Using a consumption function to explain the Lucas Critique to undergraduate students

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Abstract

Purpose – The purpose of this paper is to contribute to the teaching of undergraduate macroeconomics.
Design/methodology/approach – To suggest a roadmap, based on a consumption function, to be used by instructors willing to teach the Lucas Critique subject.
Findings – Therefore, this paper proposes a lesson, which consists of three parts, to help undergraduates better understand the subject: (1) a grading exercise to bring the topic closer to students' lives; (2) a Keynesian and an optimal consumption function, followed by an example based on an unemployment insurance policy; and (3) two optional topics consisting of extensions of the optimal consumption function and some empirical results related to the Lucas Critique.
Originality/value – The Lucas Critique influenced the evolution of research in macroeconomics, but it is not easily grasped in a classroom.

Keywords Undergraduate teaching, Lucas critique, Consumption function, Policy evaluation

Paper type Research paper

1. Introduction

In the 1960s, macroeconomists used econometric models consisting of several equations divided into four categories: (1) identities, such as those derived from national accounts; (2) institutional rules, such as tax rates; (3) technological constraints, such as production functions; and (4) behavioral equations, such as consumption functions relating disposable income and consumption. After estimating the systems of equations, researchers used to examine the quality of all specifications applied by their ability to mimic historical data. This was also the criterion used for including equations and variables to find the best, which was then used for forecasting and assessing the impact of alternative economic policies (Kydland & Prescott, 1991).

Lucas (1976) criticized using the systems of equations to evaluate alternative economic policies. This attitude led to the so-called Lucas Critique, whose foundation was based on the assumption that agents alter their behavior whenever economic policies change. In this vein, Lucas (1976) provided an explanation for the case in which coefficients in a system of equations were nonconstant when policy rules changed (Ericsson & Irons, 1995). Ultimately,

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if an econometric model is not invariant to alternative policies, simulations using such a model can be misleading about the consequences of each policy (Lucas, 1976). In Lucas' words:

> given that the structure of an econometric model consists of optimal decision rules of economic agents, and that optimal decision rules vary systematically with changes in the structure of series relevant to the decision maker, it follows that any change in policy will systematically alter the structure of econometric models (Lucas, 1976, p. 41).

It is important to emphasize, as, in Lucas (1976), that forecasting is an investigation based on deviations between the estimated value and the real value of the variable of interest. This is different from evaluating alternative policies, which depends on the difference between the "true" structure, when there is no other policy, and the new "true" structure of a particular policy. Therefore, policy evaluation is not a forecasting exercise, but an analysis of the expected effects of each alternative policy considered. This assessment, based on an econometric model, makes sense only if the model's structure is invariant to alternative policies analyzed.

The Lucas Critique is a conjecture, which can be either rejected or not. But one must not forget that agents have the potential to respond to new policies in different ways and make proper changes. Therefore, even when it is not easy to empirically confirm it, the Lucas Critique is still theoretically relevant, and it needs to be understood by undergraduate students in general, as well as economists and economic agents. However, teaching a lesson on the Lucas Critique subject might not be an easy task, as it is not easily grasped by the classroom.

Therefore, the aim of this article is to propose a Lucas Critique lesson roadmap to be used by instructors in an advanced undergraduate macroeconomics class. The lesson consists of three parts. The first one is a grading exercise to show students that the subject is closer to their real-life than they think. In the second part of the lesson, we explain the Lucas Critique by means of a Keynesian Consumption Function and an Optimal Consumption Function based on a version of the Permanent Income Hypothesis (PIH). We derive the marginal propensity to consume in Hall's (1978) version of the PIH, which depends on the probability of unemployment. We then illustrate the Lucas Critique by analyzing how consumers respond to an unemployment insurance policy. Finally, in the third part of the lesson, we present two optional topics, which are an extension of the optimal consumption function and some empirical results related to the Lucas Critique.

Besides this introduction, the article is organized as follows. Section 2 is dedicated to showing some literature on teaching macroeconomics to undergraduates. Section 3 presents a Lucas Critique lesson plan based on the proposed material. Section 4 brings the topic to students' lives by means of an instructor's grading system. Section 5 reports the Keynesian Consumption Function, a behavioral equation subject to the Lucas Critique. Section 6 reviews the discussion between the Lucas Critique and the system of equations adopted by macroeconomists in the 1960s. Section 7 brings the discussion related to Optimal Consumption and Hall's version of the PIH, in which we add both unemployment risk and insurance policy. Section 8 brings some empirical results related to the Lucas Critique, which can be used as an additional presentation of the topic. The last section draws some conclusions.

2. Some literature background on teaching macroeconomics

Several articles have been written on teaching macroeconomics at an undergraduate level. For instance, Becker (2000) argued that improving the teaching quality in economics is an important tool to help reverse a downward trend in the number of undergraduates majoring in economics in the USA. The author offered some suggestions on how to change the concepts taught and addressed how students could be assessed and how pedagogical practices could be evaluated.

Taylor (2000) argued that teaching modern macroeconomics at the principles level is possible and appropriate but also recognized that there are "*many alternative ways to teach macroeconomics and that what works well for one teacher and his or her students may not be attractive to others.*" Brevik and Gärtner (2007) focused on how to use the graphical approach to teach real business cycles to undergraduates. The authors saw the graphical analysis as a meaningful way to provide students with a first grasp of the supply side of the economy.

Bofinger, Mayer and Wollmershäuser (2009) concentrated on how to teach New Keynesian open economy macroeconomics at the intermediate level. Their model was able to provide "*an efficient tool kit for the discussion of the costs and benefits of fixed and flexible exchange rates, which also was at the core of the Mundell-Fleming model.*" Shiller (2010) found some student dissatisfaction with the lectures in macroeconomics taught at universities, especially in explaining the 2008–2009 financial crisis. According to the author, this frustration was not intense, as most undergraduates acknowledged their teachers' effort to deal with conceptual complexities in economics, but professors ought to always clarify the reasons for the theoretical differences of other times.

De Araujo, O'Sullivan and Simpson (2013) discussed what should be taught in intermediate macroeconomics, especially in relation to the Keynesian/classical divide. The authors considered the pros and cons of each approach and how they were dealt with in textbooks. Gärtner, Griesbach and Jung (2013) worked on a survey among undergraduate instructors in Europe and the USA, on teaching macroeconomics after the global financial crisis. The results showed that macro models were taught in the same way as before the crisis, except for public debt dynamics. However, interest grew in topics related to financial markets, case studies, economic history and history of economic thought.

