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The dual effect of idiosyncratic volatility on stock pricing and return

Zhuo (June) Cheng and Jing (Bob) Fang
The Hong Kong Polytechnic University, Kowloon, China

Abstract

Purpose – This study aims to examine what underlies the estimated relation between idiosyncratic volatility and realized return.

Design/methodology/approach – Idiosyncratic volatility has a dual effect on stock pricing: it not only affects investors' expected return but also affects the efficiency of stock price in reflecting its value. Therefore, the estimated relation between idiosyncratic volatility and realized return captures its relations with both expected return and the mispricing-related component due to its dual effect on stock pricing. The sign of its relation with the mispricing-related component is indeterminate.

Findings – The estimated relation between idiosyncratic volatility and realized return decreases and switches from positive to negative as the estimation sample consists of proportionately more ex ante overvalued observations; it increases and switches from negative to positive as the estimation sample consists of proportionately more ex post overvalued observations. In sum, the relation of idiosyncratic volatility with the mispricing-related component dominates its relation with expected return in its estimated relation with realized return. Moreover, its estimated relation with realized return varies with research design choices and even switches sign due to their effects on its relation with the mispricing-related component.

Originality/value – The novelty of the study is evident in the implication of its findings that one cannot infer the sign of the relation of idiosyncratic volatility with expected return from its estimated relation with realized return.

Keywords Idiosyncratic volatility, Stock pricing efficiency, Realized return, Expected return, Asset pricing puzzle

Paper type Research paper

1. Introduction

Whether a stock's expected return depends on idiosyncratic volatility is an unresolved asset pricing puzzle (Hou & Loh, 2016). Traditional asset pricing theories predict either no relation between idiosyncratic volatility and expected return under the assumption of complete and frictionless markets and perfect portfolio diversification (Black, 1972; Lintner, 1965; Sharpe, 1964) or a positive relation under the assumption of limited portfolio diversification (Levy, 1978; Merton, 1987). However, Ang, Hodrick, Xing and Zhang (2006) and several following studies find a negative relation between idiosyncratic volatility and realized return. If realized return is an adequate expected return proxy, considered an asset pricing puzzle, the finding of a negative relation between idiosyncratic volatility and realized return poses a direct challenge to those asset pricing theories.

JEL Classification — G12, G14, G40.

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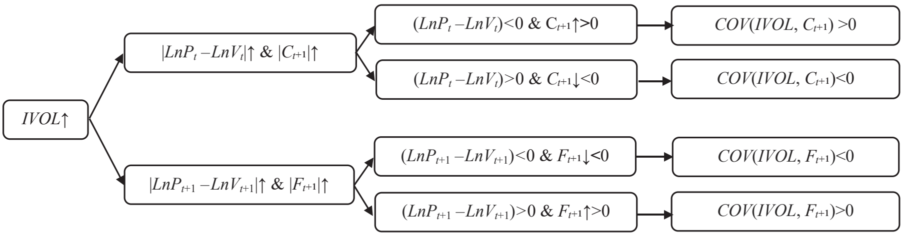
Existing research has focused on proposing explanations for the negative relation and on evaluating the proposed explanations (Hou & Loh, 2016) [1]. This focus downplays the fact that quite a few studies find a positive or no relation between idiosyncratic volatility and realized return (e.g. Fama & MacBeth, 1973; Lehmann, 1990). That is, findings about the relation between idiosyncratic volatility and realized return are indeed inconsistent across studies. Moreover, some studies with inconsistent findings employ highly overlapped samples and similar – or even identical – measures and testing methods. Arguably, what drives this inconsistency is just as puzzling as the finding of a negative relation.

Evidently, research is needed to investigate what drives this inconsistency. Answers to this question help to resolve the idiosyncratic volatility puzzle. We aim to shed light on this inconsistency. To do so, we first show that the dual effect of idiosyncratic volatility on stock pricing causes its estimated relation with realized return to be an inadequate indicator of its relation with expected return. Idiosyncratic volatility has a dual effect on stock pricing because it affects both investors' expected return and the efficiency of stock price in reflecting its underlying value. Idiosyncratic volatility is positively related to arbitrage risks that deter arbitrage and thus hinder the reduction of mispricing (Pontiff, 2006). In sum, idiosyncratic volatility has a dual effect on stock pricing because it not only shapes investors' expected return but also lowers stock pricing efficiency.

Due to this dual effect, the estimated relation of idiosyncratic volatility (*IVOL*) with realized return ($R_{t \to t+1}$) captures its relations with both expected return and with the mispricing-related component of $R_{t \to t+1}$. In principle, the mispricing-related component stems from the correction of ex ante mispricing ($C_{t \to t+1}$) or from the formation of ex post mispricing ($F_{t \to t+1}$) or from both. For the correction of ex ante undervaluation, $C_{t \to t+1}$ is positive and increases with the degree of ex ante undervaluation corrected, while for the correction of ex ante overvaluation, $C_{t \to t+1}$ is negative and decreases with the degree of ex ante overvaluation corrected; for the formation of ex post undervaluation, $F_{t \to t+1}$ is negative and decreases with the degree of ex post undervaluation formed, while for the formation of ex post overvaluation, $F_{t \to t+1}$ is positive and increases with the degree of ex post overvaluation formed. Because *IVOL* is positively related to arbitrage risks, it is positively related to the degree of mispricing, both ex ante and ex post. Taken together, evidently *IVOL* is positively related to $C_{t \to t+1}$ among ex ante undervalued stocks and negatively related to $C_{t \to t+1}$ among ex ante overvalued stocks since a higher value of *IVOL* is associated with greater ex ante mispricing, both undervaluation and overvaluation; and it is negatively related to $F_{t \to t+1}$ among ex post undervalued stocks and positively related to $F_{t \to t+1}$ among ex post overvalued stocks, since a higher value of *IVOL* is associated with greater ex post mispricing, both undervaluation and overvaluation. Figure 1 depicts the process through which *IVOL* is linked to $C_{t \to t+1}$ and $F_{t \to t+1}$.

Figure 1 shows that *IVOL*'s relation with $R_{t \to t+1}$ is a potentially biased estimate of its relation with expected return due to its relations with $C_{t \to t+1}$ and $F_{t \to t+1}$. To infer *IVOL*'s relation with expected return from its estimated relation with $R_{t \to t+1}$, the crucial task is to calibrate the sign and magnitude of its relation with the mispricing-related component ($C_{t \to t+1}$ and $F_{t \to t+1}$). However, this task is empirically unfeasible because all estimates of value are noisy and thus "we can never know how far away price is from value" (Black, 1986, p. 533). Nevertheless, *IVOL*'s relations with $C_{t \to t+1}$ and $F_{t \to t+1}$ suggest a testable hypothesis that, ceteris paribus, its estimated relation with $R_{t \to t+1}$ decreases with the proportion of ex ante overvalued observations in the sample and increases with the proportion of ex post overvalued observations in the sample.

Consistent with the hypothesis, we find robust evidence that *IVOL*'s estimated relation with $R_{t \to t+1}$ decreases and switches sign from positive to negative as the estimation sample consists of proportionately more ex ante overvalued observations and increases and switches sign from negative to positive as the estimation sample consists of proportionately more



Note(s): This figure depicts the process through which idiosyncratic volatility (*IVOL*) is linked to the mispricing-correction component (C*t*→*t*+1) and the mispricing-formation component (F*t*→*t*+1) of realized return (R*t*→*t*+1). ↑ and ↓ denote larger and smaller, respectively. *COV* denotes covariance. *LnPt* is the natural logarithm of the market value of equity at time *t* and *LnVt* is the natural logarithm of the intrinsic value of equity at time *t*. |*LnPt* - *LnVt*| denotes the absolute value of *LnPt* - *LnVt* and measures the degree of ex ante mispricing. |*LnPt*+1 - *LnVt*+1| denotes the absolute value of *LnPt*+1 - *LnVt*+1 and measures the degree of ex post mispricing. $|C_{t+1}|$ denotes the absolute value of C_{t+1}, and $|F_{t+1}|$ denotes the absolute value of F_{t+1}

Figure 1.
Idiosyncratic volatility and the mispricing-related component

ex post overvalued observations. Our finding suggests that we cannot draw a reliable inference about *IVOL*'s relation with expected return from its estimated relation with $R_{t\to t+1}$. Indeed, the sign switching for *IVOL*'s estimated relation with $R_{t\to t+1}$ suggests that its relations with $C_{t\to t+1}$ and $F_{t\to t+1}$ dominate its relation with expected return in its estimated relation with $R_{t\to t+1}$. If not, one would not observe such sign switching other than in the virtually inconceivable situation that its relation with expected return varies similarly in response to change in the proportion of ex ante (ex post) overvalued observations in the sample. One thus cannot infer the sign – let alone the magnitude – of *IVOL*'s relation with expected return from its estimated relation with $R_{t\to t+1}$. Evidently, the documented negative relation between *IVOL* and $R_{t\to t+1}$ does not necessarily contradict the predictions of traditional asset pricing theories and hence may not be an asset pricing puzzle. Moreover, we show that existing methods cannot address the bias. In sum, the estimated relation of idiosyncratic volatility (*IVOL*) with realized return ($R_{t\to t+1}$) is an inadequate indicator of its relation with expected return.

Our findings demonstrate that *IVOL*'s estimated relation with $R_{t\to t+1}$ is inherently unstable due to the nonlinearity of its relations with $C_{t\to t+1}$ and $F_{t\to t+1}$. We reason that ostensibly immaterial variation in research design choices can cause *IVOL*'s estimated relation with $R_{t\to t+1}$ to vary dramatically and even switch sign due to their effects on its estimated relations with $C_{t\to t+1}$ and $F_{t\to t+1}$. First, a research design choice can change the proportions of ex ante and ex post undervalued and overvalued observations in the sample. Second, it can change the weight of ex ante significantly undervalued observations and ex post significantly overvalued observations in *IVOL*'s estimated relation with $R_{t\to t+1}$ due to its effect on the distribution of $R_{t\to t+1}$. A well-known regularity about $R_{t\to t+1}$ is that it is right-skewed. We show that large positive returns driving the right-skewness of $R_{t\to t+1}$ stem from the correction of ex ante significant undervaluation and the formation of ex post significant overvaluation to a large extent. This is not surprising because $C_{t\to t+1}$ for the correction of ex ante undervaluation and $F_{t\to t+1}$ for the formation of ex post overvaluation can be very large, whereas $C_{t\to t+1}$ for the correction of ex ante overvaluation and $F_{t\to t+1}$ for the formation of ex post undervaluation are at most -1.

We show that *IVOL*'s estimated relation with $R_{t\to t+1}$ is sensitive to ostensibly immaterial variation in oft-employed research design choices in a predictable manner consistent with our reasoning. Prior studies have generally excluded observations with very low price, arguing that the price movements of those stocks are susceptible to microstructure biases. However, these studies may differ regarding the price threshold for exclusion. Moreover, these studies rarely specify the timing for measuring stock price. We show that *IVOL*'s estimated relation with $R_{t\to t+1}$ varies dramatically and even switches sign in a predictable manner in response to ostensibly immaterial variation in the price threshold for exclusion ($0 vs $1 vs $5) and in the timing of measuring stock price (at the beginning and at the end of the holding period and average stock price for the holding period). Prior studies may also exclude small stocks, arguing that data quality of those stocks is low due to their illiquidity and ensuing noise in their stock pricing and that their economic significance is trivial. In practice, studies have great discretion over the size threshold for exclusion and rarely specify the timing of measuring size. We show that *IVOL*'s estimated relation with $R_{t\to t+1}$ varies dramatically and even switches sign in a predictable manner in response to variation in the size threshold for exclusion and in the timing of measuring firm size.

