helps to reveal what teachers learned (MKT, KQ . . .) and how they learned (IMTPG, CHAT, CoP . . .) simultaneously in great detail. Third, some of the theories are enriched through the exploration of networking theories at the empirical level. For example, the IMTPG is enriched in Group Domain by combining MKT (da Ponte *et al.*) and in Domain of Practice by locally integrating CoP (Qi *et al.*). The papers in this special issue are all related to mathematics teaching and learning. Nevertheless, since many of these theoretical perspectives and models are not specific to mathematics, we expect that networking theories could be adopted to research of LS in other subjects. It is our hope that this special issue serves as a starting point for researchers in the LS field to explore how networking theories contributes to the development of theory and practice of LS internationally.

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