Editorial

1. Explanation of terms of Grey models for decision-making

1.1 Grey decision making

The decision making is that the action which should be taken is determined by actual situations and intended target. The nature of the decision means "decision-make" or "countermeasure-make". There are the broad sense and narrow sense to understand the decision. In a broad sense, the decision is a whole process of a series of activities such as identifying problems, collect data, determine the target, scheme, the evaluation and implementation, feedback and correction, etc. In a narrow sense, the decision making refers to the planning option of the decision-making process, customarily referred to as the "clapper". Others simply understand the decision as the planning option under the uncertainty condition, namely, to make a choice. It largely depends on the personal experience, attitude and determination of decision makers, as to what risks they may take. The grey decision is made in the situation that the decision model has grey elements or the normal decision model and grey model is combined, and the key research is the scheme selection problem (Deng, 1985; Liu and Lin, 2006).

1.2 Grey target for decision

According to the preset decision goal, the satisfaction value of decision-making effects is called the grey target for decision. The grey target for decision is essentially the area of satisfaction effect under the relative optimisation sense. In many cases, it is impossible to achieve an absolute optimal situation, so people often settle for the second best and generally can accept basic satisfactory results. In the situation of single objective decisions, the grey target for decision is an interval on the axis. In the multi-objective decision-making situation, the grey target for decision is a flat area or higher dimensional hyperplane area (Deng, 1990; Liu *et al.*, 2015).

1.3 One-dimension grey target for decision

Set $d_1^{(k)}$and $d_2^{(k)}$ as the upper and lower critical values of decision effect, respectively, under the target k, namely, that wish the decision-making effect value under target k should be between $d_1^{(k)}$and $d_2^{(k)}$. And $S^1 = \{r | d_1^{(k)} \leqslant r \leqslant d_2^{(k)}\}$ is the one-dimension grey target for decision. For example, the countries formulate the development plans and hope stable economic growth at around 8 per cent. If we regard the 8 per cent as the interval between 7.6 and 8.5 per cent, so $S^1 =$[7.6 per cent, 8.5 per cent] is a one-dimension grey target for decision. Once it appears that the economic growth rate is lower than 7.6 per cent or higher than 8.5 per cent, we can think of the economy as heating up or cooling down, so the countries need to take corresponding control measures (Deng, 1985; Liu and Lin, 2011).

1.4 Two dimensions grey target for decision

Set $d_1^{(1)}$ and $d_2^{(1)}$ as the situation effect critical values of target 1, $d_1^{(2)}$and $d_2^{(2)}$, respectively, as the situation effect critical value of target 2, so:

$$S^2 = \left\{ \left(r^{(1)}, r^{(2)}\right) \middle| d_1^{(1)} \leqslant r^{(1)} \leqslant d_2^{(1)}, d_1^{(2)} \leqslant r^{(2)} \leqslant d_2^{(2)} \right\},$$

which is the two-dimension grey target for decision. In fact, a two-dimension grey target for decision is a matrix on the plane. For example, some people think that the standard height of

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a young man is 178–186 cm, and the weight is 65–75 kg. So $S^2 = \{(r^{(1)}, r^{(2)}) | 178 \leqslant r^{(1)} \leqslant 186,\ 65 \leqslant r^{(2)} \leqslant 75\}$ is the corresponding two-dimension grey target for decision (Deng, 1985; Liu, Yingjie, Jeffrey, 2016; Liu, Yingjie, Naiming, Jeffrey, 2016; Liu, Tao, Xie *et al.*, 2016; Liu *et al.*, 2017).

1.5 s-dimensions decision grey target

Set $d_1^{(1)}, d_2^{(1)}$ as $d_1^{(2)}, d_2^{(2)}, \ldots, d_1^{(s)}, d_2^{(s)}$, respectively, as the situation effect threshold for the target 1, 2, …, s, called the s-dimensional hyper-plane area.