Our study contributes to research on the relation between idiosyncratic volatility (*IVOL*) and realized return ($R_{t\to t+1}$) in at least two aspects. First, our study shows that because of its dual effect on stock pricing, *IVOL*'s estimated relation with $R_{t\to t+1}$ captures both its relation with expected return and its relations with the mispricing-correction component ($C_{t\to t+1}$) and the mispricing-formation component ($F_{t\to t+1}$). Hence its estimated relation with $R_{t\to t+1}$ is an inadequate indicator of its relation with expected return. Indeed, our findings suggest that its relations with $C_{t\to t+1}$ and $F_{t\to t+1}$ dominate its relation with expected return in its estimated relation with $R_{t\to t+1}$. One thus cannot infer the sign of its relation with expected return from its estimated relation with $R_{t\to t+1}$. Evidently, the negative relation between idiosyncratic volatility and realized return documented in some studies does not necessarily contradict the prediction of traditional asset pricing theories and hence may not be an asset pricing puzzle.

Using a different ex ante overvaluation likelihood measure, Stambaugh, Yu, and Yuan (2015) also find that *IVOL*'s estimated relation with $R_{t\to t+1}$ decreases and switches sign from positive to negative as the estimation is moved from the bottom to the top quintile of their ex ante overvaluation likelihood measure. Our study complements and extends their study in at least two aspects. First, using a different but arguably superior ex ante overvaluation likelihood measure, our study affirms their major finding. More importantly, our study shows that the relation of *IVOL* with the mispricing-formation component also plays a significant role in shaping the overall relation between *IVOL* and realized return.

Second, our study sheds light on the inconsistency of findings about *IVOL*'s estimated relation with $R_{t\to t+1}$. Our study shows that ostensibly immaterial variations in oft-employed research design choices can cause *IVOL*'s estimated relation with $R_{t\to t+1}$ to vary dramatically and even switch sign in a predictable manner due to their effects on its estimated relations with $C_{t\to t+1}$ and $F_{t\to t+1}$. That is, our study shows that variations in research design choices drive the inconsistency of findings across studies. Extending prior studies that expose the sensitivity of *IVOL*'s estimated relation with $R_{t\to t+1}$ to variations in research design choices, our study provides a conceptual framework for understanding how and why *IVOL*'s estimated relation with $R_{t\to t+1}$ varies with research design choices.

The rest of the paper is organized as follows. Section 2 describes the dual effect of idiosyncratic volatility on stock pricing and develops the testable hypothesis. Section 3 presents the research design. Section 4 reports and discusses results from the main test and robustness tests. Section 5 reports and discusses results from additional analyses. Section 6 concludes.

2. Research hypothesis

2.1 The dual effect of idiosyncratic volatility on stock pricing

Idiosyncratic volatility (*IVOL*) has a dual effect on stock pricing. First, it can shape stock pricing by affecting investors' expected return. Assuming that markets are complete and frictionless and investors are well-diversified, the classic capital asset pricing model (CAPM) implies no relation between a stock's expected return and its *IVOL* because no factors other than beta capture the cross-section variation in expected returns under CAPM. However, theories built on the realistic assumption of limited portfolio diversification imply a positive relation between a stock's expected return and its *IVOL* because total risks – including idiosyncratic risks – matter to investors with limited portfolio diversification (Levy, 1978; Merton, 1987). If investors expect a high return for holding a stock with high *IVOL*, the high expected return is equivalent to a low stock price given expected future cash flows available to stockholders.

Second, *IVOL* can shape stock pricing by affecting the efficiency of stock price in reflecting its underlying equity value. *IVOL* is positively related to arbitrage risks (Pontiff, 2006; Stambaugh *et al.*, 2015). For arbitrageurs who can neutralize their exposure to benchmark risks, *IVOL* is more closely related to arbitrage risk than total volatility (Stambaugh *et al.*, 2015). Pontiff (2006) shows that a mean-variance investor's desired position size for a given level of mispricing is smaller when a stock's *IVOL* is higher because higher *IVOL* is associated with a higher likelihood of substantial adverse price moves. Evidently, *IVOL* is positively related to the degree of mispricing, both ex ante and ex post and both undervaluation and overvaluation.

Consistent with firms with high *IVOL* having low stock pricing efficiency, we find that *IVOL* is negatively related to the informational efficiency of stock price, as gauged by the absolute value of the first-order autocorrelation of daily returns (*AbsAutoCorr*) ad price delay (*PriceDelay*) (see Figure IA1 of the Internet Appendix). A smaller *AbsAutoCorr* indicates that the pricing process is closer to a random walk, making stock price more informationally efficient (Chordia, Roll & Subrahmanyam, 2008). *PriceDelay* captures the average delay of price movements in response to information (Hou & Moskowitz, 2005) [2]. A larger *PriceDelay* indicates greater information delay.

In sum, *IVOL* has a dual effect on stock pricing by affecting both investors' expected return and stock pricing efficiency. Next, we elaborate how this dual effect shapes the relation between idiosyncratic volatility and realized return.

2.2 Hypothesis development

We reason that the estimated relation of idiosyncratic volatility (*IVOL*) with realized return, due to its dual effect on stock pricing, is a potentially biased estimate of its relation with expected return. Our reasoning builds on two interrelated regularities. The first is that the price of a stock can deviate substantially from its value due to the speculative nature of stock price (Black, 1986; Shiller, 2014). This seems to be the norm rather than the exception, as observed and presented to members of American Finance Association by Fischer Black in his 1985 presidential address (Black, 1986) [3]. Fischer Black based his observation on his hands-on experience outside the academic ivory tower [4].

Hence realized return – the outcome of speculative stock price movements – consists of a mispricing-related component in addition to expected return (Black, 1986; Elton, 1999; Shiller, 2014). The mispricing-related component stems from the correction of ex ante mispricing, the formation of ex post mispricing, or both. Realized return ($R_{t\to t+1}$) from time t to time $t+1$ thus can be decomposed into four components: expected return, a mispricing-correction component ($C_{t\to t+1}$), a mispricing-formation component ($F_{t\to t+1}$), and anything else ($O_{t\to t+1}$).

$$R_{t\to t+1} = E_t(R_{t\to t+1}) + C_{t\to t+1} + F_{t\to t+1} + O_{t\to t+1} \tag{1}$$

where $R_{t\to t+1}$ is realized return from time t to time $t+1$; $E_t(R_{t\to t+1})$ is expected return given information at time t; $C_{t\to t+1}$ is the mispricing-related component that stems from the correction of ex ante mispricing; $F_{t\to t+1}$ is the mispricing-related component that stems from the formation of ex post mispricing; and $O_{t\to t+1}$ is the component of $R_{t\to t+1}$ other than $E_t(R_{t\to t+1})$, $C_{t\to t+1}$, and $F_{t\to t+1}$. For the correction of ex ante undervaluation, $C_{t\to t+1}$ is positive and increases with the degree of ex ante undervaluation corrected, while for the correction of ex ante overvaluation, it is negative and decreases with the degree of ex ante overvaluation corrected; for the formation of ex post undervaluation, $F_{t\to t+1}$ is negative and decreases with the degree of ex post undervaluation formed, while for the formation of ex post overvaluation, it is positive and increases with the degree of ex post overvaluation formed.

The second regularity is that because $IVOL$ is positively related to the degree of mispricing, both ex ante and ex post and both undervaluation and overvaluation, its estimated relation with $R_{t\to t+1}$ captures both its relation with $E_t(R_{t\to t+1})$ ant its relations with $C_{t\to t+1}$ and $F_{t\to t+1}$. Without loss of generality, we formalize the second regularity in the following equations:

$$E_t(R_{t\to t+1}) = \delta + \beta * IVOL + \varepsilon_{t\to t+1},\ E(\varepsilon_{t\to t+1}) = 0,\ E(\varepsilon_{t\to t+1}IVOL) = 0, \tag{2}$$

$$R_{t\to t+1} = \delta + \beta\ *\ IVOL + \varepsilon_{t\to t+1} + C_{t\to t+1} + F_{t\to t+1} + O_{t\to t+1} \tag{3}$$

$$plim\widehat{\beta} = \beta + \frac{COV(C_{t\to t+1},\ IVOL)}{Var(IVOL)} + \frac{COV(F_{t\to t+1},\ IVOL)}{Var(IVOL)} + \frac{COV(O_{t\to t+1},\ IVOL)}{Var(IVOL)} \tag{4}$$

where $plim$ is the probability limit operator, $COV(\cdot)$ denotes covariance, and $Var(\cdot)$ denotes variance. In Equation (2), we presume that there is a linear relation between $IVOL$ and $E_t(R_{t\to t+1})$, which is captured by β. $\varepsilon_{t\to t+1}$ is the residual with a mean of 0. We get Equation (3) by expanding $E_t(R_{t\to t+1})$ in Equation (1) according to Equation (2). Equation (3) can be viewed as a simplified version of the model specification that we estimate to examine the relation (β) between $IVOL$ and expected return $E_t(R_{t\to t+1})$. In Equation (4), $\widehat{\beta}$ is the OLS estimate of β.

Assuming that $COV(O_{t\to t+1},\ IVOL) = 0$, we get

$$plim\widehat{\beta} = \beta + \frac{COV(C_{t\to t+1},\ IVOL)}{Var(IVOL)} + \frac{COV(F_{t\to t+1},\ IVOL)}{Var(IVOL)} \tag{5}$$

Equation (5) shows that $\widehat{\beta}$ is a potentially biased estimate of β due to $COV(C_{t\to t+1},\ IVOL)$ and $COV(F_{t\to t+1},\ IVOL)$. Worse, the sign of $COV(C_{t\to t+1},\ IVOL)$ and $COV(F_{t\to t+1},\ IVOL)$ is indeterminate. Have LnP_t denote the natural logarithm of market value of equity at time t and LnV_t denote the natural logarithm of the intrinsic value of equity at time t. Then, $|LnP_t - LnV_t|$ measures the degree of ex ante mispricing at time t and $|LnP_{t+1} - LnV_{t+1}|$ measures the degree of ex post mispricing at time $t+1$. Because $IVOL$ is positively related to the degree of mispricing, both ex ante and ex post, we have $COV(IVOL,\ |LnP_t - LnV_t|) > 0$ and $COV(IVOL,\ |LnP_{t+1} - LnV_{t+1}|) > 0$. Therefore,

$$IVOL\uparrow \to |LnP_t - LnV_t|\uparrow \to |C_{t\to t+1}|\uparrow$$

If $LnP_t - LnV_t < 0$ (i.e. ex ante undervalued), $C_{t\to t+1}\uparrow(>0)$. Therefore, $COV(C_{t\to t+1},\ IVOL|LnP_t - LnV_t < 0) > 0$.

If $LnP_t - LnV_t > 0$ (i.e. ex ante overvalued), $C_{t\to t+1}\downarrow(<0)$. Therefore, $COV(C_{t\to t+1},\ IVOL|LnP_t - LnV_t < 0) < 0$.
and

$$IVOL\uparrow \to |LnP_{t+1} - LnV_{t+1}|\uparrow \to |F_{t\to t+1}|\uparrow$$

If $LnP_{t+1} - LnV_{t+1} < 0$ (i.e. ex post undervalued), $F_{t\to t+1}\downarrow(<0)$. Therefore, $COV(F_{t\to t+1},\ IVOL|LnP_{t+1} - LnV_{t+1} < 0) < 0$.

If $LnP_{t+1} - LnV_{t+1} > 0$ (i.e. ex post overvalued), $F_{t\to t+1}\uparrow(>0)$. Therefore, $COV(F_{t\to t+1}, IVOL|LnP_{t+1} - LnV_{t+1} > 0) > 0$.
where $\uparrow$ denotes larger; $\downarrow$ denotes smaller; and $|x|$ denotes the absolute value of x.

It is so far evident that $IVOL$'s covariance with $C_{t\to t+1}$ and with $F_{t\to t+1}$ causes its estimated relation ($\widehat{\beta}$) with $R_{t\to t+1}$ to be a potentially biased estimate of its relation (β) with expected return ($E_t(R_{t\to t+1})$). To draw a reliable inference about $IVOL$'s relation (β) with $E_t(R_{t\to t+1})$ from its estimated relation ($\widehat{\beta}$) with $R_{t\to t+1}$, we need to calibrate the sign and magnitude of the bias resulting from its covariance with $C_{t\to t+1}$ ($COV(C_{t\to t+1}, IVOL)$) and with $F_{t\to t+1}$ ($COV(F_{t\to t+1}, IVOL)$). However, it is empirically unfeasible to do so, since "we can never know how far away price is from value" (Black, 1986, p. 533). Nevertheless, Equation (5) suggests a testable hypothesis that, ceteris paribus, $IVOL$'s estimated relation with $R_{t\to t+1}$ decreases with the proportion of ex ante overvalued observations in the sample and increases with the proportion of ex post overvalued observations in the sample.

