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Simulation of nonuniform transmission lines

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Abstract

Purpose – The purpose of the paper is the simulation of nonuniform transmission lines.

Design/methodology/approach – The method involves a Magnus expansion and a numerical Laplace transform. The method involves a judicious arrangement of the governing equations so as to enable efficient simulation.

Findings – The results confirm an effective and efficient numerical solver for inclusion of nonuniform transmission lines in circuit simulation.

Originality/value – The work combines a Magnus expansion and numerical Laplace transform algorithm in a novel manner and applies the resultant algorithm for the effective and efficient simulation of nonuniform transmission lines.

Keywords Numerical analysis, Transient analysis

Paper type Research paper

Introduction

The ongoing increase in signal frequencies and increase in the density of circuits have the consequential requirement for accurate modelling of interconnects. Models of interconnects in circuits need to include the nonuniformity in geometry so as to enable accurate analysis and design (Antonini, 2012). Another application of nonuniform transmission lines is in microwave filters that are used in many applications, for example, mobile and satellite communications and test and measurement systems (Hashash *et al.*, 2018; Arnedo *et al.*, 2012; Attamimi and Alaydrus, 2015). While such filters may be designed using cascaded distributed transmission lines, abrupt changes in structure can lead to parasitic effects and consequently to errors. To avoid these errors, nonuniform transmission lines are used in microwave filters. Non-uniform transmission lines are also present in power systems (Gunawardana, 2022). Many approaches have been proposed for time-domain modelling of nonuniform transmission lines. Antonini (2012) proposes a method based on Green's function of a uniform transmission line. The result is a rational macromodel that is suitable for the time-domain simulation. Afrooz and Abdipour (2012) present a Finite Difference Time Domain technique that is unconditionally stable and describes a method for handling modulated signals in an efficient manner. Brančík and Ševčík (2011) present a method based on an implicit Wendroff method. Jurić-Grgić (2015) uses a finite element approach. Manfredi *et al.* (2016) present a perturbative approach whereby the variations in the transmission line parameters are seen as perturbations of their average value. Chernobryvko (2014) also uses a perturbative approach focussing on multiconductor lines. Some frequency-domain approaches have also been presented, for example those of Gómez and Escamilla (2013) and Moreno *et al.* (2005). An approach based on the state-transition matrix and using the Peano–Baker Series is proposed in Momeni *et al.* (2020).

The first section of the present paper shall describe the theory governing the proposed method. The following section shall give examples to highlight its efficacy. Finally, conclusions and suggestions for future work shall be given.

Methodology

Consider a section of a nonuniform transmission line, as shown in Figures 1 and 2. Let x be the direction of propagation.

The Telegrapher's equations are:

$$\begin{aligned} \frac{\partial}{\partial x}V(s,x) &= -(R(x)+sL(x))I(s,x) = -Z(s,x)I(s,x) \\ \frac{\partial}{\partial x}I(s,x) &= -\big(G(x)+sC(x)\big)V(s,x) = -Y(s,x)V(s,x) \end{aligned} \tag{1}$$

$V(s, x)$ and $I(s, x)$ are the voltage and current along the line. s is the Laplace parameter. $R(x)$, $L(x)$, $G(x)$ and $C(x)$ are the per-unit resistance, inductance, conductance and capacitance along the