$S^s = \{(r^{(1)}, r^{(2)}, \ldots, r^{(s)}) | d_1^{(1)} \leqslant r^{(1)} \leqslant d_2^{(1)}, d_1^{(2)} \leqslant r^{(2)} \leqslant d_2^{(2)}, \ldots, d_1^{(s)} \leqslant r^{(s)} \leqslant d_2^{(s)}\}$ is the s-dimensional decision grey target (Deng, 1985; Liu, Yingjie, Jeffrey, 2016; Liu, Yingjie, Naiming, Jeffrey, 2016; Liu, Tao, Xie *et al.*, 2016; Liu *et al.*, 2017).

1.6 s-dimensions spherical grey target

Set $r_0 = (r_0^{(1)}, r_0^{(2)}, \ldots, r_0^{(s)})$ as optimal effect vector, say $R^s = \{(r^{(1)}, r^{(2)}, \ldots, r^{(s)}) | (r^{(1)} - r_0^{(1)})^2 + (r^{(2)} - r_0^{(2)})^2 + \ldots + (r^{(s)} - r_0^{(s)})^2 \leqslant R^2\}$ is the s-dimensions spherical grey target which use $r_0 = (r_0^{(1)}, r_0^{(2)}, \ldots, r_0^{(s)})$ as the bull's-eye and R as radius (Deng, 1985; Liu, Yingjie, Jeffrey, 2016; Liu, Yingjie, Naiming, Jeffrey, 2016; Liu, Tao, Xie *et al.*, 2016, Liu *et al.*, 2017).

1.7 Bull's-eye distance

Set $r_1 = (r_1^{(1)}, r_1^{(2)}, \ldots, r_1^{(s)}) \in R$, and say $|r_1 - r_0| = [(r_1^{(1)} - r_0^{(1)})^2 + (r_1^{(2)} - r_0^{(2)})^2 + \ldots + (r_1^{(s)} - r_0^{(s)})^2]^{1/2}$ is the bull's-eye distance of vector r_1. Bull's-eye distance value reflects the merits of the decision-making vector.

Unlike the rectangular grey target, the spherical decision-making grey target can determine the merits of the decision-making vector by the comparison of the bull's-eye distance number (Deng, 1985; Liu, Yingjie, Jeffrey, 2016; Liu, Yingjie, Naiming, Jeffrey, 2016; Liu, Tao, Xie *et al.*, 2016, Liu *et al.*, 2017).

1.8 The effect measure of benefit-type objective

Set k a efficiency target, that is the bigger the sample values of decision targeted effects, the better; set grey target for decision under target k as $u_{ij}^{(k)} \in [u_{i_0 j_0}^{(k)}, \max_i \max_j \{u_{ij}^{(k)}\}]$, so $u_{i_0 j_0}^{(k)}$ is the target threshold effect of k, then:

$$r_{ij}^{(k)} = \frac{u_{ij}^{(k)} - u_{i_0 j_0}^{(k)}}{\max_i \max_j \left\{u_{ij}^{(k)}\right\} - u_{i_0 j_0}^{(k)}},$$

which is called the effect measure of benefit-type objective. The effect measure of benefit-type objective reflects the proximity of the effect sample values and maximum effect sample values and the extent away from the target threshold effect (Liu *et al.*, 2013, 2017, Liu, Yingjie, Jeffrey, 2016; Liu, Yingjie, Naiming, Jeffrey, 2016; Liu, Tao, Xie *et al.*, 2016).

1.9 The effect measure of cost-type objective

Set k a the cost-type target, that is the smaller the sample values of decision targeted effects, the better; set grey target for decision under target k as $u_{ij}^{(k)} \in [\min_i \min_j \{u_{ij}^{(k)}\},\ u_{i_0 j_0}^{(k)}]$, so $u_{i_0 j_0}^{(k)}$ is the target threshold effect of k, then:

$$r_{ij}^{(k)} = \frac{u_{i_0 j_0}^{(k)} - u_{ij}^{(k)}}{u_{i_0 j_0}^{(k)} - \min_i \min_j \left\{u_{ij}^{(k)}\right\}},$$

which is called the effect measure of cost-type objectives. The effect measure of cost-type objectives reflects the proximity of the effect sample values and minimum effect sample values and the extent away from the target threshold effect. (Liu *et al.*, 2013, 2017; Liu, Yingjie, Jeffrey, 2016; Liu, Yingjie, Naiming, Jeffrey, 2016; Liu, Tao, Xie *et al.*, 2016).