Next, we test this hypothesis. Finding evidence supporting the hypothesis will cast doubt on the appropriateness of inferring $IVOL$'s relation with expected return from its estimated relation with $R_{t\to t+1}$. Moreover, if we ever find sign switching for $IVOL$'s estimated relation with $R_{t\to t+1}$ – from positive (negative) to negative (positive) as the estimation sample consists of proportionately more ex ante (ex post) overvalued observations – we know that $IVOL$'s relations with $C_{t\to t+1}$ and $F_{t\to t+1}$ dominate its relation with expected return in its estimated relation with $R_{t\to t+1}$. If not, we would not observe such sign switching other than in the virtually inconceivable situation that $IVOL$'s relation with expected return varies and even switches sign similarly in response to change in the proportion of ex ante (ex post) overvalued observations in the sample. Hence one cannot infer the sign – let alone the magnitude – of $IVOL$'s relation with expected return from its estimated relation with $R_{t\to t+1}$.

3. Research design

3.1 Measuring overvaluation likelihood

The crucial task for the hypothesis testing is to measure the overvaluation likelihood of observations. Following Rhodes-Kropf, Robinson, and Viswanathan (2005), we use the difference between the natural logarithm of the market value of equity and the natural logarithm of the estimated intrinsic value of equity as our primary measure of overvaluation likelihood (hereafter LnP/V). Detailed in Appendix 1, this estimated intrinsic value of equity (log) is a function of accounting items; the coefficients are rolling time-series averages of annual estimates. Rhodes–Kropf *et al.* (2005) provide initial evidence for the validity of LnP/V as an overvaluation likelihood measure, showing that the pattern of merger and acquisition activities varies with LnP/V as theoretically predicted. We further show that observations in the top and bottom LnP/V quintiles differ systematically along dimensions that prior studies find vary with observations' valuation status (i.e. undervalued or overvalued).

Any estimation of a stock's valuation status is subject to the misclassification problem, since "all estimates of value are noisy" (Black, 1986, p. 533). But that may not be a concern in our research setting since misclassification only runs against finding evidence for the hypothesis. More importantly, what we need in our setting is an instrument that reasonably captures the relative overvaluation likelihood rather than the exact valuation status. LnP/V seems well suited, since the comparison results reported in Appendix 1, together with Rhodes–Kropf *et al.* (2005) finding, demonstrate that observations with larger LnP/V are more likely to be overvalued than observations with smaller LnP/V. Moreover, we show that our inference is robust to the use of two alternative measures of overvaluation likelihood.

3.2 Measuring idiosyncratic volatility

Our primary measure of idiosyncratic volatility (*IVOL*) is the standard deviation of residuals from a regression that takes daily excess returns as a function of daily excess market returns and daily returns to the small-minus-big, high-minus-low, momentum, robust-minus-weak, and conservative-minus-aggressive factors. *IVOL* is computed using daily data from 07/01 of $t-1$ to 06/30 of t, $t = 1966$ to 2015. Moreover, we show that our inference is robust to two alternative ways of computing *IVOL*.

3.3 The regression model for hypothesis testing

The regression model for the hypothesis testing is:

$$R_{i,t\to t+1} = Intercept + \lambda_1 * IVOL_{i,t} + \sum_{j=2}^{j=5} \lambda_j * IVOL_{i,t} * LnP/V(t){:}\ Q_j + \sum_{j=2}^{j=5} \delta_j * LnP/V(t){:}\ Q_j + Controls + Industry\,FE + \varepsilon_{i,t\to t+1} \quad (6a)$$

$$R_{i,t\to t+1} = Intercept + \lambda_1 * IVOL_{i,t} + \sum_{j=2}^{j=5} \lambda_j * IVOL_{i,t} * LnP/V(t+1){:}\ Q_j + \sum_{j=2}^{j=5} \delta_j * LnP/V(t+1){:}\ Q_j + Controls + Industry\,FE + \varepsilon_{i,t\to t+1} \quad (6b)$$

where i is firm i and t is year t; $R_{i,t\to t+1}$ is stock return over 07/01 of t through 06/30 of $t+1$, $t = 1966$ to 2015; *IVOL* is the idiosyncratic volatility measure; $LnP/V(t)$ ($LnP/V(t+1)$) is the difference between the natural logarithm of the market value of equity on 06/30 of t ($t+1$) and the natural logarithm of the estimated intrinsic value of equity obtained using the latest accounting information available by 06/30 of t ($t+1$) (see Appendix 1); and $LnP/V(t){:}\ Q_j$ ($LnP/V(t+1){:}\ Q_j$) is an indicator variable that equals 1 if $LnP/V(t)$ ($LnP/V(t+1)$) is in the j-th quintile (0 otherwise), $j = 1$ to 5, in which we sort observations with no missing values for $R_{i,t\to t+1}$, *IVOL* and $LnP/V(t)$ ($LnP/V(t+1)$) into five equal groups by $LnP/V(t)$ ($LnP/V(t+1)$) on 06/30 of t ($t+1$). Consistent with the hypothesis, the construction of $LnP/V(t+1)$ uses the latest information available by 06/30 of $t+1$. This is not an issue since we are proposing any trading strategy. Instead, we examine what underlies the estimated relation between idiosyncratic volatility and realized return.

Controls stands for control variables. We refer to Fama and French (2008) to identify them. Specifically, we control for these equity attributes: firm size (*Size*), the book-to-market ratio (*B/M*), momentum (*Momentum*), net stock issues (*NetStkIssue*), zero net stock issues (*ZeroNetStkIssue*), negative total accruals (*NegTtlAcc*), positive total accruals (*PosTtlAcc*), asset growth (*AssetGrowth*), positive profitability (*PosIB/BE*) and loss (*NegIB*). Definitions of these control variables are provided in Appendix 2.

Because firms differing in idiosyncratic volatility may differ in their exposure to systematic risk factors, we control for firms' sensitivity to six risk factors identified in Carhart (1997) and Fama and French (2015). Specifically, we control for *Beta-MktRf*, *Beta-SMB*, *Beta-HML*, *Beta-MOM*, *Beta-CMA*, and *Beta-RMW*; these are factor loadings on the market factor, the small-minus-big factor (*SMB*), the high-minus-low factor (*HML*), the momentum factor (*MOM*), the conservative-minus-aggressive factor (CMA), and the robust-minus-weak factor (*RMW*), respectively. Because expected return varies across industries (Fama & French, 1997), we also control for industry fixed effects, *Industry FE*. We define industry membership according to the Fama-French 49 industries.

We apply the Fama-MacBeth regression to estimate Equations (6a) and (6b), since it is "standard in tests of asset pricing models" (Fama, 2014, p. 1478). Our Fama-MacBeth regression estimate is the time-series average of annual OLS coefficient estimates (Fama & MacBeth, 1973). We use standard errors adjusted for Newey-West autocorrelations of three lags to compute T-statistics.

The hypothesis predicts that in Equation (6a), $\lambda_5 < 0$ and $\lambda_1 > (\lambda_1 + \lambda_5)$, while in Equation (6B), $\lambda_5 > 0$ and $\lambda_1 < (\lambda_1 + \lambda_5)$. We focus on the contrast between observations in the top and the bottom LnP/V quintiles because nP/V, as a noisy measure of the price-to-value ratio, is arguably better able to differentiate the valuation status for observations in these two quintiles than for observations in the middle three quintiles. While we do not expect a monotonic transition from λ_1 to $\lambda_1 + \lambda_5$, we expect $\lambda_1 + \lambda_j$, $j = 2, 3, 4,$ to be between λ_1 and $\lambda_1 + \lambda_5$.

3.4 Data, sample and descriptive statistics

We obtain accounting data from Compustat; equity data from CRSP; and the Fama-French industry group classifications and factor return data, including risk-free rates, from Kenneth R. French's online data library. To mitigate the concern about data snooping and mining, we use all firm-year observations with the required variables available.

Table 1 reports descriptive statistics of variables for the sample used in the main test. The sample consists of 180,717 firm-year observations from 1966 through 2015 [5]. We winsorize all continuous variables except $R_{t\to t+1}$ at the 1st and 99th percentiles of their cross-sectional distributions each year. Summary statistics reported in Panel A are comparable with those reported in prior studies.

Panel B reports Pearson and Spearman correlations. Three sets of correlations deserve attention. First, the correlations of $R_{t\to t+1}$ with control variables are consistent with findings of prior studies. Second, $LnP/V(t)$ and $LnP/V(t+1)$ are significantly positively correlated: 0.73 (Pearson) and 0.70 (Spearman), suggesting that a firm's overvaluation likelihood is persistent over time. Third, consistent with $LnP/V(t)$ measuring the ex ante overvaluation likelihood $LnP/V(t)$ and $R_{t\to t+1}$ are negatively correlated: -0.10 (Pearson) and -0.11 (Spearman); consistent with $LnP/V(t+1)$ measuring the ex post overvaluation likelihood $LnP/V(t+1)$ and $R_{t\to t+1}$ are positively correlated: 0.30 (Pearson) and 0.33 (Spearman).

4. Results

4.1 Main results

Table 2 presents results from the main test of the hypothesis. Table 2 shows that the coefficient estimates for $LnP/V(t)$: Q_j ($LnP/V(t+1)$: Q_j), $j = 2$ to 5, are significantly negative (positive) and increase in magnitude. The transition pattern of these coefficient estimates is consistent with the notion that observations with larger LnP/V are more likely to be overvalued than observations with smaller LnP/V.

When the model specification ignores that $IVOL$'s estimated relation with $R_{t\to t+1}$ varies with the proportion of ex ante (ex post) overvalued observations in the sample, the overall estimated relation is positive but statistically insignificant [6]. This appears to be consistent with the finding of some prior studies (e.g. Fama & MacBeth, 1973). Panel A shows that with(out) control variables, $IVOL$'s estimated relation with $R_{t\to t+1}$ monotonically decreases from 2.9027 ($t = 3.47$) (2.9829 ($t = 2.52$)) to -1.9359 ($t = -1.85$) (-1.6024 ($t = -1.14$)) as the estimation is moved from the bottom $LnP/V(t)$ quintile to the top one; Panel B shows that with(out) control variables, $IVOL$'s estimated relation with $R_{t\to t+1}$ monotonically increases from -1.8086 ($t = -3.95$) (-1.8627 ($t = -2.49$)) to 9.1157 ($t = 6.11$) (9.9579 ($t = 5.84$)) as the estimation is moved from the bottom $LnP/V(t+1)$ quintile to the top one. These results support the hypothesis.