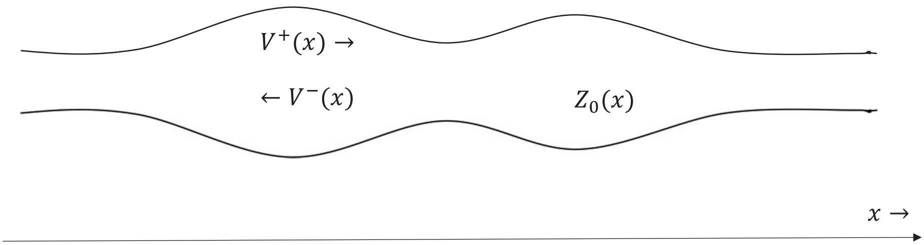

Figure 1. Nonuniform transmission line of varying width

Source: Author's own work

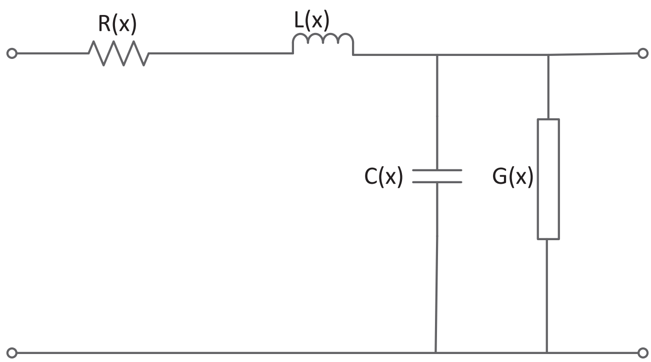

Figure 2. Nonuniform transmission line per-unit length parameters

Source: Author's own work

line. $Z(s, x) = R(x) + sL(x)$ is the per-unit length impedance of the line. $Y(s, x) = G(x) + sC(x)$ is the per unit length admittance of the line.

The solution of the equations in (1) are of the form:

$$\begin{aligned} V(s,x) &= V^{+}(s,x) + V^{-}(s,x) \\ I(s,x) &= I^{+}(s,x) + I^{-}(s,x) \end{aligned} \tag{2}$$

$V^{+}(s, x)$, $V^{-}(s, x)$ are the forward and backward waves on the transmission line.

Bearing in mind the approaches taken in Tang and Zhang (2011), Zhang *et al.* (2011) and Pereira (2014), the following form of equations shall be considered, with $a^{+}(s, x)$ and $a^{-}(s, x)$ defined as follows:

$$\begin{aligned} V(s,x) &= Z_0^{\frac{1}{2}}(s,x)\left(a^{+}(s,x) + a^{-}(s,x)\right) \\ I(s,x) &= Z_0^{-\frac{1}{2}}(s,x)\left(a^{-}(s,x) - a^{+}(s,x)\right) \end{aligned} \tag{3}$$

This form of equation is selected as it is appropriate for the subsequent analysis.
The propagation constant is:

$$\gamma(s,x) = \sqrt{Z(s,x)Y(s,x)} = \sqrt{\left(R(x) + sL(x)\right)\left(G(x) + sC(x)\right)}$$

The characteristic impedance is:

$$\begin{aligned} Z_0(s,x) &= Y_0^{-1}(s,x) \\ Y_0(s,x) &= \left(R(x) + sL(x)\right)^{-1}\sqrt{\left(R(x) + sL(x)\right)\left(G(x) + sC(x)\right)} \end{aligned} \tag{4}$$

Then:

$$\frac{\partial}{\partial x}\begin{bmatrix} a^{+}(s,x) \\ a^{-}(s,x) \end{bmatrix} = \begin{bmatrix} M(s,x) & K(s,x) \\ K(s,x) & -M(s,x) \end{bmatrix}\begin{bmatrix} a^{+}(s,x) \\ a^{-}(s,x) \end{bmatrix} = A\begin{bmatrix} a^{+}(s,x) \\ a^{-}(s,x) \end{bmatrix} \tag{5}$$

where:

$$M(s,x) = \frac{1}{2}\left(Z_0^{-1}(s,x)(R(x) + sL(x)) + \left(G(x) + sC(x)\right)Z_0(s,x)\right)$$

$$K(s,x) = -\frac{1}{2}Z_0^{-1}(s,x)\frac{dZ_0(s,x)}{dx}$$

$K(s, x)$ is the coupling coefficient that varies along the length of the line.

Equation (5) is then solved using an efficient numerical algorithm.