Reingewertz (2013) showed that flowcharts could be a valuable tool for teaching macroeconomics to undergraduates, as they could help present graphs and algebra in the explanation process. Valcarcel (2013) offered a single monetary economy experiment to help boost students' participation in a semester-long macroeconomics course. By adopting this experimental approach, the author was able to work with several concepts such as income, redistribution, intertemporal substitution and banking.

Costa and Garcia-Cintado (2018) put forward a systematic approach to teaching dynamic stochastic general equilibrium (DSGE) models to undergraduates by presenting the structural model, the steady-state and log-linearized equations, followed by the simulation of a productivity shock. The authors extended their model with the inclusion of the government and foreign sectors. Solis-Garcia (2018) also wrote about teaching DSGE models to undergraduate students. The author discussed how to fit the topic into a one-semester time frame and talked about his own experience in teaching the subject.

Mankiw (2019) listed six useful guidelines to help instructors, especially first-timers, teach intermediate macroeconomics: (1) suppress idiosyncrasies; (2) reinforce basic principles; (3) revel in the act of model building; (4) admit our models' limitations; (5) do not go overboard; and (6) remember the real world. Croushore (2019) discussed what should be taught in intermediate macroeconomics and said that the focus was the construction and understanding of macroeconomic models. The author also mentioned that understating the concept of general equilibrium is essential at an intermediate level.

Wulwick (1991) argued that teaching the Phillips Curve as a mapping of a stable AS curve, as many principles of macroeconomics books do, could be problematic. The author showed that the stable Phillips Curve could be mapped off a stable AS curve that was double exponential in form but not a linear or exponential AS curve.

Yamaura and Thompson (2018) used the map of Japan to teach students the AD-AS model and Phillips Curve theories. The authors were able to pinpoint some Japanese regional effects which were used as a teaching toolkit to help undergraduates understand the Japanese macroeconomic experience for the period ranging from 2005 to 2017.

Finally, Barros, Gomes and Calcini (2022) put forward a strategy to teach two-period consumption models to undergraduate students, taking into account income risk (precautionary motive). Therefore, despite not discussing the Lucas Critique, these authors worked on the topic of optimal consumption, as we are doing in this article.

3. A roadmap to teach the Lucas Critique

This section presents a roadmap in Box 1 to be used by instructors who want to adopt the proposed material to teach the Lucas Critique to undergraduates. In the first part of the lesson, the teacher could start by applying the subject to students' lives. This can be done by showing them a simple example in which different approaches can be used to calculate a student's final grade. Section 4 brings this example. Once students realize that an optimal behavior may depend on the approach adopted, the instructor could then discuss the Keynesian consumption function (Section 5), by asking if such function reacts to different economic policies. After that, the instructor could explain how the Lucas Critique affected the system of equations commonly used by macroeconomists in the 1960s (Section 6).

In the second part of the lesson, the instructor would use the contents in Sections 7.1 and 7.2 to derive an optimal consumption function, in which the marginal propensity to consume reacts to new policies (unemployment insurance). Thus, students could see a simple example in which a new policy affects the consumer's behavior. More importantly, they could realize how difficult it is to assess different economic policies.

The third part of the lesson would be optional, as it is about some refinements of the optimal consumption function (Section 7.3) and some Lucas Critique's empirical assessments (Section 8). Indeed, it is important for students to realize that the own Lucas Critique should be investigated.

Box 1 also presents the prerequisites needed for each topic, assuming that undergraduates will not derive the optimal consumption function (Equation 9). Conditional on such assumption, the prerequisites for the first and the second classes are covered by initial courses in statistics, econometrics and macroeconomics. Regarding the (optional) third part of the lesson, notions of precautionary savings and liquidity constraints are required, but any gap could be covered previously by the instructor. Therefore, the roadmap presented in Box 1 enables the contextualized teaching of Lucas Critique for undergraduate students, which is a helpful toolkit to incorporate the topic in the classroom.

Box 1. A roadmap

Class	Topics	Prerequisites
First	Apply the topic to students' lives (Section 4) Present the Keynesian Consumption Function (Section 5) Present the Lucas Critique and the System of Equations (Section 6)	Geometric and arithmetic means Linear function Notions of econometrics, especially estimation
Second	Present the Consumption Function and Unemployment Insurance Policy (Sections 7.1 and 7.2)	Random variable, expected value, and the Law of Large Numbers
Third (optional)	Discuss some Extensions of the Optimal Consumption Function (Section 7.3) Lucas Critique: Some Empirical Results (Section 8)	Notions of precautionary savings and liquidity constraints

4. The Lucas Critique and students' lives: a simple example

As depicted in Box 1, the first part of the lesson is to apply the Lucas Critique to students' everyday lives. The instructor can raise awareness and ask the class to imagine the grades given in two exams. The final grade can be calculated by applying either the arithmetic mean (AM) or geometric mean (GM), as follows:

$$AM = 0.5N_1 + 0.5N_2 \tag{1}$$

$$GM = \sqrt{N_1 N_2} \tag{2}$$

where N_1 and N_2 are the grades from the first and second exams, respectively. A satisfactory passing grade would be 5 or higher, in both cases.

The next step is to ask students which alternative – arithmetic or the geometric mean – they prefer. Furthermore, ask them why. After that, Table 1 can be presented to the whole class. There are different scenarios (from A to K) for grades N_1 and N_2, but the arithmetic mean is always 5.00, which implies that a student passes the exams under such a metric. However, the geometric mean reaches the passing grade only in Scenario A.

Ask students again about their preferred alternative. Under arithmetic mean, a very high grade (either N_1 or N_2) implies that a student will pass the subject easily. But the same is not true under a geometric average. In such a case, even when $N_1 = 10$, the student does not receive a passing grade if he/she fails to meet the minimum performance levels in the second exam.

Given that, ask students if their behavior would change if the instructor decided to use the geometric mean, instead of the arithmetic mean. An affirmative answer illustrates that people react differently to each policy, which is the main idea behind the Lucas Critique. Essentially, Lucas (1976) argued that alternative policies would induce agents to adjust their behavior.