Table 1. Descriptive statistics

Panel A: Summary statistics

Variable	Mean	SD	P25	P50	P75
$R_{t\rightarrow t+1}$	0.1594	0.6691	−0.1816	0.0718	0.3520
IVOL	0.0288	0.0180	0.0162	0.0240	0.0361
LnP/V(t)[a]	0.1942	0.9062	−0.3186	0.2122	0.7165
LnP/V $(t+1)$	0.1639	0.9107	−0.3429	0.1934	0.6963
Size	4.9194	2.1750	3.3185	4.8055	6.4364
B/M	−0.4667	0.9514	−1.0013	−0.4156	0.1031
Momentum	0.1506	0.5343	−0.1594	0.0742	0.3372
ZeroNetStkIssue	0.1360	0.3428	0.0000	0.0000	0.0000
NetStkIssue	0.0688	0.2414	0.0000	0.0047	0.0358
NegTtlAcc	−0.0205	0.0575	−0.0134	0.0000	0.0000
PosTtlAcc	0.0292	0.0558	0.0000	0.0000	0.0369
AssetGrowth	0.0733	0.2921	−0.0208	0.0671	0.1666
NegIB	0.2124	0.4090	0.0000	0.0000	0.0000
PosIB/BE	0.1101	0.1131	0.0214	0.0975	0.1547
Beta-MktRf	0.8762	0.6376	0.4677	0.8691	1.2504
Beta-SMB	0.6840	0.8426	0.1335	0.6071	1.1611
Beta-HML	0.1716	1.1155	−0.3786	0.2014	0.7704
Beta-MOM	−0.0493	0.7111	−0.3874	−0.0349	0.3003
Beta-CMA	0.0168	1.2006	−0.5810	0.0249	0.6148
Beta-RMW	−0.0893	1.2275	−0.6540	−0.0112	0.5555

Panel B: Correlations

Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
$R_{t\rightarrow t+1}$ (1)		*0.04*	*−0.10*	*0.30*	*−0.08*	*0.08*	*−0.03*	*0.02*	*−0.05*	*−0.02*	*−0.02*	*−0.05*	0.00	*0.01*	*−0.02*	*0.01*	*0.02*	*−0.01*
IVOL (2)	*−0.10*		*−0.12*	*−0.12*	*−0.47*	*−0.03*	*−0.01*	*0.08*	*0.15*	*−0.22*	*0.15*	*−0.11*	*0.43*	*−0.24*	*0.05*	*0.20*	*−0.04*	*−0.07*
LnP/V(t) (3)	*−0.11*	*−0.12*		*0.73*	*0.45*	*−0.82*	*0.32*	*−0.18*	*0.11*	*0.01*	*0.02*	*0.11*	*0.04*	*0.10*	*0.14*	*0.01*	*−0.07*	*0.08*
LnP/V(t + 1) (4)	*0.33*	*−0.14*	*0.70*		*0.39*	*−0.65*	*0.18*	*−0.16*	*0.03*	0.00	0.00	*0.05*	*−0.01*	*0.14*	*0.11*	0.00	*−0.06*	*0.04*
Size (5)	0.00	*−0.51*	*0.49*	*0.43*		*−0.32*	*0.12*	*−0.29*	0.00	*0.14*	*−0.17*	*0.08*	*−0.21*	*0.21*	*0.20*	*−0.22*	*−0.02*	*0.04*
B/M (6)	*0.13*	*−0.03*	*−0.79*	*−0.60*	*−0.35*		*−0.33*	*0.18*	*−0.13*	*0.05*	*−0.12*	*−0.12*	*−0.03*	*−0.30*	*−0.12*	*−0.05*	*0.12*	*−0.11*

(*continued*)

Table 1.

Panel B: Correlations

Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
Momentum (7)	*0.02*	*−0.16*	*0.33*	*0.19*	*0.19*	*−0.31*		0.00	*−0.01*	*−0.01*	*−0.01*	*0.05*	*−0.09*	*0.10*	*0.05*	*0.05*	*0.01*	*0.13*
ZeroNetStkIssue (8)	*0.02*	*0.07*	*−0.19*	*−0.18*	*−0.29*	*0.20*	0.00		*−0.11*	*−0.02*	*−0.01*	*−0.05*	*0.03*	*−0.03*	*−0.11*	*0.01*	*0.03*	*−0.02*
NetStkIssue (9)	*−0.10*	*0.15*	*0.19*	*0.11*	*0.07*	*−0.23*	*−0.03*	*−0.30*		*−0.23*	*0.20*	*−0.27*	*0.15*	*−0.11*	*0.04*	*0.05*	*−0.04*	*0.03*
NegTtlAcc (10)	*0.01*	*−0.16*	0.00	0.00	*0.07*	*0.02*	*−0.01*	*−0.02*	*−0.04*		*0.19*	*0.33*	*−0.22*	*0.09*	*−0.01*	*−0.06*	*0.02*	*0.02*
PosTtlAcc (11)	*−0.05*	*0.13*	0.00	*−0.01*	*−0.12*	*−0.10*	*−0.05*	*−0.02*	*0.03*	*0.63*		*0.08*	0.00	*0.07*	*0.05*	*0.09*	*−0.05*	*0.01*
AssetGrowth (12)	*−0.03*	*−0.09*	*0.11*	*0.07*	*0.11*	*−0.19*	*0.07*	*−0.08*	*−0.02*	*0.24*	*0.25*		*−0.23*	*0.19*	*0.07*	*0.02*	*−0.03*	*0.06*
NegIB (13)	*−0.08*	*0.40*	*0.03*	*−0.01*	*−0.20*	0.00	*−0.16*	*0.03*	*0.13*	*−0.21*	*−0.07*	*−0.32*		*−0.51*	*−0.01*	*0.07*	*−0.04*	*−0.07*
PosIB/BE (14)	*0.07*	*−0.33*	*0.06*	*0.11*	*0.24*	*−0.26*	*0.17*	*−0.07*	*−0.10*	*0.17*	*0.13*	*0.39*	*−0.71*		*0.06*	*−0.05*	*−0.02*	*0.07*
Beta-MktRf (15)	*−0.04*	*0.11*	*0.17*	*0.13*	*0.23*	*−0.15*	*0.01*	*−0.12*	*0.11*	*−0.02*	*0.07*	*0.08*	*−0.02*	*0.08*		*0.49*	*0.23*	*−0.09*
Beta-SMB (16)	*−0.03*	*0.26*	0.00	*−0.02*	*−0.20*	*−0.04*	0.00	0.00	*0.08*	*−0.03*	*0.09*	*0.02*	*0.06*	*−0.04*	*0.45*		*0.24*	*−0.08*
Beta-HML (17)	*0.02*	*−0.04*	*−0.08*	*−0.07*	*−0.02*	*0.14*	*0.02*	*0.03*	*−0.04*	*0.03*	*−0.06*	*−0.04*	*−0.04*	*−0.01*	*0.19*	*0.23*		*0.01*
Beta-MOM (18)		*−0.05*	*0.09*	*0.04*	*0.03*	*−0.11*	*0.14*	*−0.02*	*0.05*	*0.01*	*0.01*	*0.07*	*−0.06*	*0.09*	*−0.07*	*−0.06*	*0.03*	
Beta-CMA (19)	*0.01*	0.00	*−0.01*	*−0.01*	*−0.04*	*0.02*	*0.01*	*0.02*	*−0.05*	*−0.03*	0.00	*−0.06*	*0.03*	*−0.04*	*0.02*	*0.02*	*−0.31*	0.00
Beta-RMV (20)	*0.04*	*−0.12*	*−0.02*	*0.01*	0.00	*0.03*	*0.03*	*0.02*	*−0.08*	*0.03*	0.00	*0.02*	*−0.11*	*0.11*	*0.09*	*0.25*	*0.32*	*−0.04*

Note(s): This table presents descriptive statistics of variables for the 180,717 firm-year observations used in the main test. The sample period is 1966 through 2015. Panel A presents summary statistics. In Panel B, Pearson (Spearman) correlations are in the upper (lower) triangle. Correlations that are significantly different from 0 at p-value < 5% are in italicface. Variable definitions are provided in Appendix 2. All continuous variables except $R_{t \to t+1}$ are winsorized at the 1st and 99th percentiles of their cross-sectional distributions each year

[a]The means of *LnP/V(t)* and *LnP/V* $(t+1)$ are zero by construction. Their reported means here are different from zero because we calculate both using the largest possible sample, but our final sample is smaller after requiring the availability of control variables

Table 2. Idiosyncratic volatility and return

Panel A: Idiosyncratic volatility and the correction of ex ante mispricing

Variable	(1)	(2)	(3)	(4)	(5)	(6)
IVOL (λ_1)	0.8626 (0.99)	0.7017 (0.78)	2.9829* (2.52)	0.8143 (1.39)	0.7622 (1.34)	2.9027** (3.47)
*IVOL * LnP/V(t):Q2* (λ_2)			−2.3642** (−3.02)			−2.5912** (−4.56)
*IVOL * LnP/V(t):Q3* (λ_3)			−2.5416** (−3.28)			−2.9431** (−5.22)
*IVOL * LnP/V(t):Q4* (λ_4)			−3.1003** (−4.54)			−3.8869** (−7.66)
*IVOL * LnP/V(t):Q5* (λ_5)			−4.5853** (−6.02)			−4.8386** (−7.65)
LnP/V(t):Q2		−0.0475** (−5.86)	−0.0311** (−3.57)		−0.0304** (−4.20)	−0.0147* (−2.01)
LnP/V(t):Q3		−0.0601** (−6.45)	−0.0462** (−4.96)		−0.0317** (−4.19)	−0.0177* (−2.56)
LnP/V(t):Q4		−0.0751** (−5.90)	−0.0622** (−4.94)		−0.0391** (−3.62)	−0.0268** (−2.92)
LnP/V(t):Q5		−0.1075** (−5.95)	−0.0923** (−5.28)		−0.0508** (−3.73)	−0.0385** (−3.11)
N	187,231	187,231	187,231	180,717	180,717	180,717
R^2	0.0313	0.0432	0.0471	0.1267	0.1278	0.1306
Controls	No	No	No	Yes	Yes	Yes
Industry FE	No	No	No	Yes	Yes	Yes
$\lambda_1 + \lambda_2$			0.6187 (0.44)			0.3115 (0.31)
$\lambda_1 + \lambda_3$			0.4413 (0.31)			−0.0404 (−0.04)
$\lambda_1 + \lambda_4$			−0.1173 (−0.09)			−0.9842 (−1.01)
$\lambda_1 + \lambda_5$			−1.6024 (−1.14)			−1.9359† (−1.85)

Panel B: Idiosyncratic volatility and the formation of ex post mispricing

Variable	(1)	(2)	(3)	(4)
IVOL (λ_1)	2.1212* (2.50)	−1.8672* (−2.49)	0.7895 (1.50)	−1.8086** (−3.95)
*IVOL * LnP/V(t + 1):Q2* (λ_2)		2.0528** (2.84)		1.4121** (3.56)
*IVOL * LnP/V(t + 1):Q3* (λ_3)		2.8230** (4.03)		1.6470** (3.64)
*IVOL * LnP/V(t + 1):Q4* (λ_4)		5.2171** (7.54)		3.4584** (6.66)
*IVOL * LnP/V(t + 1):Q5* (λ_5)		11.8252** (7.72)		10.9243** (7.70)
LnP/V(t + 1):Q2	0.1576** (10.02)	0.1282** (9.69)	0.2946** (9.14)	0.2723** (9.48)

(*continued*)

Table 2.

Panel B: Idiosyncratic volatility and the formation of ex post mispricing

Variable	(1)	(2)	(3)	(4)
LnP/V(t + 1):Q3	0.2528** (11.95)	0.2218** (12.39)	0.4676** (9.80)	0.4389** (10.34)
LnP/V(t + 1):Q4	0.3418** (13.80)	0.3170** (14.38)	0.6385** (10.90)	0.6103** (11.59)
LnP/V(t + 1):Q5	0.5330** (10.95)	0.5106** (10.84)	0.9480** (10.42)	0.9209** (10.61)
N	187,231	187,231	180,717	180,717
R^2	0.1283	0.1441	0.2915	0.3032
Controls	No	No	Yes	Yes
Industry FE	No	No	Yes	Yes
$\lambda_1 + \lambda_2$		0.1855 (0.18)		−0.3965 (−0.65)
$\lambda_1 + \lambda_3$		0.9558 (0.93)		−0.1616 (−0.25)
$\lambda_1 + \lambda_4$		3.3498** (3.29)		1.6498* (2.38)
$\lambda_1 + \lambda_5$		9.9579** (5.84)		9.1157** (6.11)

Note(s): This table presents results of the main test. The dependent variable ($R_{t \to t+1}$) is stock return over 07/01 of t to 06/30 of $t + 1$, $t = 1966$ to 2015. *IVOL* is the idiosyncratic volatility measure, defined in Appendix 2. $LnP/V(t)$ is the difference between the natural logarithm of the market value of equity on 06/30 of t and the natural logarithm of the estimated intrinsic value of equity obtained using the latest accounting information available by 06/30 of t (see Appendix 1). $LnP/V(t): Qi$ is an indicator variable that equals 1 if $LnP/V(t)$ is in the i-th annual quintile (0 otherwise), $i = 1$ to 5. *Industry FE* stands for industry fixed effects. *Controls* stands for control variables, defined in Appendix 2. T-statistics in parentheses are adjusted for Newey-West autocorrelations of three lags. **, *, and † denote statistical significance at the 1, 5, and 10% levels, respectively, using a 2-tailed test

Evidently, to draw a reliable inference about *IVOL*'s relation with expected return ($E_t(R_{t\to t+1})$) from its estimated relation with $R_{t\to t+1}$, we need to calibrate the sign and magnitude of its relations with $C_{t\to t+1}$ and $F_{t\to t+1}$, which is empirically unfeasible (Black, 1986). Importantly, the sign switching for *IVOL*'s estimated relation with $R_{t\to t+1}$ suggests that its relation with the mispricing-related component ($C_{t\to t+1}$ and $F_{t\to t+1}$) dominates its relation with expected return ($E_t(R_{t\to t+1})$) in its estimated relation with $R_{t\to t+1}$. If not, we would not observe the sign switching other than in the virtually inconceivable situation that *IVOL*'s relation with expected return varies similarly between ex ante (ex post) undervalued and overvalued observations. We thus cannot infer the sign – let alone the magnitude – of *IVOL*'s relation with expected return from its estimated relation with $R_{t\to t+1}$. That is, we have no way to know whether the negative estimated relation between *IVOL* and $R_{t\to t+1}$ documented in some studies contradicts the prediction of classic asset pricing theories.