The numerical algorithm involves two steps. The first is solution in the spatial domain, x. For this purpose, the approach in Vaibhav (2019) and Blanes and Moan (2006) is used. It is a fourth-order algorithm based on an equispaced division of the x axis. The divisions are x and the nth point is x_n.

The solution of (5) is:

$$\begin{bmatrix} a^{+}(s,x) \\ a^{-}(s,x) \end{bmatrix} = e^{\Lambda(x,x_n)} \begin{bmatrix} a^{+}(s,x_n) \\ a^{-}(s,x_n) \end{bmatrix} \tag{6}$$

For the remainder of the paper, explicit dependence of variables on x, s shall be omitted in some equations to enable ease of understanding.

$\Lambda(x, x_n)$ can be represented by the Magnus series (Magnus, 1954):

$$\Lambda(x,x_n) = \sum_{i=1}^{\infty} \Lambda_i \tag{7}$$

The series is made up of an infinite series of terms, and the subscript is to distinguish each term.

The method proposed by Blanes and Moan (2006) and used by Vaibhav (2019) involved a truncation of this series to obtain a fourth-order method. The first two terms in the Magnus series are:

$$\begin{aligned} \Lambda_1 &= \int_{x_n}^{x} A(\tau)d\tau \\ \Lambda_2 &= \frac{1}{2}\int_{x_n}^{x} d\tau_1 \int_{x_n}^{\tau_1} [A(\tau_1), A(\tau_2)]d\tau_2 \end{aligned} \tag{8}$$

where A is as defined in equation (5).

A Taylor series expansion of $A(x)$ about $x_{1/2} = x_n + \frac{\Delta x}{2}$ is performed and let:

$$A^0 = \int_{x_n}^{x_n+\Delta x} A(\tau)d\tau$$

$$A^1 = \frac{1}{\Delta x} \int_{x_n}^{x_n+\Delta x} (x - x_{1/2})A(\tau)d\tau \tag{9}$$

Then a fourth-order method is given by:

$$\Lambda^{order4} = A^0 + [A^1, A^0] \tag{10}$$

$[\alpha, \beta]$ is the commutator bracket, $[\alpha, \beta] = \alpha\beta - \beta a$.

Gauss-Legendre-Lobatto quadrature is used to evaluate the integrals in (9). The method may be derived by matching the zeroth, first and second moments. The ith moment is defined as:

$$m_i = \int_{x_n}^{x_n+\Delta x} x^i dx \tag{11}$$

The numerical quadrature method is:

$$\int_{x_n}^{x_n+\Delta x} f(x)dx = \sum_{k=1,2,3} w_k f(x_k) \tag{12}$$

The weights are determined as follows:

$$w_1 {x_n}^i + w_2\left(x_n + \frac{\Delta x}{2}\right)^i + w_3(x_n + \Delta x)^i = m_{i,} \qquad i = 0, 1, 2 \tag{13}$$

Solving equation (13) yields:

$$w_1 = \frac{\Delta x}{6},\ w_2 = \frac{4\ \Delta x}{6},\ w_3 = \frac{\Delta x}{6}$$

Using this result, the integrals in (9) may be determined as follows:

$$\int_{x_n}^{x_n+\Delta x} A(\tau)d\tau = \frac{\Delta x}{6}(A_1 + 4A_2 + A_3) \tag{14}$$

$$\begin{gathered} A_1 = A(x_n), A_2 = A\left(x_n + \frac{\Delta x}{2}\right), A_3 = A(x_n + \Delta x) \\ \frac{1}{\Delta x}\int_{x_n}^{x_n+\Delta x} (x - x_{1/2})A(\tau)d\tau = \frac{\Delta \mathrm{x}}{12}(A_3 - A_1) \end{gathered} \tag{15}$$

$$\Lambda_{n+1}^{order4} = \Lambda^4(x_n + \Delta x, x_n) = \frac{\Delta x}{6}(A_1 + 4A_2 + A_3) + \frac{\Delta x^2}{72}([A_1 + 4A_2 + A_3, A_3 - A_1]) \tag{16}$$

The first and second terms in (16) correspond to the first and second terms in (10) when the Gauss-Legendre-Lobatto quadrature is used. The superscripts denote the terms in (9) and (10) while the subscript refers to the definitions in (14).