At this point, it is important to assess whether the class has really understood the concept behind the Lucas Critique. An excellent strategy is to ask students if historical data on N_1 and N_2 from a previous semester could be used to evaluate whether the adoption of the geometric mean affects the class passing/failure rates. They should answer that such a strategy is not valid if the students' behavior were affected by the new method used to calculate the final grade. For instance, if the adoption of a geometric mean leads students to study more, historical data on N_1 and N_2 underestimate the future grades under the geometric mean. In this vein, this statistical approach may fail. Indeed, according to Lucas (1976), because alternative policies induce agents to adjust their behavior, even stable econometric specifications based on historical data become a misleading guide to the real consequences of alternative economic policies.

5. Presenting the Keynesian Consumption Function

In order to start presenting the Keynesian Consumption Function, remind students that Keynes (1936) postulated the following consumption function:

Table 1. Arithmetic mean (AM) and geometric mean (GM) for different scenarios

	Scenarios										
	A	B	C	D	E	F	G	H	I	J	K
Grades N_1	5.00	6.00	4.00	7.00	3.00	8.00	2.00	9.00	1.00	10.00	0.00
Grades N_2	5.00	4.00	6.00	3.00	7.00	2.00	8.00	1.00	9.00	0.00	10.00
Arithmetic mean	*5.00*	*5.00*	*5.00*	*5.00*	*5.00*	*5.00*	*5.00*	*5.00*	*5.00*	*5.00*	*5.00*
Geometric mean	*5.00*	4.90	4.90	4.58	4.58	4.00	4.00	3.00	3.00	0.00	0.00

Note(s): Italic cells indicate that the passing grade was achieved

$$C = c_0 + c_1 Y \tag{3}$$

in which consumption, C, depends linearly on disposable income, Y. Constants $c_0 > 0$ and $c_1 \in (0, 1)$ are, respectively, autonomous consumption and marginal propensity to consume. This behavioral equation states that a fixed fraction of disposable income is consumed, regardless of the policies adopted by the government. Because such consumption function is taught in basic macroeconomic courses, the consumption decision is a useful topic to illustrate the Lucas Critique.

If the parameters c_0 and c_1 are constant over time, they could be estimated by simple econometric estimators using historical data. However, it does not mean that we could assess how different policies, aimed at affecting disposable income immediately, affect consumption.

Ask students to imagine that a policymaker has the consumption level as a target variable and he/she can affect its future value by means of M alternative policies, each one indexed by $m = 1, \ldots, M$. Imagine a certain future disposable income for each policy, $Y_{t+1}^{(m)}, m = 1, \ldots, M$.

Explain to the class that if we ignored the Lucas Critique, we would estimate the impact of each policy on consumption as follows:

$$C_{t+1}^{(m)} = \widehat{\beta}_0 + \widehat{\beta}_1 Y_{t+1}^{(m)} \tag{4}$$

where $\widehat{\beta}_0$ and $\widehat{\beta}_1$ are, respectively, estimates of c_0 and c_1 based on historical data.

However, different policies would potentially induce a behavior change, for instance, an adjustment in marginal propensity to consume. This parameter would then be specific for each policy, that is, $c_1^{(m)}, m = 1, \ldots, M$. Using estimates from historical data, each policy evaluation would be based on an estimate of c_1, rather than $c_1^{(m)}$. The greater the difference between these two quantities, the worse the evaluation of alternative policies. Therefore, if the Lucas Critique holds, fixed behavioral equations prevent alternative policies from being accurately assessed.

6. Presenting the Lucas Critique and system of equations

Explain to the class that to understand the impact of Lucas Critique on macroeconomics research, it is necessary to briefly describe how macroeconometric models were used in the 1960s. Until then, they contained several equations (identities, institutional rules, technological constraints and behavioral equations), as follows:

$$\begin{aligned}
y_{1,t} &= \beta_{1,0} + \sum_{j=1}^{J} \beta_{1,j} y_{j,t-1} + \sum_{k=1}^{K} \gamma_{1,k} x_{k,t-1} + \varepsilon_{1,t} \\
y_{2,t} &= \beta_{2,0} + \sum_{j=1}^{J} \beta_{2,j} y_{j,t-1} + \sum_{k=1}^{K} \gamma_{2,k} x_{k,t-1} + \varepsilon_{2,t} \\
&\vdots \\
y_{J,t} &= \beta_{L,0} + \sum_{j=1}^{J} \beta_{J,j} y_{j,t-1} + \sum_{k=1}^{K} \gamma_{J,k} x_{k,t-1} + \varepsilon_{L,t}
\end{aligned} \tag{5}$$

where $y_{1,t}, \ldots, y_{L,t}$ are L endogenous variables in period t; $x_{1,t}, \ldots, x_{K,t}$ are K exogenous variables; $\varepsilon_{1,t}, \ldots, \varepsilon_{L,t}$ are error terms; $\beta_{j,l}$ and $\gamma_{j,k}, j = 1, \ldots, J, l = 1, \ldots, L, k = 1, \ldots, K$, are unknown parameters.

This example is a simple specification in which each endogenous variable depends on the first lag of all endogenous and exogenous variables. Usually, a system of equations has a lot of lags in endogenous and exogenous variables, and the current value of exogenous variables is used as covariates. Therefore, macroeconomists used to estimate several specifications of such system. Appendix 1 presents a formal representation of the system of equations and how the Lucas Critique affects such approach.

Explain to the class that if a policymaker has a target variable $y_{1,t}$ and, as before, he/she can affect its future value by means of M alternative policies. These economic policies affect some of the exogenous variables $x_{1,t}, \ldots, x_{K,t}$, which affect the future trajectory of $y_{1,t}$. Such effect depends on the system of equations' coefficients.

According to Lucas (1976), the structure of an econometric model is not invariant to alternative policies, because the optimal agents' choice is not invariant to alternative policies. Faced with changes in the economic environment, these agents would make different decisions rather than following behavioral rules, such as a Keynesian consumption function, in which the marginal propensity to consume is constant. In this vein, the Lucas Critique means that some of the parameters $\beta_{j,l}$ and $\gamma_{j,k}$ are not invariant to alternative economic policies.

As a result, Lucas criticized the use of the system of equations to perform policies evaluations by stating that "*simulations using these models can, in principle, provide no useful information as to the actual consequences of alternative economic policies*" (Lucas, 1976, p. 20).