1.10 The lower effect measure of moderate value type objective

Set k a moderate type target, that is the closer the sample values towards a moderate value A, the better; set grey target for decision under target k as $u_{ij}^{(k)} \in [A-u_{i_0j_0}^{(k)}, A+u_{i_0j_0}^{(k)}]$, so $A-u_{i_0j_0}^{(k)}, A+u_{i_0j_0}^{(k)}$, respectively, are the lower effect threshold and the upper effect threshold under target k, when $u_{ij}^{(k)} \in [A-u_{i_0j_0}^{(k)}, A]$, say:

$$r_{ij}^{(k)} = \frac{u_{ij}^{(k)} - A + u_{i_0j_0}^{(k)}}{u_{i_0j_0}^{(k)}},$$

which is the lower effect measure of moderate value type objective.

The lower effect measure of moderate value type objective reflect the proximity of the effect sample values that are less than the moderate value A and the moderate value and the extent away from the lower threshold effect (Liu *et al.*, 2013, 2017; Liu, Yingjie, Jeffrey, 2016; Liu, Yingjie, Naiming, Jeffrey, 2016; Liu, Tao, Xie *et al.*, 2016).

1.11 The upper effect measure of moderate value type objective

Set k a moderate type target, that is the closer the sample values towards a moderate value A, the better; set the grey target for decision under target k as $u_{ij}^{(k)} \in [A-u_{i_0j_0}^{(k)}, A+u_{i_0j_0}^{(k)}]$, so $A-u_{i_0j_0}^{(k)}$ and $A+u_{i_0j_0}^{(k)}$, respectively, are the lower effect threshold and the upper effect threshold under target k, when $u_{ij}^{(k)} \in [A, A+u_{i_0j_0}^{(k)}]$, say:

$$r_{ij}^{(k)} = \frac{A + u_{i_0j_0}^{(k)} - u_{ij}^{(k)}}{u_{i_0j_0}^{(k)}},$$

which is the upper effect measure of moderate value type objective.

The upper effect measure of the moderate value type objective reflects the proximity of the effect sample values more than the moderate value A and the moderate value and the extent away from the upper threshold effect (Liu *et al.*, 2013, 2017; Liu, Yingjie, Jeffrey, 2016; Liu, Yingjie, Naiming, Jeffrey, 2016; Liu, Tao, Xie *et al.*, 2016).

1.12 Out of target

There are the following four kinds of situations:

(1) the effect value of benefit-type objective less than the threshold $u_{i_0j_0}^{(k)}$, that is $u_{ij}^{(k)} < u_{i_0j_0}^{(k)}$;

(2) the effect value of cost-type objective more than the threshold $u_{i_0j_0}^{(k)}$, that is $u_{ij}^{(k)} > u_{i_0j_0}^{(k)}$;

(3) the effect value of moderate type objective less than the lower effect threshold $A-u_{i_0j_0}^{(k)}$, that is $u_{ij}^{(k)} < A-u_{i_0j_0}^{(k)}$; and

(4) the effect value of moderate type objective more than the upper effect threshold $A+u_{i_0j_0}^{(k)}$, that is $u_{ij}^{(k)} > A+u_{i_0j_0}^{(k)}$.

These are called off-target (Liu *et al.*, 2013, 2017; Liu, Yingjie, Jeffrey, 2016; Liu, Yingjie, Naiming, Jeffrey, 2016; Liu, Tao, Xie *et al.*, 2016).