4.2 Robustness

4.2.1 Alternative overvaluation likelihood measures. We try two alternative overvaluation likelihood measures. One is the market-to-book ratio (*MTB(t)*), in which the market value of equity is measured on 06/30 of *t* and the book value of equity is computed using the latest accounting information available by 06/30 of *t*. The book-to-market ratio (*BTM(t)*), the inverse of *MTB(t)*, has consistently been found to be positively related to realized return. Piotroski and So (2012) find that *BTM(t)* has predictive power for realized return only for firms for which the expectation implied by *BTM(t)* is incongruent with the strength of the firm's fundamentals. Their finding is consistent with the view that observations with larger *MTB(t)* are more likely to be overvalued than those with smaller *MTB(t)*.

The other is the price-to-value ratio, based on the residual income valuation model (*P/V–F&L(t)*); the numerator (*P*) is the market value of equity on 06/30 of *t* and the denominator (*V–F&L*) is the estimated intrinsic value of equity obtained by incorporating model-based earnings predictions and the industry-specific cost of equity into Frankel and Lee's (1998) empirical implementation of the residual income valuation model introduced in Ohlson (1995). We adopt Hou, van Dijk, and Zhang's (2012) model-based approach to forecasting earnings [7]. Following Frankel and Lee (1998), we apply Fama and French's (1993) three-factor model to estimate the industry-specific cost of equity. Frankel and Lee (1998) find a statistically reliable positive relation between their *V/P* estimate, the inverse of *P/V-F&L*, and realized return, suggesting that observations with larger *P/V-F&L* are more likely to be overvalued than those with smaller *P/V-F&L*.

We report results based on *MTB* and *P/V-F&L* respectively in Tables IA2 and IA3 of the Internet Appendix. *IVOL*'s estimated relation with $R_{t\to t+1}$ is significantly smaller (larger) in the top quintile than in the bottom quintile of *MTB* or *P/V-MPEG* measured on 06/30 of *t* ($t+1$) and is in between for the middle three quintiles. Evidently, results based on *MTB* and *P/V-F&L* support the hypothesis.

4.2.2 Alternative idiosyncratic volatility measures. We try two alternative ways of computing *IVOL*. Our primary measure of *IVOL* is computed using daily data from 07/01 of $t-1$ to 06/30 of *t*. To alleviate the potential concern that this measure may not well capture investors' expectation about future idiosyncratic volatility on 06/30 of *t* since it seems to rely on distant data, we use data from 04/01 of *t* through 06/30 of *t* to compute idiosyncratic volatility (*IVOL3MON*). We use monthly data from 07/01 of $t-5$ through 06/30 of *t* with at least 12 observations to compute *IVOL* (*IVOL5Year*).

We report results based on *IVOL3MON* and *IVOL5Year* respectively in Tables IA4 and IA5 of the Internet Appendix. Results based on *IVOL3MON* and *IVOL5Year* also well support the hypothesis.

4.2.3 Controlling for return skewness and stock liquidity. Prior studies propose various explanations for the negative estimated relation between *IVOL* and realized return that is first documented by Ang *et al.* (2006). It is hard to conceive how these explanations account for the nonlinear relation between *IVOL* and realized return documented in this study. However, to show that our results cannot be accounted for by these explanations, we control for return skewness and stock liquidity. Hou and Loh (2016) show that explanations based on investors' lottery preferences and market frictions are most promising in explaining the negative estimated relation between *IVOL* and realized return. Return skewness captures investors' lottery preferences and stock liquidity measures market frictions (Hou & Loh, 2016).

We compute return skewness (*RetSkewness*) using daily return data from 04/01 of t to 06/30 of t and compute stock liquidity (*StkLiq*) as $-1 \times$ the natural logarithm of Abdi and Ranaldo's (2017) effective bid-ask spread estimate using daily close, high, and low prices from 07/01 of $t-1$ to 06/30 of t. Table IA6 of the Internet Appendix shows that controlling for *RetSkewness* and *StkLiq* has no material impact on our inference.

4.2.4 Other robustness analyses. We run four more robustness analyses. In the first robustness analysis, we follow Brennan, Chordia, and Subrahmanyam (1998) and use the risk-adjusted return as the dependent variable. In the second robustness analysis, we use monthly realized return as the dependent variable. Table IA7 the Internet Appendix shows that our inference remains the same under these two alternative quantifications of realized return.

In the third robustness analysis, we examine whether the occurrence of economic recessions affect our results since economic recessions and ensuing crises may affect market-wide mispricing. As shown in Table IA8, our results hold regardless of whether economic recessions occur before and after the portfolio formation.

Finally, we run size-weighted Fama-MacBeth regressions to test the hypothesis. In untabulated results, we find that the hypothesis holds. This suggests that the results are not unduly influenced by microcap stocks.

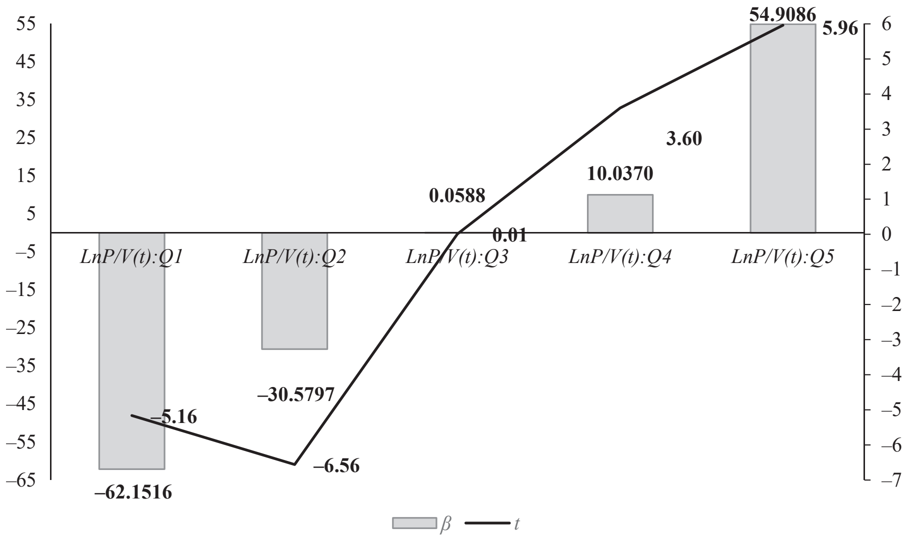
5. Additional analyses

5.1 Idiosyncratic volatility and mispricing: direct evidence

Our hypothesis development builds on the regularity that *IVOL* and the degree of mispricing are positively related. $|LnP_t - LnV_t|$, the absolute difference between LnP_t and LnV_t, measures the degree of mispricing where $Ln(\cdot)$ is the natural logarithm transformation operator, P_t is the market value of equity at time t, and V_t is the intrinsic value of equity at time t. A larger value of $|LnP_t - LnV_t|$ means greater mispricing. Because *IVOL* and the degree of mispricing are positively related, we have $COV(IVOL, |LnP_t - LnV_t|) > 0$ where $COV(\cdot)$ denotes covariance. For overvalued stocks, $|LnP_t - LnV_t| = LnP_t - LnV_t = LnP_t/V_t$ while for undervalued stocks, $|LnP_t - LnV_t| = -(LnP_t - LnV_t) = -LnP_t/V_t$. Therefore, for overvalued stocks, $COV(IVOL, LnP_t/V_t) > 0$ while for undervalued stocks, $COV(IVOL, LnP_t/V_t) < 0$. This suggests a testable hypothesis that the greater the proportion of overvalued observations in the sample, the larger the estimated relation between *IVOL* and LnP_t/V_t.

$LnP/V(t)$, the overvaluation likelihood measure, can be taken as the sum of LnP_t/V_t and a value estimation error [8]. We therefore expect to observe that *IVOL*'s estimated relation with $LnP/V(t)$ is larger for firms in the top $LnP/V(t)$ quintile than for firms in the bottom $LnP/V(t)$ quintile, which Figure 2 shows to be the case. To generate Figure 2, we run the Fama and MacBeth (1973) regression to estimate the following equation for each $LnP/V(t)$ quintile: $Pct = \gamma + \beta * IVOL + \varepsilon$, where *Pct* is the annual rank of $LnP/V(t)$ scaled to have a minimum of 0 and a maximum of 100. Figure 2 shows that *IVOL*'s estimated relation with *Pct* monotonically increases from -62.1516 ($t = -5.16$) to 54.9086 ($t = 5.96$) as the estimation is moved from the bottom $LnP/V(t)$ quintile to the top one.

We substitute $M/B(t)$ or $P/V - F\&L(t)$ for $LnP/V(t)$ and redo the analysis. We report the results in Figure IA2 of the Internet Appendix. Panel A shows that *IVOL*'s estimated



Note(s): This figure depicts variation in the estimated relation of idiosyncratic volatility (IVOL) with LnP/V(t) across quintiles formed by sorting on *LnP*/*V*(*t*). IVOL is the idiosyncratic volatility measure, defined in Appendix B.*LnP*/*V*(*t*) is the difference between the natural logarithm of the market value of equity on 06/30 of *t* and the natural logarithm of the estimated intrinsic value of equity obtained using the latest accounting information available by 06/30 of *t* (see Appendix A). *LnP*/*V*(*t*):Qi indicates the i-th quintile of *LnP*/*V*(*t*), *i* = 1 to 5. To generate this figure, we run the Fama-MacBeth regression to estimate the following equation separately for each *LnP*/*V*(*t*) quintile: $Pct = \gamma + \beta$ * IVOL + ε where Pct is the annual rank of LnP/V(t) scaled to have a minimum of 0 and a maximum of 100. *T*-statistics are adjusted for Newey-West autocorrelations of three lags

Figure 2. Idiosyncratic volatility and mispricing: direct evidence

relation with the annual rank of $M/B(t)$ monotonically increases from −50.7943 ($t = -7.02$) to 48.2048 ($t = 5.71$) as the computation is moved from the bottom $M/B(t)$ quintile to the top one. Panel B shows that *IVOL*'s estimated relation with the annual rank of $P/V - F\&L(t)$ monotonically increases from −50.9163 ($t = -3.84$) to 82.2379 ($t = 7.76$) as the computation is moved from the bottom $P/V - F\&L(t)$ quintile to the top one.

Collectively, Figure 2 and Figure IA2 provide direct evidence that idiosyncratic volatility and the degree of mispricing are positively related.

5.2 Sensitivity to research design choices

Research on the relation between idiosyncratic volatility (*IVOL*) and $R_{t\to t+1}$ is characterized by inconsistent findings. *IVOL*'s estimated relation with $R_{t\to t+1}$ is inherently unstable, due to the nonlinearity of *IVOL*'s relation with $C_{t\to t+1}$ and with $F_{t\to t+1}$. We reason that variation in research design choices across studies may drive the inconsistency by shaping *IVOL*'s estimated relations with $C_{t\to t+1}$ and $F_{t\to t+1}$.