In addition to the Lucas Critique, the system of equations was also criticized for empirical reasons. According to Lucas and Sargent (1979), such models predicted, in the late 1960s, that high inflation would be compatible only with low unemployment. However, this was not the pattern observed in the 1970s, when a combination of high inflation and unemployment occurred, a phenomenon known as stagflation. According to Kydland and Prescott (1991), this phenomenon constituted a predictive failure for models based on systems of equations. Indeed, Leeper (1995) argued that Lucas Critique provides a plausible explanation of the empirical breakdown of econometric models in the 1970s.

7. Optimal consumption: the permanent income hypothesis – PIH

The second part of the lesson is to present to the classroom the notion of optimal consumption. As a prerequisite, students must have some background on random variables, expected values and the law of large numbers.

Explain that you will present Hall's (1978) version of the PIH, in which the optimal consumption behavior depends on the state of the economy. For instance, it depends on the expected future income and interest rate. After that, make clear that you will use Hall's framework to exemplify the Lucas Critique by means of an unemployment insurance policy.

7.1 Presenting the optimal consumption function

Explain that, although Hall's (1978) model assumes several simplifying assumptions, it is useful to understand how economic policies that affect the data generating process of the disposable income cause modifications in the optimal consumption decision. This happens because economic policies that impact the path of future income lead consumers to adjust their consumption. And this is Lucas Critique's idea.

Hall (1978) assumed that consumers maximize their lifetime utility, subject to a budget constraint. Thus, the consumer welfare is given by:

$$U_t = E_t\left[\sum_{i=0}^{\infty} \beta^i u(C_{t+i})\right] \tag{6}$$

where C_t is consumption in period t, $E_t[\cdot]$ refers to the expectation conditional on available information in period t, $\beta \in (0, 1)$ is the intertemporal discount factor and $u(\cdot)$ is the instantaneous utility function.

Hall (1988) assumed that a consumer transfers wealth between periods by means of an asset with a constant (gross) rate of return, $R = 1 + r$, with $r > 0$. Thus, the budget constraint is given by:

$$A_{t+1} = (A_t + Y_t - C_t)R \tag{7}$$

where A_t and Y_t are, respectively, wealth and disposable income in period t. Thus, disposable income is added to wealth, and consumption is subtracted from it. The return over this amount determines the wealth one period ahead.

Hall (1978) also assumed that the instantaneous utility is quadratic. Thus, the maximization of welfare (6), subject to the budget constraint (7), implies that optimal consumption follows a random walk process [1]:

$$C_{t+1} = C_t + \varepsilon_{t+1} \tag{8}$$

where ε_{t+1} is an innovation, $E_t[\varepsilon_{t+1}] = 0$. This result, together with intertemporal budget constraint, entails the following consumption function [2]:

$$C_t = \frac{r}{1+r}\left[A_t + \sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i E_t[Y_{t+i}]\right] \tag{9}$$

Make sure students are aware that the consumption level depends positively on financial wealth, A_t, and on labor wealth (expected present value of future disposable labor income). Therefore, consumption is determined by permanent income (9), that is, by the expected income flow generated by financial wealth and labor. This feature connects Hall's work with Friedman's (1957) PIH.

7.2 Assessing an unemployment insurance policy

Explain that investigating how consumers respond to an unemployment insurance policy can be a good illustration of how the Lucas Critique can be demonstrated in a context where Hall's optimal consumption behavior depends on the probability of someone being unemployed.

To begin with, assume that, at each period, either the consumer is employed and receives available income $Y > 0$, or he/she is unemployed. Also, assume that, at each period, the probability of the consumer being employed is $p \in (0, 1)$. Consequently, unemployment occurs with probability $1 - p$. Given such an assumption, a consumer disposable income is a random variable with the following distribution:

$$Y_t = \begin{cases} Y, & \text{with probability } p \\ 0, & \text{with probability } 1 - p \end{cases} \tag{10}$$

Suppose that an unemployment insurance policy is implemented, that is, there will be an income transfer program for the unemployed consumers. By assumption, this insurance is proportional to the disposable income a consumer would have if he/she were employed. However, this program is financed by the consumers themselves, that is, they pay a tax while they are employed. Thus, disposable income is now described by the following distribution:

$$Y_t = \begin{cases} (1-\tau)Y, & \text{with probability } p \\ \gamma(1-\tau)Y, & \text{with probability } 1 - p \end{cases} \tag{11}$$

Parameter $\tau \in (0, 1)$ represents the tax rate for the unemployment insurance to be viable and financed, where $(1-\tau)Y$ is the new disposable income of an employed consumer. Parameter $\gamma \in (0, 1)$ indicates the percentage of this disposable income that the unemployed consumer gets as a transfer benefit.

Consider an economy with N consumers. In period t, a fraction $f_{e,t}$ of consumers are employed, and one of them has disposable income $(1-\tau)Y$. The remaining fraction of consumers, $f_{u,t} = 1 - f_{e,t}$, is unemployed. Suppose N is large enough so that the law of large numbers applies. Thus, a fraction of employed individuals converge to p, $f_{e,t} \overset{P}{\to} p$ and a fraction of unemployed individuals converge to $1-p$, $f_{u,t} \overset{P}{\to} 1-p$. In any period, government spending on unemployment insurance benefits converges to the following expression:

$$G_S = (1-p)N\gamma(1-\tau)Y \tag{12}$$

where $(1-p)N$ is the probability of being unemployed times the number of agents in the economy, totaling the number of unemployed consumers who are eligible to collect unemployment insurance benefits, which is equal to $\gamma(1-\tau)Y$. The government revenue needed to cover all benefits is given by

$$G_R = pN\tau Y \tag{13}$$

where pN is the probability of being employed times the number of agents in the economy, totaling the number of employed consumers who pay the additional tax to fund unemployment insurance benefits, which equals to τY.

The government budget is still balanced even with the new unemployment insurance program, in each period, if expenditure (12) is equal to tax collection (13). In this case, the conclusion is that:

$$\gamma = \frac{p}{1-p}\frac{\tau}{1-\tau} \tag{14}$$

As $\tau \in (0, 1)$ and $p \in (0, 1)$, to ensure that $\gamma < 1$, the condition $\tau + p < 1$ needs to be imposed [3]. When this restriction is considered, the unemployment insurance benefit is a fraction of disposable income compatible with the tax collection.