Computing matrix exponentials is computationally expensive and hence, the structure of the $\Lambda(x, x_n)$ matrix must be investigated for savings in computation as was done in Vaibhav (2019) and Blanes and Moan (2006).

For the equation in (5), (16) becomes:

$$\Lambda_{n+1}^{order4} = \frac{\Delta x}{6}\begin{bmatrix} M_n + 4M_{n+\frac{1}{2}} + M_{n+1} & K_n + 4K_{n+\frac{1}{2}} + K_{n+1} \\ K_n + 4K_{n+\frac{1}{2}} + K_{n+1} & -\left(M_n + 4M_{n+\frac{1}{2}} + M_{n+1}\right) \end{bmatrix} + \frac{\Delta x^2}{72}\begin{bmatrix} M_{n+1} - M_n & K_{n+1} - K_n \\ K_{n+1} - K_n & -(M_{n+1} - M_n) \end{bmatrix} \tag{17}$$

$$K_n = K(x_n), K_{n+\frac{1}{2}} = K\left(x_n + \frac{\Delta x}{2}\right), K_{n+1} = K(x_n + \Delta \mathrm{x}),$$

$$M_n = M(x_n), M_{n+\frac{1}{2}} = M\left(x_n + \frac{\Delta x}{2}\right), M_{n+1} = M(x_n + \Delta \mathrm{x}).$$

Because of its structure, $det\left(\exp\left(\Lambda_{n+1}^{order4}\right)\right) = 1$. Hence, for a single transmission line:

$$e^{\Lambda_{n+1}^{order4}} = \begin{bmatrix} cosh(\Gamma_{n+1}) & 0 \\ 0 & cosh(\Gamma_{n+1}) \end{bmatrix} + \frac{sinh(\Gamma_{n+1})}{\Gamma_{n+1}}\Lambda_{n+1} \tag{18}$$

where:

$$\Gamma_{n+1} = \pm\sqrt{-det\left(\Lambda_{n+1}^{order4}\right)}$$

Thus an efficient method for computing the exponential of Λ_{n+1} has been determined.

For 2-conductor lines, the matrix Λ_{n+1}^{order4} has the following structure:

$$\Lambda_{n+1}^{order4} = \begin{bmatrix} a & b & c & d \\ b & a & d & c \\ c & d & -a & -b \\ d & c & -b & -a \end{bmatrix} \tag{19}$$

For this specific structure of matrix, the matrix exponential can be computed as:

$$e^{\Lambda_{n+1}^{order4}} = \begin{bmatrix} e^{\lambda_1}\left(\dfrac{(\Lambda_{n+1}-\lambda_2 I)(\Lambda_{n+1}-\lambda_3 I)(\Lambda_{n+1}-\lambda_4 I)}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)(\lambda_1-\lambda_4)}\right) \\ +e^{\lambda_2}\left(\dfrac{(\Lambda_{n+1}-\lambda_1 I)(\Lambda_{n+1}-\lambda_3 I)(\Lambda_{n+1}-\lambda_4 I)}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)(\lambda_2-\lambda_4)}\right) \\ +e^{\lambda_3}\left(\dfrac{(\Lambda_{n+1}-\lambda_1 I)(\Lambda_{n+1}-\lambda_2 I)(\Lambda_{n+1}-\lambda_4 I)}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)(\lambda_3-\lambda_4)}\right) \\ +e^{\lambda_4}\left(\dfrac{(\Lambda_{n+1}-\lambda_1 I)(\Lambda_{n+1}-\lambda_2 I)(\Lambda_{n+1}-\lambda_3 I)}{(\lambda_4-\lambda_1)(\lambda_4-\lambda_2)(\lambda_4-\lambda_3)}\right) \end{bmatrix} \tag{20}$$

$$\lambda_{1,2} = \pm\sqrt{(a+b)^2 + (c+d)^2}$$

$$\lambda_{3,4} = \pm\sqrt{(a-b)^2 + (c-d)^2}$$

This enables the $a^{\pm}(s, x)$ to be determined for each x_n for a particular s in an efficient manner.