1.13 Normality assumptions of effect measures

It is done in order to make all kinds of effect measures of objectives to meet regulatory, which is:

$$r_{ij}^{(k)} \in [-1, 1].$$

For a benefit-type objective, might as well set $u_{ij}^{(k)} \geqslant -\max_i \max_j \{u_{ij}^{(k)}\} + 2u_{i_0j_0}^{(k)}$; for a cost-type objective, might as well set $u_{ij}^{(k)} \leqslant -\min_i \min_j \{u_{ij}^{(k)}\} + 2u_{i_0j_0}^{(k)}$; and for situations in which the effect value of moderate type objective is less than the lower effect threshold $A - u_{i_0j_0}^{(k)}$, might as well set $u_{ij}^{(k)} \geqslant A - 2u_{i_0j_0}^{(k)}$.

For the situation in which the effect value of moderate type objective is more than the upper effect threshold $A + u_{i_0j_0}^{(k)}$, might as well set $u_{ij}^{(k)} \leqslant A + 2u_{i_0j_0}^{(k)}$ (Liu *et al.*, 2013, 2017; Liu, Yingjie, Jeffrey, 2016; Liu, Yingjie, Naiming, Jeffrey, 2016; Liu, Tao, Xie *et al.*, 2016).

1.14 Uniform effect measure

Uniform effect measures the effect measure of decision objective to meet the following conditions:

$$r_{ij}^{(k)}(i = 1, 2, \ldots, n;\ j = 1, 2, \ldots,\ m;\ k = 1, 2, \ldots, s),$$

where there are the following uniform effect measures: $r_{ij}^{(k)}$ is non-dimensional; the effect is more ideal, $r_{ij}^{(k)}$ is larger; and $r_{ij}^{(k)} \in [-1, 1]$. In the case of k target, $r_{ij}^{(k)} \in [0, 1]$; In the case of k off-target, $r_{ij}^{(k)} \in [-1, 0]$.

Accordingly, it is not difficult to give the definition consistent effect measure vectors and a uniform effect measure matrix (Liu *et al.*, 2013, 2017; Liu, Yingjie, Jeffrey, 2016; Liu, Yingjie, Naiming, Jeffrey, 2016; Liu, Tao, Xie *et al.*, 2016).

1.15 Synthetic effect measure

Set $\eta_k(k = 1, 2, \ldots, s)$ as the decision-making power of the target k, and $\sum_{k=1}^{s} \eta_k = 1$ and $r_{ij}^{(k)}(i = 1, 2, \ldots, n; j = 1, 2, \ldots, m; k = 1, 2, \ldots, s)$ are the uniform effect measures, then say $\sum_{k=1}^{s} \eta_k \cdot r_{ij}^{(k)}$ is the synthetic effect measure of the decision target, still referred to as $r_{ij} = \sum_{k=1}^{s} \eta_k \cdot r_{ij}^{(k)}$.

Synthetic effect measure $r_{ij}^{(k)} \in [0, 1]$ belongs to the target situation. Synthetic effect measure $r_{ij}^{(k)} \in [-1, 0]$ belongs to missed situations. In the case of target, we can also compare the level of the synthetic effect measure $r_{ij}(i = 1, 2, \ldots, n; j = 1, 2, \ldots, m)$ to judge pros and cons of the cases a_i $(i = 1, 2, \ldots, n)$, counter-measures b_j $(j = 1, 2, \ldots, m)$ and decision $s_{ij} = (a_i, b_j)$ $(i = 1, 2, \ldots, n, j = 1, 2, \ldots, m)$ (Liu *et al.*, 2013, 2017; Liu, Yingjie, Jeffrey, 2016; Liu, Yingjie, Naiming, Jeffrey, 2016; Liu, Tao, Xie *et al.*, 2016).