We can think of at least two reasons for which research design choices affect *IVOL*'s estimated relations with $C_{t\to t+1}$ and $F_{t\to t+1}$. First, a research design choice can change the proportions of ex ante and ex post undervalued and overvalued observations in the sample. Second, it can change the weight of ex ante significantly undervalued observations and ex post significantly overvalued observations in *IVOL*'s estimated relation with $R_{t \to t+1}$ due to its effect on the statistical properties of $R_{t \to t+1}$. A well-known regularity of $R_{t \to t+1}$ is that it is right-skewed. $C_{t \to t+1}$ for the correction of ex ante undervaluation and $F_{t \to t+1}$ for the formation of ex post overvaluation can be very large, but $C_{t \to t+1}$ for the correction of ex ante overvaluation and $F_{t \to t+1}$ for the formation of ex post undervaluation are at most -1. Therefore, large positive returns driving the right-skewness of $R_{t \to t+1}$ high likely stem from the correction of ex ante significant undervaluation and the formation of ex post significant overvaluation.

To demonstrate the validity of the second reason, we estimate *IVOL*'s relation with the continuously compounded return ($LnR_{t \to t+1}$). $LnR_{t \to t+1}$ is the natural logarithm of 1 plus $R_{t \to t+1}$ [9]. $LnR_{t \to t+1}$ is expected to be less right-skewed than $R_{t \to t+1}$, since the logarithm transformation reduces the influence of large positive returns on the statistical distribution. We provide the contrast between $R_{t \to t+1}$ and $LnR_{t \to t+1}$ in Table IA9 of the Internet Appendix. As expected, $R_{t \to t+1}$ is highly right-skewed while $LnR_{t \to t+1}$ is slightly left-skewed. Gauged by the standard deviation and the difference between the 99th and 1st percentiles of their pooled distributions, the variance of $R_{t \to t+1}$ is much larger for firms in the bottom $LnP/V(t)$ quintile than for firms in the top $LnP/V(t)$ quintile and for firms in the top $LnP/V(t+1)$ quintile than for firms in the bottom $LnP/V(t+1)$ quintile. This is consistent with the notion that $C_{t \to t+1}$ for the correction of ex ante undervaluation and $F_{t \to t+1}$ for the formation of ex post overvaluation can be very large but that $C_{t \to t+1}$ for the correction of ex ante overvaluation and $F_{t \to t+1}$ for the formation of ex post undervaluation are at most -1. In contrast, the variance of $LnR_{t \to t+1}$ is slightly larger for firms in the top $LnP/V(t)$ quintile than for firms in the bottom $LnP/V(t)$ quintile and is significantly smaller for firms in the top $LnP/V(t+1)$ quintile than for firms in the bottom $LnP/V(t+1)$ quintile.

The contrast between $LnR_{t \to t+1}$ and $R_{t \to t+1}$ suggests that observations incurring the correction of ex ante significant undervaluation and observations incurring the formation of ex post significant overvaluation are weighted econometrically less in the estimated relation when $LnR_{t \to t+1}$ is the dependent variable than when $R_{t \to t+1}$ is the dependent variable. *IVOL* and $C_{t \to t+1}$ are positively related among ex ante undervalued observations and negatively related among ex ante overvalued observations; and *IVOL* and $F_{t \to t+1}$ are positively related among ex post overvalued observations and negatively related among ex post undervalued observations. Therefore, *IVOL*'s overall estimated relation with realized return is expected to be comparatively smaller when $LnR_{t \to t+1}$ is the dependent variable than when $R_{t \to t+1}$ is the dependent variable.

We report the results based on $LnR_{t \to t+1}$ in Table 3. For comparison, we also report the results based on $R_{t \to t+1}$. To ensure comparability, we use standardized $LnR_{t \to t+1}$ ($StdLnR_{t \to t+1}$) and standardized $R_{t \to t+1}$ ($StdR_{t \to t+1}$) with a mean of 0 and a standard deviation of 1 as the dependent variables. Table 3 shows that results based on $LnR_{t \to t+1}$ supports the hypothesis. This is not surprising since there is a one-to-one mapping between $LnR_{t \to t+1}$ and $R_{t \to t+1}$. Importantly, Table 3 shows that, consistent with our expectation, *IVOL*'s estimated relation with realized return is comparatively smaller when $LnR_{t \to t+1}$ is the dependent variable than when $R_{t \to t+1}$ is the dependent variable. For instance, *IVOL*'s overall estimated relation with $StdR_{t \to t+1}$ is 1.2142 ($t = 1.39$) while its overall estimated relation with $StdLnR_{t \to t+1}$ is -4.4337 ($t = -4.17$). The proportions of ex ante and ex post undervalued and overvalued observations in the sample are the same regardless of whether $LnR_{t \to t+1}$ or $R_{t \to t+1}$ is the dependent variable. Therefore, the contrast between $LnR_{t \to t+1}$ and $R_{t \to t+1}$ regarding their estimated relations with *IVOL* is driven by the difference between their statistical properties and the ensuing weight of observations incurring the correction of ex ante significant undervaluation and observations incurring the formation of ex post significant overvaluation in their estimated relation with *IVOL*.

Table 3. Raw return vs the continuously compounded return

Variable	$StdR_{t\to t+1}$	$StdLnR_{t\to t+1}$	$StdR_{t\to t+1}$	$StdLnR_{t\to t+1}$	$StdR_{t\to t+1}$	$StdLnR_{t\to t+1}$
IVOL (λ_1)	1.2142 (1.39)	−4.4337** (−4.17)	4.3283** (3.47)	−1.1580 (−0.93)	−2.6969** (−3.95)	−7.1910** (−6.55)
*IVOL * LnP/V(t):Q2* (λ_2)			−3.8638** (−4.56)	−2.8613** (−4.28)		
*IVOL * LnP/V(t):Q3* (λ_3)			−4.3886** (−5.22)	−4.1540** (−4.82)		
*IVOL * LnP/V(t):Q4* (λ_4)			−5.7958** (−7.66)	−6.3433** (−8.06)		
*IVOL * LnP/V(t):Q5* (λ_5)			−7.2149** (−7.65)	−8.4960** (−7.88)		
*IVOL * LnP/V(t + 1):Q2* (λ_2)					2.1056** (3.56)	2.7174** (3.91)
*IVOL * LnP/V(t + 1):Q3* (λ_3)					2.4558** (3.64)	3.0296** (3.85)
*IVOL * LnP/V(t + 1):Q4* (λ_4)					5.1569** (6.66)	4.9885** (5.64)
*IVOL * LnP/V(t + 1):Q5* (λ_5)					16.2895** (7.70)	11.7653** (5.84)
N	180,717	180,717	180,717	180,717	180,717	180,717
R^2	0.1267	0.1614	0.1306	0.1657	0.3032	0.3670
LnP/V(t):Qi/LnP/V(t + 1):Qi	No	No	Yes	Yes	Yes	Yes
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
$\lambda_1 + \lambda_2$			0.4644 (0.31)	−4.0193** (−2.83)	−0.5912 (−0.65)	−4.4736** (−3.44)
$\lambda_1 + \lambda_3$			−0.0603 (−0.04)	−5.3121** (−3.50)	−0.2410 (−0.25)	−4.1614** (−3.08)
$\lambda_1 + \lambda_4$			−1.4676 (−1.01)	−7.5013** (−5.08)	2.4601* (2.38)	−2.2025 (−1.56)
$\lambda_1 + \lambda_5$			−2.8867† (−1.85)	−9.6541** (−5.85)	13.5927** (6.11)	4.5743† (1.99)

Note(s): This table presents results of the analysis that uses the continuously compounded return as the dependent variable. $R_{t\to t+1}$ is stock return over 07/01 of t to 06/30 of $t+1$, $t = 1966$ to 2015. $LnR_{t\to t+1}$, the continuously compounded return, is the natural logarithm of 1 plus $R_{t\to t+1}$. $StdR_{t\to t+1}$ ($StdLnR_{t\to t+1}$) is standardized $R_{t\to t+1}$ ($LnR_{t\to t+1}$) with a mean of 0 and a standard deviation of 1. *IVOL* is the idiosyncratic volatility measure, defined in Appendix 2. $LnP/V(t)$ is the difference between the natural logarithm of the market value of equity on 06/30 of t and the natural logarithm of the estimated intrinsic value of equity obtained using the latest accounting information available by 06/30 of t (see Appendix 1). $LnP/V(t)$: *Qi* is an indicator variable that equals 1 if $LnP/V(t)$ is in the i-th quintile (0 otherwise), $i = 1$ to 5. *Industry FE* stands for industry fixed effects. *Controls* stands for control variables and are defined in Appendix 2. T-statistics in parentheses are adjusted for Newey-West autocorrelations of three lags. **, *, and † denote statistical significance at the 1, 5, and 10% levels, respectively, using a 2-tailed test

We next examine the effect of screen for price on *IVOL*'s estimated relation with $R_{t\to t+1}$. Asset pricing studies have generally excluded observations with very low price, arguing that price movements of such observations are susceptible to microstructure biases. However, these studies differ regarding the price threshold for exclusion. Moreover, these studies rarely specify the timing of measuring stock price.

Table IA10 of the Internet Appendix reports results of the analysis that examines the effect of screen for price on the sample composition and on the property of $R_{t\to t+1}$. Panel A shows the effect of screen for the ex ante stock price (*Price(t)*). *Price(t)* is stock price on 06/30 of *t*. As the ex ante price threshold for exclusion increases from \$0 to \$1 to \$5, observations incurring the correction of ex ante significant undervaluation and observations incurring the formation of ex post significant overvaluation are weighted comparatively less and observations incurring the formation of ex post undervaluation are weighted comparatively more in *IVOL*'s estimated relation with $R_{t\to t+1}$. *IVOL* is positively related to $C_{t\to t+1}$ among ex ante undervalued stocks and to $F_{t\to t+1}$ among ex post overvalued stocks and negatively related to $F_{t\to t+1}$ among ex post undervalued stocks. *IVOL*'s overall estimated relation with $R_{t\to t+1}$ is thus expected to decrease with increase in the ex ante price threshold for exclusion.

Panel B of Table IA10 shows the effect of screen for ex post stock price (*Price* $(t+1)$). *Price* $(t+1)$ is stock price on 06/30 of $t+1$. As the ex post price threshold for exclusion increases from \$0 to \$1 to \$5, observations incurring the correction of ex ante significant undervaluation are weighted comparatively more and observations incurring the formation of ex post undervaluation are weighted comparatively less in *IVOL*'s estimated relation with $R_{t\to t+1}$. *IVOL* is positively related to $C_{t\to t+1}$ among ex ante undervalued observations and negatively related to $F_{t\to t+1}$ among ex post undervalued stocks. *IVOL*'s overall estimated relation with $R_{t\to t+1}$ is thus expected to increase with increase in the ex post price threshold for exclusion.

Panel C of Table IA10 shows the effect of screen for average stock price (*AvgPrice*). *AvgPrice* is the average of *Price(t)* and *Price* $(t+1)$. Panel C shows that the effect of screen for average stock price (*AvgPrice*) resembles but is weaker than the effect of screen for ex post stock price. *IVOL*'s overall estimated relation with $R_{t\to t+1}$ is thus expected to increase with increase in the average price threshold for exclusion but less so than with increase in the ex post price threshold for exclusion.

Table 4 reports results of the analysis that examines the effect of screen for price on *IVOL*'s overall estimated relation with $R_{t\to t+1}$. As expected, with(out) control variables, when the ex ante price threshold for exclusion increases from \$0 to \$1 to \$5, *IVOL*'s overall estimated relation with $R_{t\to t+1}$ decreases from 0.8143 (t = 1.39) (0.8626 (t = 0.99)) to 0.2863 (t = 0.48) (0.4196 (t = 0.48)) to −1.1268 (t = −2.08) (−0.6503 (t = −0.73)); when the ex post price threshold for exclusion increases from \$0 to \$1 to \$5, *IVOL*'s overall estimated relation with $R_{t\to t+1}$ increases from 0.8143 (t = 1.39) (0.8626 (t = 0.99)) to 2.4014 (t = 3.66) (2.6923 (t = 3.14)) to 7.9500 (t = 8.03) (8.4361 (t = 7.51)); and when the average price threshold for exclusion increases from \$0 to \$1 to \$5, *IVOL*'s overall estimated relation with $R_{t\to t+1}$ increases from 0.8143 (t = 1.39) (0.8626 (t = 0.99)) to 1.6416 (t = 2.61) (1.7637 (t = 2.03)) to 4.8574 (t = 5.60) (4.8036 (t = 4.63)).