Numerical Laplace transform

The second part of the algorithm involves computation of the inverse Laplace transform. The advantage of using the inverse Laplace transform is that it enables ease of inclusion of frequency-dependent effects. In addition, it avoids the difficulties of Fourier transform approaches when step responses are of interest (Griffith and Nakhla, 1990). Many methods have been proposed for numerical inversion of the Laplace transform for example (Griffith and Nakhla, 1990; Cohen, 2007). A recent work recommending and indicating its suitability for state-of-the-art circuit simulation is Gad (2022). The work (Gad, 2022) involves a rational approximation of the exponential function. In the current work, the approach of Wilcox (1978) is used. This approach uses the midpoint integration rule and uses the FFT thus resulting in an efficient routine. To account for Gibbs oscillations, the sigma-factor, σ, suggested in Wilcox (1978) is used. If the inverse Laplace transform of $F(s)$ is required, then the suggested approach involves multiplying this by σ:

$$\sigma = \frac{sin(\omega\pi/\ \Omega)}{\omega\pi/\ \Omega} \tag{21}$$

where Ω *rad/s* is the frequency beyond which the frequency spectrum of the time-domain signal may be assumed to be neglected. This method was selected as it is efficient and includes a method to overcome Gibbs oscillations. Future work will compare different numerical inverse Laplace transform routines.

Examples

The first example is a two-conductor coupled nonuniform transmission line similar to that in Manfredi *et al.* (2016). The length of the line is $l = 4$ cm. The conductance is set to zero, as is done in other publications, for example, Antonini (2012) and Momeni *et al.* (2020).

$$L(x) = \begin{bmatrix} 200 & 20 \\ 20 & 200 \end{bmatrix} e^{x/l} \mathrm{nH/m}$$

$$C(x) = \begin{bmatrix} 60 & -6 \\ -6 & 60 \end{bmatrix} e^{-x/l} pF/m$$

$$R(x) = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix} \Omega/m$$

Figure 3 shows the unit-step response at the end of the excited line. The proposed method is compared with that obtained with a very fine segmentation technique. The proposed method uses $\Delta x = 1$ cm. The segment technique uses 128 sections. Figure 4 shows the voltages at the receiving end of the second conductor. Figure 5 shows the response at the receiving end of the excited conductor to a pulse with a rise and fall time of 25 ps. The duration of the pulse is also 25 ps. Figure 6 shows the response at the receiving end when the input is a 20 GHz sinewave.

The results indicate the efficacy of the proposed method.

In a manner similar to Momeni *et al.* (2020), the accuracy and efficiency of the techniques are compared as follows: The finite discretisation technique is applied with 8,000 sections to obtain an "exact" solution. This is denoted v_{ex}. The same technique is then applied with a varying number of sections to obtain a result within a tolerance ϵ. This is denoted v_{sect}.