In the current period, each consumer is either employed or unemployed. Those employed have disposable income $(1-\tau)Y$, while those unemployed collect the government's benefit $\gamma(1-\tau)Y$. However, in future periods, employed and unemployed individuals face the same expected disposable income, which is given by

$$E_t[Y_{t+i}] = p \times (1-\tau)Y + (1-p) \times \gamma(1-\tau)Y \tag{15}$$

for $i > 0$. By using expression (14), we can conclude that $E_t[Y_{t+i}] = pY$, for $i > 0$.

Therefore, the expected disposable income does not depend on parameters that characterize the unemployment insurance benefit, γ and τ, given that such policy simply transfers income from the employed to the unemployed, without modifying a country's GDP (p% of the consumers work, each one generating income Y). Finally, replacing expressions (14) and (15) in consumption function (9), and considering employed and unemployed consumers in period t, we are able to obtain, respectively, the following expressions:

$$C_t^E = \frac{r}{1+r}A_t + \frac{p + r - r\tau}{1+r}Y \tag{16}$$

$$C_t^U = \frac{r}{1+r}A_t + \frac{(1-p+\tau r)p}{(1-p)(1+r)}Y \tag{17}$$

where C_t^E and C_t^U are, respectively, the consumption of employed and unemployed consumers. The former has disposable income $(1-\tau)Y$, with a marginal propensity to consume given by $\frac{p+r-r\tau}{(1+r)(1-\tau)}$ (see equation 16). The latter receives transfer $\gamma(1-\tau)Y$, with a marginal propensity to consume equal $\frac{1-p+\tau r}{\tau(1+r)}$ (see equation 17). With fixed p, each percentage γ determines the tax rate τ percentage – according to equation (14) – that affects the marginal propensity to consume of employed and unemployed consumers. Therefore, the marginal propensity to consume is not invariant to the unemployment insurance policy, which means that consumers react to changes in economic policy.

After going through the equations, it is important to show the classroom a comparative static exercise, as presented in Table 2, in which the marginal propensity to consume varies according to parameters that are exogenous from the consumers' point of view. The assumption is that the probability of the consumer being employed (p) is 90%, with an interest rate (r) equal to 4%. Thus, for each percentage of disposable income that an unemployed consumer gets as a transfer benefit (γ), we calculate the new tax rate (τ), using equation (14) and a fixed p. We vary γ from 10% to 100%. Hence, in the worst-case scenario, the unemployed benefit is only 10% of the usual disposable income, but in the best-case scenario, unemployment does not lead to a lower income. As shown in Table 2, the higher the unemployment benefit (γ), the higher the new tax rate (τ) needed to fund unemployment insurance programs.

While employed workers' marginal propensity to consume increases with unemployment benefit (γ), the opposite happens for the unemployed. However, when γ is equal to 100%, both types have the same marginal propensity to consume because they have the same disposable income.

Ask students if they can guess why the propensity to consume of the unemployed is so large for lower values of unemployment benefits. When unemployed, the consumers' disposable income decreases from $(1-\tau)Y$ to $\gamma(1-\tau)Y$ (except when $\gamma = 1$). Besides, the lower γ, the lower the unemployment benefit. But consumers would like to smooth their consumption path. They save when income is high and use their savings or loans to fund consumption when income is low. When γ is low, current income is also low, but consumers still smooth their consumption path and, as a result, the consumption-income ratio becomes extremely high. This is the reason why the marginal propensity to consume for the unemployed is high when γ is low.

Tell the classroom that the comparative static exercise in Table 1 helps to understand how consumers react to the unemployment insurance policy. However, an evaluation of

Unemployment insurance benefit parameter (γ)	Implied new tax rate (τ)	Implied marginal propensity to consume of employed	Implied marginal propensity to consume of unemployed
0.100	0.011	0.913	8.788
0.200	0.022	0.923	4.462
0.300	0.032	0.933	3.019
0.400	0.043	0.942	2.298
0.500	0.053	0.952	1.865
0.600	0.063	0.962	1.577
0.700	0.072	0.971	1.371
0.800	0.082	0.981	1.216
0.900	0.091	0.990	1.096
1.000	0.100	1.000	1.000

Note(s): Fixed parameters: employed probability (p) = 0.9; interest rate (r) = 0.04

Table 2. Marginal propensity to consume

alternative policies depends on the difference between the "true" structure, when there is no other policy and the new "true" structure with a specific policy. To illustrate this, we assume an economy without any type of unemployment insurance for 10 periods, $t = 0, \ldots, 9$, which means that $\gamma = 0$. Thus, until Period 9, the consumption behavior is described by equations (16) and (17) with $\tau = 0$. Assuming that the employed probability (p) is 90%, the interest rate (r) is 4% and the income (Y) is 1, we simulate the consumption unemployment status and calculate the consumption level [4]. The results are displayed in Table 3.

The consumption level ranges from 0.890 to 0.933. The lowest value occurs when the consumer loses his/her job in period 7. If the policymaker does not adopt an unemployment insurance benefit, the data generating process remains the same – equations (16) and (17) with $\tau = 0$ – and in period 10 the consumption will be 0.905 if the consumer remains employed, and 0.857 if he/she becomes unemployed. However, if the policymaker adopts an unemployment insurance benefit in period 10, the data generating process becomes equations (16) and (17) with $\tau > 0$. Assuming that $\gamma = 0.5$, which implies $\tau = 0.053$, the new "true" economy leads consumption to be 0.902, if the consumer remains employed, and becomes 0.884, if the consumer is no longer employed. Therefore, prior to Period 10, when the policymaker needs to decide whether the insurance policy will be adopted, he/she should consider that consumers may react (differently) to each alternative policy. Indeed, as the policy is adopted, Table 3 shows the future level of consumption for each status in the labor market.

In our model, it is easy to calculate how consumers react to unemployment insurance because we know the data generating process and the deep parameters that characterize the marginal propensity to consume (p, r, Y, γ and τ). Thus, for each value for γ, we can calculate the new "true" economy. As it can be seen in Table 2, for $\gamma = 0.50$, the consumer decides to consume less if he/she remains employed, which is expected since he/she pays an additional tax. However, if he/she becomes unemployed, the consumption level is larger, increasing by 3.13% (from 0.857 to 0.884). This result is also expected because, instead of zero income, the consumer ends up with transfer $\gamma(1-\tau)Y$.

