A modified Li-He's variational principle for plasma

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Abstract

Purpose – It is extremely difficult to establish a variational principle for plasma. Kalaawy obtained a variational principle by using the semi-inverse method in 2016, and Li and He suggested a modification in 2017. This paper aims to search for a generalized variational formulation with a free parameter.

Design/methodology/approach – The semi-inverse method is used by suitable construction of a trial functional with some free parameters.

Findings – A modification of Li-He's variational principle with a free parameter is obtained.

Originality/value – This paper suggests a new approach to construction of a trial-functional with some free parameters.

Keywords Variational theory, Semi-inverse method, Burger equation, Plasma, Editorial

Paper type Research paper

1. Introduction

In 2016, Kalaawy obtained the following equation for plasma (El-Kalaawy, 2016):

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left[\frac{2}{3}Au^{3/2} - C\frac{\partial u}{\partial x}\right] = 0 \tag{1}$$

and derived a variational principle by using the semi-inverse method (He, 2004; He, 2017; Wu and He, 2018) through introducing two special functions.

In 2017, Li and He found another variational principle, which reads (Li and He, 2017):

$$J_{Li-He}(u, \Phi) = \iint\left\{\frac{1}{2}\frac{\partial \Phi}{\partial x}\frac{\partial \Phi}{\partial t} + \frac{2}{3}Au^{3/2}\frac{\partial \Phi}{\partial x} - \frac{1}{2}C\left(\frac{\partial \Phi}{\partial x}\right)^2 - \frac{2}{5}Au^{5/2}\right\}dxdt \tag{2}$$

where Φ satisfies the following relations:

$$\frac{\partial \Phi}{\partial x} = u \tag{3}$$

$$\frac{\partial \Phi}{\partial t} = -\frac{2}{3}Au^{3/2} + C\frac{\partial u}{\partial x} \tag{4}$$

2. General variational principle with a free parameter

The semi-inverse method is widely used to search for variational principles directly from governing equations (El-Kalaawy, 2016; El-Kalaawy, 2017; Biswas *et al.*, 2017). By the

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semi-inverse method (He, 2004; He, 2017; Wu and He, 2018), we can construct a trial-functional in the form:

$$J(u,\Phi)=\iint\left\{u\frac{\partial\Phi}{\partial t}+\left[\frac{2}{3}Au^{3/2}-C\frac{\partial u}{\partial x}\right]\frac{\partial\Phi}{\partial x}+F\right\}dxdt \tag{5}$$

where F is a known function of u and its derivative, however we cannot identify F, so equation (5) has to be modified as:

$$J_{New}(u,\Phi)=\iint\left\{mu\frac{\partial\Phi}{\partial t}+n\frac{\partial\Phi}{\partial x}\frac{\partial\Phi}{\partial t}+aAu^{1/2}\left(\frac{\partial\Phi}{\partial x}\right)^2+bAu^{3/2}\frac{\partial\Phi}{\partial x}-C\frac{\partial u}{\partial x}\frac{\partial\Phi}{\partial x}+F\right\}dxdt \tag{6}$$

where m, n, a and b are constants.

The stationary conditions of equation (6) are as follows:

$$-m\frac{\partial u}{\partial t}-2n\frac{\partial^2\Phi}{\partial x\partial t}-\frac{\partial}{\partial x}\left[2aAu^{1/2}\frac{\partial\Phi}{\partial x}+bAu^{3/2}-C\frac{\partial u}{\partial x}\right]=0 \tag{7}$$

and

$$m\frac{\partial\Phi}{\partial t}+\frac{1}{2}aAu^{-1/2}\left(\frac{\partial\Phi}{\partial x}\right)^2+\frac{3}{2}bAu^{1/2}\frac{\partial\Phi}{\partial x}+C\frac{\partial}{\partial x}\left(\frac{\partial\Phi}{\partial x}\right)+\frac{\delta F}{\delta u}=0 \tag{8}$$

In view of equations (3) and (4), we have:

$$-(m+2n)\frac{\partial u}{\partial t}-\frac{\partial}{\partial x}\left[(2a+b)Au^{3/2}-C\frac{\partial u}{\partial x}\right]=0 \tag{9}$$

$$mC\frac{\partial u}{\partial x}+\left(\frac{1}{2}a+\frac{3}{2}b-\frac{2}{3}m\right)Au^{3/2}+C\frac{\partial u}{\partial x}+\frac{\delta F}{\delta u}=0 \tag{10}$$

Equation (9) should be equivalent to equation (1), this requires:

$$2a+b=1 \tag{11}$$

$$m+2n=1 \tag{12}$$

To identify F in equation (10), we set:

$$m=-1 \tag{13}$$

Equation (10) becomes:

$$\left(\frac{1}{2}a+\frac{3}{2}b+\frac{2}{3}\right)Au^{3/2}+\frac{\delta F}{\delta u}=0 \tag{14}$$

From equation (14), F can be identified as:

$$F = -\frac{2}{5}\left(\frac{1}{2}a + \frac{3}{2}b + \frac{2}{3}\right)Au^{5/2} = -\frac{2}{5}\left(\frac{13}{6} - \frac{5}{2}a\right)Au^{5/2} \tag{15}$$

We, therefore, obtain the following variational principle:

$$J_{New}(u,\Phi) = \iint\left\{\frac{\partial\Phi}{\partial x}\frac{\partial\Phi}{\partial t} - u\frac{\partial\Phi}{\partial t} + aAu^{1/2}\left(\frac{\partial\Phi}{\partial x}\right)^2 + (1-2a)Au^{3/2}\frac{\partial\Phi}{\partial x} - C\frac{\partial u}{\partial x}\frac{\partial\Phi}{\partial x} - \frac{2}{5}\left(\frac{13}{6} - \frac{5}{2}a\right)Au^{5/2}\right\}dxdt$$

where a is a free parameter.

3. Conclusion

In this paper, we suggest a general construction of a trial functional with some parameters. The semi-inverse method is a powerful tool to establish variational principles from the governing equations. The variational principle is a foundation of the variational iteration method (Anjum and He, 2019; He, 2006), which is now widely applied in fractional calculus (Baleanu *et al.*, 2018; Dogan Durgun and Konuralp, 2018; Inc *et al.*, 2018; Jafari *et al.*, 2018; Wang *et al.*, 2018).

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About the author

Ji-Huan He is an expert on Nonlinear Science and Nanotechnology. He is the owner of some famous analytical methods, such as the semi-inverse method, the variational iteration method, the homotopy perturbation method, the exp-function method and He's frequency formulation. He has published more than 390 articles with an *h*-index of 66. Ji-Huan He can be contacted at: hejihuan@suda.edu.cn