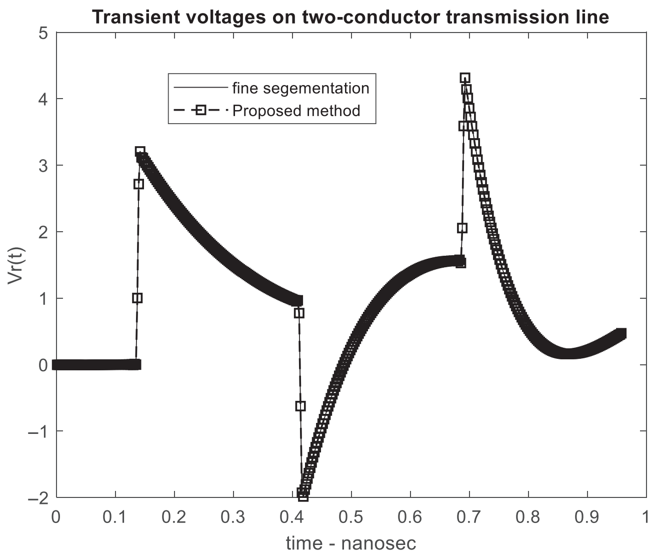

Figure 3. Unit-step responses at the receiving end of the excited conductor

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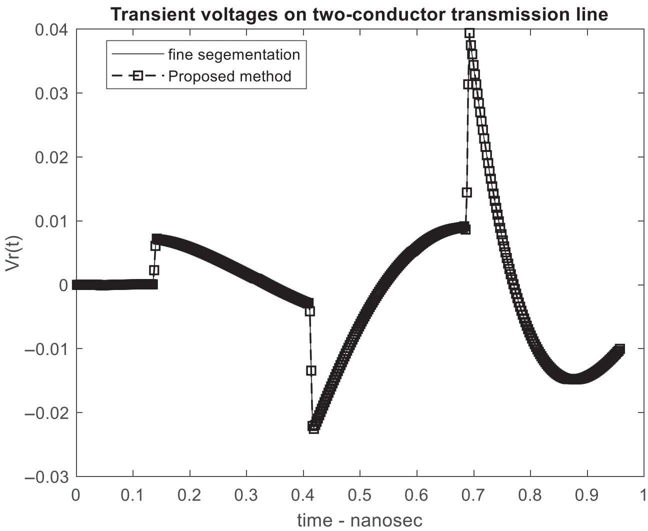

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Figure 4. Unit-step responses at the receiving end of the second conductor

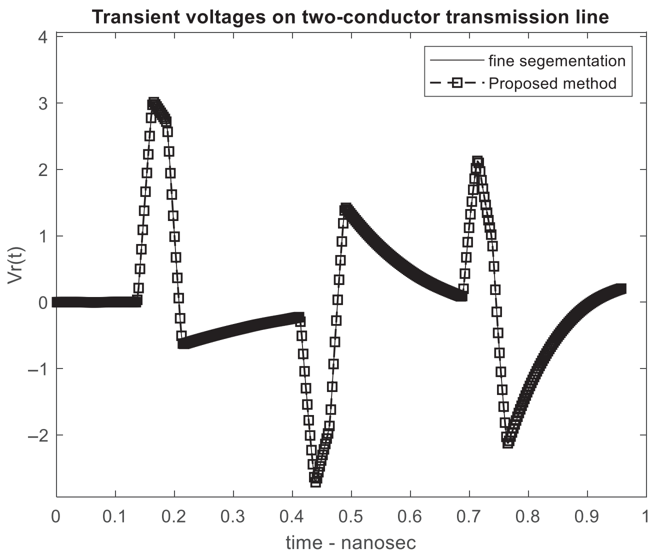

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Figure 5. Pulse responses at the receiving end of the excited conductor