Finally, it is important to pinpoint to the class the two main lessons learned from this example. First, agents may react to new policies by adjusting their behavior. Even though this is an empirical issue, when long historical data is used, the data generating process

Period (t)	Employment status	Consumption and unemployment policy No unemployment benefit ($\gamma = 0$)	
0	Employed	0.905	
1	Employed	0.910	
2	Employed	0.914	
3	Employed	0.919	
4	Employed	0.924	
5	Employed	0.929	
6	Employed	0.933	
7	Unemployed	0.890	
8	Employed	0.895	
9	Employed	0.900	
Future consumption conditional on employment status without and with unemployment benefit		*Consumption without benefit ($\gamma = 0$)*	*Consumption with benefit ($\gamma = 0.5$)*
10	Employed	0.905	0.902
	Unemployed	0.857	0.884

Note(s): The following parameters are adopted: $p = 0.90$, $r = 0.04$, $Y = 1$
Initial wealth equals zero ($A_0 = 0$)

Table 3. Consumption simulation and a new policy

should not be taken as constant over time. Therefore, structural stability tests should be applied. Second, instead of behavioral equations without microfoundations, the focus should be on structural models in which agents take the economic environment into account.

7.3 Extensions of the optimal consumption function

As depicted in Box 1, extending the discussion on optimal consumption function is optional, depending on the timeframe allotted to the topic. Start by explaining that Hall's (1978) model was used to illustrate how consumers react to economic policies. The simplifying assumptions adopted by the author allow us to obtain an analytical solution for the consumption function. However, this benefit comes at a cost. First, the precautionary motive is neglected because future income's variance is irrelevant. Indeed, consumption function (9) depends on both the expected (average) value of future disposable income and current wealth. Despite unemployment insurance being a simple income transfer, the variance of disposable income decreases with such policy, as it varies between $\gamma(1-\tau)Y > 0$ and $(1-\tau)Y < Y$, instead of varying between zero and Y. Therefore, under utility functions that capture precautionary motive, insurance policy could lead consumers to decrease their savings rate, which means that marginal propensity to consume would increase.

Second, liquidity constraints are neglected because Hall (1978) assumed that future disposable income is collateral to finance current consumption. Liquidity constraints induce consumers to save more (reducing the marginal propensity to consume) because they have a limited access to credit. However, if there is an unemployment insurance benefit, consumers will not need to save so much, because it is as if the government replaced (albeit partially) the credit market, financing consumption when consumers are unemployed. In this sense, an unemployment insurance benefit program would increase the marginal propensity to consume, if there were credit restrictions.

Even under restrictive assumptions, we show that the marginal propensity to consume is not invariant to the economic environment. By relaxing the simplifying assumptions mentioned, the Lucas Critique would become even more relevant, at least on theoretical grounds.

For readers interested in the subject, additional useful references on consumption functions are Caballero (1990), who derives a closed-form solution for consumption in which the variance of income forecast error decreases the consumption level, and Alessie and Lusardi (1997) and Gomes (2020), who also take in account the precautionary motive but add habit formation in the discussion.

8. The Lucas Critique: some empirical results

Also, as depicted in Box 1, the empirical literature related to the Lucas Critique is optional. Depending on the teacher's needs, it can be left to another time or even transferred to the beginning of the lesson.

The Lucas Critique had a significant impact on how macroeconomic models were built from then on. Since the 1970s, considerable research efforts have been made toward using models with microfoundations and rational expectations, as done by Hall (1978). In such cases, agents maximize an objective function, subject to restrictions, instead of following behavioral rules.

Still, the Lucas Critique is a conjecture that must be empirically tested. For instance, Favero and Hendry (1992) reviewed the relevance of the Lucas Critique, including its methodology, the role of incomplete information and superexogeneity tests. The authors investigated the US money demand and concluded that the Lucas Critique is not suitable for the model. Ericsson and Irons (1995) also examined the empirical relevance of the Lucas

Critique, by analyzing a list of all articles that cited Lucas (1976). The authors found no empirical evidence in favor of the Lucas Critique.

Lindé (2001) made use of a real-business-cycle model to examine the topic and found that a superexogeneity test was not able to identify the importance of the Lucas Critique in small samples. Lindé (2002) made use of a dynamic general equilibrium monetary policy backward-looking model to analyze the suitability of the Lucas Critique. The results showed parameter instability, but not in small samples.

Rudebusch (2005) analyzed two US monetary policy results which contradicted the Lucas Critique: the changing behavior of US monetary authorities in the past decades, but, on the other hand, parameter stability detected by empirical models. The author argued that these opposing results could be reconciled with the Lucas Critique "*by showing that the apparent policy invariance of reduced forms is consistent with the magnitude of historical policy shifts and the relative insensitivity of the reduced forms of plausible forward-looking macroeconomic specifications to policy shifts*."

Castelnuovo (2008) employed two different calibrated Taylor rules for the Swedish economy and reduced-form Phillips Curve to artificial data. The results did not reject the statistical significance of the Lucas Critique, but with limited economic importance. Lubik and Surico (2010) made use of a DSGE sticky-price model to analyze the empirical importance of the Lucas Critique. By calibrating their model to capture historical shifts in the US monetary policy rule, the authors found some empirical relevance for the Lucas Critique. Karimova, Simsek and Orhan (2020) used the global financial crisis to empirically test the Lucas Critique for US data (from 1990 to 2015). The authors built a conditional money demand model and found support for the Lucas Critique.

This brief review of empirical literature makes clear that testing the Lucas Critique empirically does not lead to unanimous results, with most articles showing no relevance for the phenomenon. Even so, there is no doubt about its theoretical value and how important it is to be grasped by students and researchers in general.

9. Conclusion

The aim of this article was to introduce a very important subject in Economics, the Lucas Critique, in such a way that could be easier for undergraduates to grasp Lucas' main ideas. The Lucas Critique concerns the (implicit) assumption that the structure of econometric models used in the 1960s would be invariant to alternative economic policies. According to Lucas (1976), this hypothesis neglected the fact that agents optimize their decisions by taking the economic environment into account. In this vein, faced with a new policy, agents would behave differently, and thus the structure of estimated econometric models, based on historical data, would not be stable.