We next examine the effect of screen for size on *IVOL*'s estimated relation with $R_{t\to t+1}$. Asset pricing studies may also exclude small stocks, arguing that data quality of those stocks is low due to their illiquidity and ensuing noise in stock pricing and that their economic significance is trivial. In practice, studies have great discretion over the size threshold for exclusion. Moreover, these studies rarely specify the timing of measuring size.

Table IA11 of the Internet Appendix reports results of the analysis that examines the effect of screen for size on the sample composition and on the property of $R_{t\to t+1}$. As shown in Tables IA10 and IA11, the effect of screen for the ex ante size resembles the effect of screen for

Table 4.
The effect of screen for price

Variable	All (1)	*Price(t)* ≥ \$1 (2)	*Price(t)* ≥ \$5 (3)	*Price(t + 1)* ≥ \$1 (4)	*Price(t + 1)* ≥ \$5 (5)	*AvgPrice* ≥ \$1 (6)	*AvgPrice* ≥ \$5 (7)
IVOL	0.8626 (0.99)	0.4196 (0.48)	−0.6503 (−0.73)	2.6923** (3.14)	8.4361** (7.51)	1.7637* (2.03)	4.8036** (4.63)
N	187,231	182,634	151,208	180,911	148,786	182,721	151,693
R^2	0.0313	0.0300	0.0331	0.0320	0.0532	0.0319	0.0391
Controls	No	No	No	No	No	No	No
Industry FE	No	No	No	No	No	No	No
IVOL	0.8143 (1.39)	0.2863 (0.48)	−1.1268* (−2.08)	2.4014** (3.66)	7.9500** (8.03)	1.6416* (2.61)	4.8574** (5.60)
N	180,717	176,468	146,385	174,856	144,166	176,563	146,914
R^2	0.1267	0.1290	0.1489	0.1296	0.1688	0.1288	0.1510
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Note(s): This table presents results of the analysis that examines the effect of screen for price on the estimated relation of idiosyncratic volatility (*IVOL*) with realized return ($R_{t\to t+1}$). *Price(t)* (*Price* ($t + 1$)) is stock price on 06/30 of t ($t + 1$) and *AvgPrice* is the average of *Price(t)* and *Price* ($t + 1$). The dependent variable ($R_{t\to t+1}$) is stock return over 07/01 of t through 06/30 of $t + 1$, $t = 1966$ to 2015. *IVOL* is the idiosyncratic volatility measure, defined in Appendix 2. *Industry FE* stands for industry fixed effects. *Controls* stands for control variables, defined in Appendix 2. *T*-statistics in parentheses are adjusted for Newey-West autocorrelations of three lags. **, *, and † denote statistical significance at the 1, 5, and 10% levels, respectively, using a 2-tailed test the ex ante stock price; the effect of screen for the ex post size resembles the effect of screen for the ex post stock price; and the effect of screen for average size resembles the effect of screen for average stock price. Table 5 shows that as expected *IVOL*'s overall estimated relation with $R_{t\to t+1}$ decreases with increase in the ex ante size threshold for exclusion, increases with increase in the ex post size threshold for exclusion, and increases with increase in the average size threshold for exclusion but less than with increase in the ex post size threshold for exclusion.

In practice, a study involves several research design choices. These research design choices can cancel out or reinforce each other's effect on *IVOL*'s estimated relation with $R_{t\to t+1}$ depending on the combination of the research design choices, which Table IA12 shows to be the case. It is so far evident that ostensibly immaterial variations in research design choices (e.g. the price and size threshold for exclusion, the timing of measuring price and size, and the weighting scheme) can cause the estimated relation between idiosyncratic volatility and realized return to change dramatically and even switch sign in a predictable manner due to their effects on *IVOL*'s relations with $C_{t\to t+1}$ and $F_{t\to t+1}$.

5.3 Potential solutions

IVOL's relations with $C_{t\to t+1}$ and $F_{t\to t+1}$ cause its estimated relation with $R_{t\to t+1}$ to be a potentially biased estimate of its relation with expected return. $C_{t\to t+1}$ and $F_{t\to t+1}$ are measurement errors of $R_{t\to t+1}$ as the proxy for expected return. Portfolio grouping and instrument variables are standard methods for addressing the estimation bias resulting from measurement errors, but neither seems able to remove the bias resulting from *IVOL*'s relations with $C_{t\to t+1}$ and $F_{t\to t+1}$. The instrument variable approach requires instrument variables that are correlated with *IVOL* but not with the degree of mispricing and hence not with $C_{t\to t+1}$ and $F_{t\to t+1}$; such instrumental variables seem extremely difficult – if not impossible – to find.

To apply portfolio grouping, we need to sort observations by *IVOL* and compute the portfolio average of $R_{t\to t+1}$. If $C_{t\to t+1}$ and $F_{t\to t+1}$ could cancel out at the portfolio level, portfolio grouping would remove the bias. To explore the effectiveness of portfolio grouping, we apply it to estimate *IVOL*'s relation with $R_{t\to t+1}$. Specifically, we sort observations into five groups independently on *IVOL* and $LnP/V(t)$ ($LnP/V(t+1)$) each year and then form portfolios at the intersections of *IVOL* quintiles and $LnP/V(t)$ ($LnP/V(t+1)$) quintiles. Table 6 reports the time-series average of equal-weighted returns for each portfolio. We calculate the time-series average of portfolio returns as α_p in the following regression:

$$R_{p,t\to t+1} = \alpha_p + \varepsilon_{p,t\to t+1} \quad (7)$$

where p is portfolio p; $R_{p,t\to t+1}$ is the equal-weighted average of returns over 07/01 of t through 06/30 of $t+1$ for firms in portfolio p, $t = 1966$ to 2015; and $\varepsilon_{p,t\to t+1}$ is the residual. T-statistics are adjusted for Newey-West autocorrelations of three lags.

Table 6 shows that within each *IVOL* quintile, α_p decreases monotonically as the computation is moved from the bottom to the top $LnP/V(t)$ quintile. This is consistent with $LnP/V(t)$ measuring the ex ante overvaluation likelihood. Importantly, the difference between the top and the bottom *IVOL* quintiles regarding α_p decreases monotonically from 0.1212 ($t = 2.80$) to -0.0528 ($t = -1.51$) as the computation is moved from the bottom to the top $LnP/V(t)$ quintile. Table 6 also shows that within each *IVOL* quintile, α_p increases monotonically as the computation is moved from the bottom to the top $LnP/V(t+1)$ quintile. This is consistent with $LnP/V(t+1)$ measuring the ex post overvaluation likelihood. Importantly, the difference between the top and the bottom *IVOL* quintiles regarding α_p increases monotonically from -0.1376 ($t = -4.70$) to 0.4614 ($t = 5.29$) as the computation is moved from the bottom to the top $LnP/V(t+1)$ quintile.

Table 5. The effect of screen for size

Variable	All (1)	*SizePct(t)* ≥ 10% (2)	*SizePct(t)* ≥ 30% (3)	*SizePct(t* + *1)* ≥ 10% (4)	*SizePct(t* + *1)* ≥ 30% (5)	*AvgSizePct* ≥ 10% (6)	*AvgSizePct* ≥ 30% (7)
IVOL	0.8626 (0.99)	−0.2976 (−0.33)	−1.1526 (−1.22)	4.1343** (4.55)	8.4028** (7.89)	2.4749** (2.73)	4.4690** (4.21)
N	187,231	168,097	130,999	168,490	131,051	168,489	131,051
R^2	0.0313	0.0336	0.0382	0.0373	0.0536	0.0357	0.0429
Controls	No	No	No	No	No	No	No
Industry FE	No	No	No	No	No	No	No
IVOL	0.8143 (1.39)	−0.2989 (−0.47)	−1.5151* (−2.03)	3.6225** (4.75)	7.4104** (7.26)	2.2511** (3.17)	3.9276** (4.07)
N	180,717	162,468	127,147	162,906	127,253	162,864	127,246
R^2	0.1267	0.1412	0.1680	0.1479	0.1859	0.1432	0.1703
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Note(s): This table presents results of the analysis that examines the effect of screen for size on the estimated relation of idiosyncratic volatility (*IVOL*) with realized return ($R_{t\rightarrow t+1}$). *SizePct(t)* (*SizePct* ($t+1$)) is the annual rank of firm size on 06/30 of *t* ($t+1$). *AvgSizePct* is the annual rank of the average of firm size on 06/30 of *t* and firm size on 06/30 of $t+1$. *SizePct(t)*, *SizePct* ($t+1$) and *AvgSizePct* are scaled to have a minimum of 0 and a maximum of 1. *IVOL* is idiosyncratic volatility measure, defined in Appendix 2. *Industry FE* stands for industry fixed effects. *Controls* stands for control variables, defined in Appendix 2. *T*-statistics in parentheses are adjusted for Newey-West autocorrelations of three lags. **, *, and † denote statistical significance at the 1, 5, and 10% levels, respectively, using a 2-tailed test

Table 6.
Idiosyncratic volatility and return: portfolios

	IVOL:Q1	*IVOL:Q2*	*IVOL:Q3*	*IVOL:Q4*	*IVOL:Q5*	*Q5-Q1*
All	0.1412** (6.23)	0.1471** (6.16)	0.1530** (5.51)	0.1529** (4.93)	0.1847** (4.98)	0.0435 (1.33)
LnP/V(t):Q1	0.1630** (5.46)	0.1701** (5.80)	0.1946** (6.31)	0.2001** (5.70)	0.2841** (6.22)	0.1212** (2.80)
LnP/V(t):Q2	0.1510** (6.44)	0.1602** (5.89)	0.1683** (5.49)	0.1650** (5.18)	0.1872** (4.80)	0.0362 (1.18)
LnP/V(t):Q3	0.1402** (6.65)	0.1433** (5.80)	0.1573** (5.42)	0.1610** (4.90)	0.1604** (4.46)	0.0201 (0.66)
LnP/V(t):Q4	0.1342** (6.21)	0.1395** (6.59)	0.1348** (5.14)	0.1372** (4.41)	0.1337** (3.67)	−0.0005 (−0.01)
LnP/V(t):Q5	0.1180** (5.40)	0.1244** (5.03)	0.1161** (3.98)	0.1105** (3.45)	0.0652† (1.79)	−0.0528 (−1.51)
LnP/V(t + 1):Q1	0.0167 (0.63)	−0.0211 (−0.79)	−0.0688* (−2.24)	−0.1154** (−3.07)	−0.1209** (−3.47)	−0.1376** (−4.70)
LnP/V(t + 1):Q2	0.0805** (2.95)	0.0557* (2.19)	0.0404 (1.63)	0.0342 (1.19)	0.0748* (2.10)	−0.0057 (−0.17)
LnP/V(t + 1):Q3	0.1339** (4.95)	0.1372** (5.81)	0.1311** (4.85)	0.1422** (5.09)	0.1959** (5.46)	0.0620† (1.75)
LnP/V(t + 1):Q4	0.1853** (6.49)	0.2041** (7.59)	0.2306** (7.99)	0.2464** (7.50)	0.3428** (8.05)	0.1575** (3.82)
LnP/V(t + 1):Q5	0.2341** (7.51)	0.3052** (10.31)	0.4151** (10.13)	0.5069** (8.46)	0.6955** (8.28)	0.4614** (5.29)

Note(s): This table reports the time-series average of equal-weighted returns for portfolios formed by sorting stocks independently on idiosyncratic volatility (*IVOL*) and the overvaluation likelihood measure (*LnP/V*). This table also reports the time-series average of equal-weighted returns for portfolios formed by sorting only on *IVOL*. *IVOL* is the idiosyncratic volatility measure, defined in Appendix 2. *IVOL*: *Qi* indicates the *i*-th quintile of *IVOL*, $i = 1$ to 5. $LnP/V(t)$ is the difference between the natural logarithm of the market value of equity on 06/30 of t and the natural logarithm of the estimated intrinsic value of equity obtained using the latest accounting information available by 06/30 of t, $t = 1966$ to 2015 (see Appendix 1). $LnP/V(t)$: *Qi* indicates the *i*-th quintile of $LnP/V(t)$, $i = 1$ to 5. The time-series average of portfolio returns is calculated as α_p in the following regression: $R_{p,t \to t+1} = \alpha_p + \varepsilon_{p,t \to t+1}$
where p denotes portfolio p; $R_{p,t \to t+1}$ is the equal-weighted average of returns over 07/01 of t through 06/30 of $t + 1$ for firms in portfolio p; and $\varepsilon_{p,t \to t+1}$ is the residual. T-statistics in parentheses are adjusted for Newey-West autocorrelations of three lags. **, *, and † denote statistical significance at the 1, 5, and 10% levels, respectively, using a 2-tailed test

Firms differing in idiosyncratic volatility may also differ in their exposure to traditional risk factors. In the Internet Appendix, we report risk-adjusted returns based on (1) the capital asset-pricing model (CAPM) in Table IA13 and (2) a six-factor model in Table IA14 [10]. As shown in Tables IA13 and IA14, our inference remains the same after controlling for exposure to these standard risk factors.