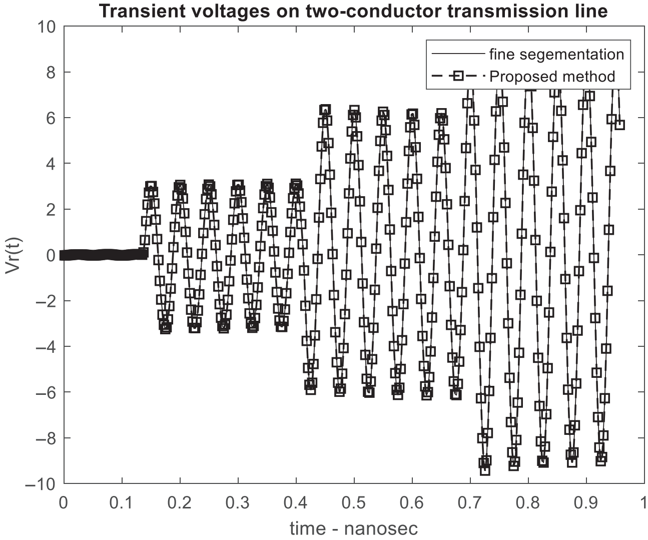


Figure 6. Sinusoidal responses at the receiving end of the excited conductor

Source: Author's own work

Finally, the proposed method is applied with varying x values to lie with the same tolerance. This is denoted as v_{proposed}:

$$\left|\frac{v_{ex} - v_{sect/proposed}}{v_{ex}}\right| < \epsilon$$

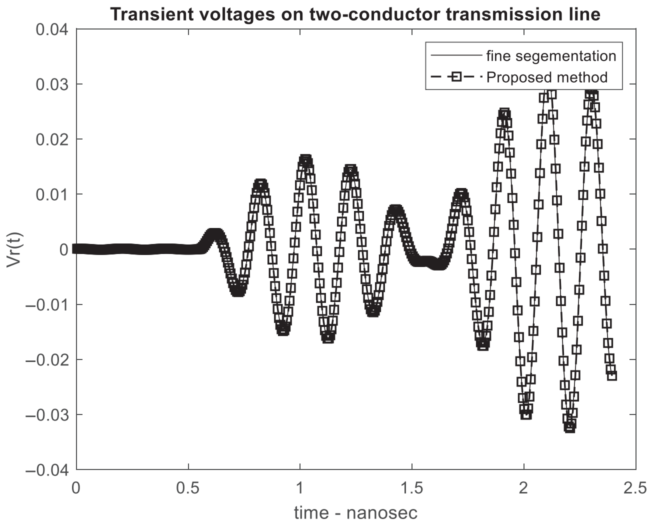
The results are noted. The speed of computation is then measured. For the two-conductor line, 128 sections are required for an accuracy of $\epsilon = O(\sim 10^{-4})$. For the proposed method, $x = 1$ cm. The computation time for the proposed method is less than 50% that for the method, with the sections indicating the efficacy of the proposed method as an alternative to traditional methods.

The second example is also similar to that in Momeni *et al.* (2020). The parameters of the line are:

$$L(x) = \begin{bmatrix} 400 & 75 \\ 75 & 400 \end{bmatrix}\left(1 + \frac{kx}{l}\right)\text{nH/m}$$

$$C(x) = \frac{\begin{bmatrix} 175 & -15 \\ -15 & 175 \end{bmatrix}}{1 + \frac{kx}{l}} pF/m$$

$$R(x) = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix} \Omega/m$$



Source: Author's own work

Figure 7. Voltages are the receiving end of the transmission line when the input is a modulated signal and when there is a load resistance

The conductance is set to zero.
The length of the line is 7 cm and $k = 1$.
The input is a modulated signal:

$$v_{in}(t) = \sin(\pi \times 10^8 t)\sin(\pi \times 10^9 t)$$

The load resistance is $R_{load} = 1\Omega$. Figure 7 shows the result at the receiving end of the line with the proposed method superimposed on the result using the method with sections. The results indicate the efficacy of the proposed technique.

Conclusions

The paper has introduced an alternative and efficient procedure for simulating nonuniform transmission lines. The method involves a Magnus expansion for the spatial variation and numerical inverse Laplace transform to determine the time-domain response. The method is accurate, and results have confirmed its efficacy, and computation times indicate its efficiency. Possible future work could involve optimising the code and memory usage. Future work could include frequency-dependent per-unit length parameters of the transmission line and nonlinear loads. In addition, further exploration is required with input signals of widely varying frequency content. Exploration of terahertz interconnects may also be a possibility.

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