Based on this, we presented a roadmap as a suggestion to be used by instructors willing to teach the subject to undergraduates. We divided the lesson into three parts. In the first part, an exam grading exercise was shown to bring the topic to students' everyday lives. Then, the Keynesian consumption function was discussed, followed by a discussion on how the Lucas Critique affected the system of equations usually used by researchers in the past. In the second part of the class, the aim was to present a derivation of an optimal consumption function, in which the marginal propensity to consume would respond to an unemployment insurance policy. Finally, as optional topics, we proposed some extensions of the optimal consumption function and some empirical results related to the Lucas Critique.

The example considered, based on a consumption function, is excellent suggestion for instructors who need to present the Lucas Critique to undergraduates. In the Keynesian case, the marginal propensity to consume is treated as constant. However, in Hall's (1978) model, this propensity could vary, due to economic environment changes. We were able to show that an unemployment insurance policy modifies the marginal propensity to consume of employed and unemployed workers.

The narrative created in this work may give the false impression that historical data are not useful for macroeconomists interested in policy evaluations. This is not the case. In models built on microfoundations, the so-called deep parameters of preferences and technology would be stable or at least more stable over time. Thus, instead of estimating the marginal propensity to consume with historical data, we should estimate the deep parameters that would affect the marginal propensity to consume. In this sense, a model would be structural only if it describes preferences and technologies relevant to the proposed research question. Hall's (1978) formulation was able to enhance the development of consumption structural models. Then, a lot of research effort was made to consolidate Hall's idea and to estimate preference parameters such as the intertemporal discount factor, relative risk aversion and intertemporal elasticity of substitution (Hansen and Singleton, 1982, 1983; Hall, 1988; Epstein & Zin, 1991). This micro-based approach, with deep parameters estimated, has become the usual approach in macroeconomics research.

Notes

1. For details on the solution of this model, see Romer (2012), chapter 8.
2. See Appendix 2.
3. The higher the p, the greater the number of consumers employed. This implies that unemployment insurance collection increases with p and, at the same time, spending on unemployment insurance decreases. In addition, when p is fixed, the higher τ is, the higher the government revenue. If the sum of these two parameters is greater than or equal to 1, then, $\gamma \geq 1$. In other words, if $\tau + p \geq 1$, then, to keep a balanced budget the government would provide unemployment insurance benefit, $\gamma(1-\tau)Y$, greater than disposable income, $(1-\tau)Y$.
4. Following equation (10), in each period, we draw whether the consumer is employed or unemployed.
5. Lucas & Sargent (1979) consider and AR model for $\vartheta_{1,t}$. For simplicity, we assume no serial correlation in $\vartheta_{1,t}$.
6. In some cases, an analysis should be performed over several periods, as agents' reaction to alternative policies may not be instantaneous. This would happen, for example, if a policy were not fully announced by policymakers. In any case, for simplicity, we assume an immediate policy effect, so that analyzing the period $t+1$ is sufficient.
7. We assume that agents' reaction to alternative policies can change only the model's coefficients. When analyzing a consumption function, we show that alternative policies lead to a change in marginal propensity to consume. In any case, the effects of alternative policies would not necessarily be limited to parameter changes. Lag numbers could be altered. Variables could also be included or excluded.

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Appendix 1

As discussed by Lucas and Sargent (1979), in the 1960s, macro-econometric models consisted of several equations and, for this reason, they became known as systems of equations. Such models can be represented as follows:

$$\boldsymbol{A}_0\boldsymbol{y}_t + \boldsymbol{A}_1\boldsymbol{y}_{t-1} + \cdots + \boldsymbol{A}_L\boldsymbol{y}_{t-L} = \boldsymbol{B}_0\boldsymbol{x}_t + \boldsymbol{B}_1\boldsymbol{x}_{t-1} + \cdots + \boldsymbol{B}_n\boldsymbol{x}_{t-n} + \vartheta_{1,t} \tag{A1}$$

where $\boldsymbol{y}_t$ is an $J \times 1$ vector of endogenous variables (determined within the system), $\boldsymbol{x}_t$ is a $K \times 1$ vector de exogenous variables and $\vartheta_{1,t}$ is an $L \times 1$ vector of errors. The unknown coefficient matrices $\boldsymbol{A}_i$, $i = 0, \ldots, L$, are $J \times J$ and $\boldsymbol{B}_i$, $i = 0, \ldots, n$, are $J \times K$ [5].

Model (A1) is in structural form (a system of simultaneous equations) and it can be rewritten using lagged values of $\boldsymbol{y}_t$, in addition to $\boldsymbol{x}_t$ and its own lags. To do so, pre-multiply the equation (A1) by $\boldsymbol{A}_0^{-1}$ reaching the following model:

$$\boldsymbol{y}_t = \boldsymbol{B}\boldsymbol{X}_t + \boldsymbol{\varepsilon}_t \tag{A2}$$

where $\boldsymbol{X}_t = [\, \boldsymbol{y}'_{t-1} \;\; \cdots \;\; \boldsymbol{y}'_{t-L} \;\; \boldsymbol{x}'_t \;\; \cdots \;\; \boldsymbol{x}'_{t-n} \,]'$ contains all regressors from model (A1) and $\boldsymbol{B} = [\, \widetilde{\boldsymbol{A}}_1 \;\; \cdots \;\; \widetilde{\boldsymbol{A}}_L \;\; \widetilde{\boldsymbol{B}}_0 \;\; \cdots \;\; \widetilde{\boldsymbol{B}}_n \,]$ contains the coefficient matrices, such that $\widetilde{\boldsymbol{A}}_i \equiv -\boldsymbol{A}_0^{-1}\boldsymbol{A}_i$, $i = 1, \ldots, L$, $\widetilde{\boldsymbol{B}}_i \equiv \boldsymbol{A}_0^{-1}\boldsymbol{B}_i$, $i = 1, \ldots, n$. The error term is $\boldsymbol{\varepsilon}_t \equiv \boldsymbol{A}_0^{-1}\vartheta_{1,t}$.

While model (A1) represents the structural form, model (A2) represents the reduced form. Under general conditions, model (A2) can be consistently estimated by Ordinary Least Squares (OLS). Econometrists are usually concerned about whether structural parameters, as in model (A1), can be recovered from a reduced form estimation, such as model (A2). This can be done as long as *a priori* restrictions (identification constraints) are applied.