1.16 Normalised clustering coefficient vector

Assuming that $\sigma_i = (\sigma_i^1, \sigma_i^2, \ldots, \sigma_i^s), i = 1, 2, \ldots, n$ are n clustering coefficient vectors, let $\delta_i^k = \sigma_i^k / \sum_{k=1}^{s} \sigma_i^k$, δ_i^k is called the normalised clustering coefficient vector of decision-making object i belonging to class k (Liu *et al.*, 2014, 2017, 2018).

1.17 The maximum component of clustering coefficient vector

Assume that $\max_{1 \leqslant k \leqslant s} \{\delta_i^k\} = \delta_i^{k^*}$, then $\delta_i^{k^*}$ is called the maximum component of clustering coefficient vector δ_i (Liu *et al.*, 2014, 2017, 2018).

1.18 The weight vector group of kernel clustering

Assume that there are s classes of decision-making, and real numbers $w_k \geqslant 0$, $k = 1, 2, \ldots, s$, then let:

$$\eta_1 = \frac{1}{\sum\limits_{k=1}^{s} w_k}(w_s, w_{s-1}, w_{s-2}, \ldots, w_1),$$

$$\eta_2 = \frac{1}{w_{s-1} + \sum\limits_{k=2}^{s} w_k}(w_{s-1}, w_s, w_{s-1}, w_{s-2}, \ldots, w_2),$$

$$\eta_3 = \frac{1}{w_{s-1} + w_{s-2} + \sum\limits_{k=3}^{s} w_k}(w_{s-2}, w_{s-1}, w_s, w_{s-1}, \ldots, w_3),$$

$$\eta_k = \frac{1}{\sum\limits_{i=s-k+1}^{s-1} w_i + \sum\limits_{i=k}^{s} w_i}(w_{s-k+1}, w_{s-k+2}, \ldots, w_{s-1}, w_s, w_{s-1}, \ldots, w_k),$$

$$\eta_{s-1} = \frac{1}{w_{s-1} + \sum\limits_{k=2}^{s} w_k}(w_2, w_3, \ldots, w_{s-1}, w_s, w_{s-1}),$$

$$\eta_s = \frac{1}{\sum\limits_{k=1}^{s} w_k}(w_1, w_2, w_3, \ldots, w_{s-1}, w_s),$$

where $\eta_k(k = 1, 2, \ldots, s)$ is called a weight vector group of kernel clustering about class k.

The s-dimensional vector $\eta_k = (\eta_k^1, \eta_k^2, \ldots, \eta_k^s)(k = 1, 2, \ldots, s)$ is the multiplication of scalar $a_k = 1/(\sum_{i=s-k+1}^{s-1} w_i + \sum_{i=k}^{s} w_i)$ with vector ζ_k, where the function of the scalar factor a_k is to ensure $\eta_k(k = 1, 2, \ldots, s)$ is a unit vector. Also, the kth component of vector factor $\zeta_k(k = 1, 2, \ldots, s)$ is w_s, which is the maximum component of ζ_k. Then the kth component w_s can be taken as a centre, and the other components on both sides of the kth component w_s descend step by step. The kth component where the largest contribution for the decision-making object belongs to grey class k, so the kth component of ζ_k should take the maximum weight w_s. The values of other components are set by the principle which states that "the component which is closest to the kth component has the largest contribution for object I belonging to class k, so it is given the largest weight; the component which is farthest from the kth component has the smallest contribution for object I belonging to class k, so it is given the smallest weight" (Liu *et al.*, 2014, 2017, 2018).

1.19 The weighted coefficient vector of kernel clustering for decision making

Assume there are n decision objects and s different grey classes, then $\omega_i^k = \eta_k \cdot \delta_i^T$ is called the weighted coefficient of kernel clustering for the decision making of object i about grey class k. And:

$$\omega_i = \left(\omega_i^1, \omega_i^2, \ldots, \omega_i^s\right); i = 1, 2, \ldots, n,$$

which is called the weighted coefficient vector of kernel clustering for the decision making of object i (Liu *et al.*, 2014, 2017, 2018).

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