Finally, Lucas and Sargent (1979) consider an autoregressive vector (VAR) to describe the evolution of exogenous variables, as follows:

$$\boldsymbol{x}_t = \boldsymbol{C}_1\boldsymbol{x}_{t-1} + \boldsymbol{C}_2\boldsymbol{x}_{t-2} + \cdots + \boldsymbol{C}_p\boldsymbol{x}_{t-p} + \vartheta_{2,t} \tag{A3}$$

where $\vartheta_{2,t}$ is a $K \times 1$ vector of errors and the unknown coefficient matrices $\boldsymbol{C}_i, i = 0, \ldots, p$, are $K \times K$. Model (A3) can also be estimated via OLS.

After selecting the best specification, forecasting exercises can be done. Focusing on the one step ahead case, the model (A2) implies the following forecast for $\boldsymbol{y}_{t+1}$:

$$\widehat{\boldsymbol{y}}_{t+1} = \widehat{\boldsymbol{B}}\widehat{\boldsymbol{X}}_{t+1} \tag{A4}$$

Forecast $\widehat{\boldsymbol{y}}_{t+1}$ depends on the forecast of $\boldsymbol{X}_{t+1}$, $\widehat{\boldsymbol{X}}_{t+1}$, which is made based on information available in period t. Given model's (A3) autoregressive structure, the exogenous variables vector $\boldsymbol{X}_{t+1}$ depends on its own lags, which are already known in period t +1. Therefore, predicting $\boldsymbol{X}_{t+1}$ should not be problematic. The problem is to obtain an appropriate estimate of $\boldsymbol{B}$, that is, $\widehat{\boldsymbol{B}}$. For that, coefficient stability is necessary. More precisely, the structure of the econometric model (A2) must not change in period $t + 1$.

Regarding policy evaluation, a policymaker would apply model (A2) to compare the path of endogenous variables for each alternative policy. Suppose that exists a finite policy set, composed of M policies. As policy variables are contained in $\boldsymbol{X}_{t+1}$, for each policy m there would be a specific regressor matrix $\boldsymbol{X}_{t+1}^{(m)}$. In principle, model (A2) implies, through $\widehat{\boldsymbol{B}}$, a certain expected path for the dependent variables for each policy: $\boldsymbol{y}_{t+1}^{(m)} = \widehat{\boldsymbol{B}}\boldsymbol{X}_{t+1}^{(m)}, m = 1, \ldots, M$. Therefore, based on $\boldsymbol{y}_{t+1}^{(m)}, m = 1, \ldots, M$, a policymaker could choose a policy that induces the most appropriate path of endogenous variables [6].

The policy assessment based on model (A2) makes sense only if an econometric model structure is invariant to all policies analyzed. Particularly, the coefficient matrix $\boldsymbol{B}$ must be fixed (invariant to the M alternative policies). However, as discussed in Section 3, Lucas (1976) criticized this hypothesis. Following Lucas' advice, each economic policy assessed would induce a new coefficient matrix, say $\boldsymbol{B}^{(m)}$, $p = 1, \ldots, P$ [7]. It is clear that $\boldsymbol{B}$ cannot be equal to $\boldsymbol{B}^{(m)}$ for all m, or even for a specific m. Since estimation $\widehat{\boldsymbol{B}}$ is treated as fixed and obtained from historical data, policy evaluation would be compromised.

Finally, two observations must be made. First, even if a model fits well with data and predicts the endogenous variables vector adequately (in the absence of new policies), policy evaluation is compromised because the coefficient matrix $\boldsymbol{B}$ is affected by the policy itself (Lucas & Sargent, 1979). Second, parameter instability is not only found in the reduced form, given that even parameters related to the structural form would not be fixed in the face of alternative policies. Therefore, imposing *a priori* constraint to identify the structural form (A.1) would not make the econometric model immune to Lucas Critique (Lucas & Sargent, 1979).

Appendix 2

Hall's (1978) model implies that consumption follows a random walk, such as:

$$C_{t+i} = C_{t+i-1} + \varepsilon_{t+i} \tag{A5}$$

By recursive substitution of the lagged consumption, the process (A5) becomes:

$$C_{t+i} = C_t + \varepsilon_{t+1} + \varepsilon_{t+2} + \cdots + \varepsilon_{t+i-1} + \varepsilon_{t+i} \tag{A6}$$

Because $E_t[\varepsilon_{t+i}] = 0$ for $i > 0$, the best prediction for future consumption is its current value:

$$E_t[C_{t+i}] = C_t \tag{A7}$$

for $i > 0$. This property is essential to derive the consumption function. But before doing that, we derive the intertemporal budget constraint. In order to do so, note that budget constraint (5) can be rewritten as follows:

$$A_t = R^{-1}A_{t+1} + C_t - Y_t \tag{A8}$$

Equation (A8) holds for any period. Therefore, wealth in period $t+1$ is given by:

$$A_{t+1} = R^{-1}A_{t+2} + C_{t+1} - Y_{t+1} \tag{A9}$$

Substituting (A9) in (A8) leads to:

$$A_t = R^{-2}A_{t+2} + R^{-1}(C_{t+1} - Y_{t+1}) + C_t - Y_t \tag{A10}$$

By repeating such procedure, we reach the following expression:

$$A_t = \lim_{n\to\infty} R^{-i}A_{t+i} + \sum_{i=0}^{\infty} R^{-i}(C_{t+i} - Y_{t+i}) \tag{A11}$$

Assuming there is no Ponzi scheme nor consumer waste wealth, then the term $\lim_{n\to\infty} R^{-i}A_{t+i}$ is null and, the intertemporal budget constraint is given by

$$\sum_{i=0}^{\infty} R^{-i}C_{t+i} = A_t + \sum_{i=0}^{\infty} R^{-i}Y_{t+i} \tag{A12}$$

The left-hand side of equation (A12) is the present value of the consumption stream, while the right-hand side is the present value of consumer resources.

Taking the expectation of both sides of equation (A12) allows us to use condition (A7) to derive the consumption function (7), as follows:

$$E_t\left[\sum_{i=0}^{\infty} R^{-i}C_{t+i}\right] = A_t + E_t\left[\sum_{i=0}^{\infty} R^{-i}Y_{t+i}\right]$$

$$\sum_{i=0}^{\infty} R^{-i}C_t = A_t + E_t\left[\sum_{i=0}^{\infty} R^{-i}Y_{t+i}\right]$$

$$C_t = \frac{r}{1+r}\left[A_t + \sum_{i=0}^{\infty} R^{-i}E_t[Y_{t+i}]\right]$$

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