In summary, the inference drawn using portfolio grouping is the same as the inference drawn using the Fama-MacBeth regression. That is, portfolio grouping cannot address the bias resulting from *IVOL*'s relations with $C_{t\to t+1}$ and $F_{t\to t+1}$.

Some studies propose methods for purging $R_{t\to t+1}$ of the measurement errors (e.g. Hou & van Dijk, 2019). These methods generally build on Campbell and Shiller's (1988) return decomposition. According to Campbell and Shiller's (1988) return decomposition, $R_{t\to t+1}$ can be decomposed into expected return, discount rate news (i.e. shocks to discount rates), and cash flow news (i.e. shocks to expected cash flows). Because the cash-flow-news variance seems to dominate the discount-rate-news variance (Chen, Da, & Zhao, 2013; Vuolteenaho, 2002), these methods focus on purging $R_{t\to t+1}$ of cash flow news. Sharing Elton's (1999) concern about the effectiveness of such methods in addressing the measurement errors of $R_{t\to t+1}$, we doubt their effectiveness in addressing the bias resulting from *IVOL*'s covariance with $C_{t\to t+1}$ and with $F_{t\to t+1}$ for at least two reasons. First, cash flow news and discount rate news seem to capture things other than $C_{t\to t+1}$ and $F_{t\to t+1}$. Conceptually, discount rate news and cash flow news are change in investors' expected return and change in investors' expectations of future cash flows, respectively. If rationally determined, they jointly capture change in investors' estimate of the intrinsic value. Second, there is no way to evaluate the effectiveness of such methods because *IVOL*'s *s* relation with expected return is unknown.

To explore the effectiveness of such methods, we follow Hou and van Dijk (2019) and control for profitability shock (*ProfitabilityShock*). *ProfitabilityShock* is the difference between profitability of $t + 1$ and the expected profitability of $t + 1$ obtained using Hou and van Dijk's (2019) method. According to Hou and van Dijk (2019), *ProfitabilityShock* captures the cash flow news. Guided by Campbell and Shiller's (1988) return decomposition, we reason that realized return is also driven by change in the intrinsic value, at least to a large extent. As turned out in this study, the intrinsic value estimate based on Rhodes–Kropf *et al.* (2005) method exhibits excellent empirical validity regardless of its simplicity. Therefore, we also control for the percentage change (*PctChgV*) in the estimated intrinsic value of equity from 06/30 of t to 06/30 of $t + 1$. In our estimation sample, *ProfitabilityShock* and *PctChgV* are positively correlated: 0.20 (Pearson) and 0.34 (Spearman). This high positive correlation is consistent with cash flow news reflecting change in investors' estimate of the intrinsic value.

Table 7 presents results of the analysis that controls for *ProfitabilityShock* and *PctChgV*. Three results deserve attention. First, both *ProfitabilityShock* and *PctChgV* are, as expected, positively related to $R_{t\to t+1}$. Second, as gauged by the magnitude of T-statistics, *PctChgV* turns out to be the most significant determinant of $R_{t\to t+1}$ among all explanatory variables. Third, controlling for *ProfitabilityShock* and *PctChgV* has no material impact on our inference, suggesting that these proposed methods cannot address the bias resulting from *IVOL*'s covariance with $C_{t\to t+1}$ and with $F_{t\to t+1}$.

Two forms of wishful thinking in addressing the bias are still possible. One is thinking that increasing the sample size may "diversify" away the bias. The other is thinking that one can construct a sample in which firms are properly priced and, as a result, the bias resulting from *IVOL*'s covariance with $C_{t\to t+1}$ and with $F_{t\to t+1}$ can be ignored. Empirically, it is always possible that the bias may accidentally cancel out, even without a large sample, and that it may be negligible, even without a specially constructed sample. However, because *IVOL*'s relation with expected return is unknown, there is no way to know whether such bias has cancelled out even if it has or to know whether such bias is negligible even if it is (Black, 1986).

In sum, it seems that existing methods cannot address the bias resulting from the covariance of idiosyncratic volatility (*IVOL*) with the mispricing-correction component ($C_{t\to t+1}$) and the mispricing-formation component ($F_{t\to t+1}$) of realized return ($R_{t\to t+1}$).

6. Conclusion

Idiosyncratic volatility (*IVOL*) has a dual effect on stock pricing: It affects stock pricing through its effect on both investors' expected return and stock pricing efficiency. Stock pricing efficiency, on average, is low for firms with high *IVOL* because high *IVOL* is associated with high arbitrage risks. That is, the extent to which stock price deviates from its underlying equity value is larger for firms with higher *IVOL*. Due to its dual effect on stock pricing, *IVOL*'s estimated relation with realized return ($R_{t\to t+1}$) captures its relations with both expected return and the mispricing-related component (the ex ante mispricing correction component ($C_{t\to t+1}$) and the ex post mispricing formation component ($F_{t\to t+1}$)). *IVOL* is positively related to $C_{t\to t+1}$ among ex ante undervalued stocks and negatively related to $C_{t\to t+1}$ among ex ante overvalued stocks; *IVOL* is negatively related to $F_{t\to t+1}$ among ex post undervalued stocks and positively related to $F_{t\to t+1}$ among ex post overvalued stocks.

We find that *IVOL*'s estimated relation with $R_{t\to t+1}$ decreases and switches sign from positive to negative as the estimation sample consists of proportionately more ex ante overvalued observations and that it increases and switches sign from negative to positive as the estimation sample consists of proportionately more ex post overvalued observations. Our finding suggests that *IVOL*'s relations with $C_{t\to t+1}$ and $F_{t\to t+1}$ dominate its relation with

Variable	(1)	(2)	Variable	(3)
IVOL (λ_1)	1.8839** (2.77)	4.0718** (4.66)	*IVOL* (λ_1)	−0.9180† (−1.78)
*IVOL * LnP/V(t): Q2* (λ_2)		−2.7312** (−6.02)	*IVOL * LnP/V(t + 1):Q2* (λ_2)	0.8882* (2.40)
*IVOL * LnP/V(t): Q3* (λ_3)		−3.3453** (−5.93)	*IVOL * LnP/V(t + 1):Q3* (λ_3)	1.3149* (2.66)
*IVOL * LnP/V(t): Q4* (λ_4)		−3.8323** (−6.33)	*IVOL * LnP/V(t + 1):Q4* (λ_4)	3.3996** (4.38)
*IVOL * LnP/V(t): Q5* (λ_5)		−5.4185** (−8.35)	*IVOL * LnP/V(t + 1):Q5* (λ_5)	12.0283** (9.48)
ProfitabilityShock	1.5800** (4.16)	1.5210** (4.23)	*ProfitabilityShock*	1.5355** (3.84)
PctChgV	0.1147** (13.24)	0.1430** (13.92)	*PctChgV*	0.2148** (16.88)
N	177,682	177,682	*N*	177,682
R^2	0.1896	0.1982	R^2	0.4043
LnP/V(t):Qi	No	Yes	*LnP/V(t + 1):Qi*	Yes
Controls	Yes	Yes	Controls	Yes
Industry FE	Yes	Yes	Industry FE	Yes

Note(s): This table presents results of the analysis that controls for profitability shocks and percentage change in the estimated intrinsic value of equity from 06/30 of *t* to 06/30 of *t* + 1. *ProfitabilityShock* is the difference between profitability of *t* + 1 and the expected profitability of *t* + 1 obtained using the method introduced in Hou and van Dijk (2019). *PctChgV* is the percentage change in the estimated intrinsic value of equity from 06/30 of *t* to 06/30 of *t* + 1. The dependent variable ($R_{t\to t+1}$) is stock return over 07/01 of *t* to 06/30 of *t* + 1, *t* = 1966 to 2015. *IVOL* is the idiosyncratic volatility measure, defined in Appendix 2. $LnP/V(t)$ is the difference between the natural logarithm of the market value of equity on 06/30 of *t* and the natural logarithm of the estimated intrinsic value of equity obtained using the latest accounting information available by 06/30 of *t*, *t* = 1966 to 2015 (see Appendix 1). $LnP/V(t)$: *Qi* is an indicator variable that equals 1 if $LnP/V(t)$ is in the *i*-th quintile (0 otherwise), $i = 1$ to 5. *T*-statistics in parentheses are adjusted for Newey-West autocorrelations of three lags. **, *, and † denote statistical significance at the 1, 5, and 10% levels, respectively, using a 2-tailed test

Table 7. Controlling for profitability shocks and percentage change in the estimated intrinsic value

expected return in its estimated relation with $R_{t\to t+1}$. One thus cannot infer the sign – let alone the magnitude – of *IVOL*'s relation with expected return from its estimated relation with $R_{t\to t+1}$. Moreover, we show that existing methods cannot address the bias resulting from *IVOL*'s relation with $C_{t\to t+1}$ and with $F_{t\to t+1}$. We further show that ostensibly immaterial variations in oft-employed research design choices can cause *IVOL*'s estimated relation with $R_{t\to t+1}$ to vary dramatically and even switch sign as a result of their effects on *IVOL*'s estimated relations with $C_{t\to t+1}$ and $F_{t\to t+1}$.

Our study contributes to research on the relation between idiosyncratic volatility (*IVOL*) and realized return ($R_{t\to t+1}$) by shedding light on the inconsistent and puzzling results about the relation. Our findings suggest that we cannot draw any reliable inference about the sign of *IVOL*'s relation with expected return from its estimated relation with $R_{t\to t+1}$. Therefore, the negative estimated relation of idiosyncratic volatility with realized return documented in some studies does not necessarily contradict the prediction of classic asset pricing theories. That is, this documented negative relation may not be an asset pricing puzzle. Our study also shows that *IVOL*'s estimated relation with realized return varies with research design choices due to their effects on its relation with the mispricing-related components, suggesting that the inconsistency of the results about *IVOL*'s estimated relation with $R_{t\to t+1}$ stems from variations in research design choices across studies. In summary, our study shows that the confusion about the relation of idiosyncratic volatility with realized return stems from neglecting its effect on stock pricing efficiency in research designs and results interpretation.

Notes

1. We choose not to provide a comprehensive literature review. Readers can refer to Hou and Loh (2016) for their excellent survey of research about the relation between idiosyncratic volatility and realized return.
2. *PriceDelay* = 1 – (R^2 of the restricted model/R^2 of the non-restricted model), where the non-restricted model is specified as $r_{i,l} = \alpha_i + \beta_i R_{m,l} + \sum_{n=1}^{4}\delta_i^{(-n)} R_{m,l-n} + \varepsilon_{i,l}$, $r_{i,l}$ is return on stock i in week l, $R_{m,l}$ is return for the CRSP value-weighted market index in week l, and the restricted model constrains $\delta_i^{(-n)} = 0$.
3. "It [Noise] keeps us from knowing the expected return on a stock or portfolio . . . We might define an efficient market as one in which price is within a factor of 2 of value, i.e. the price is more than half of value and less than twice value. The factor of 2 is arbitrary, of course. Intuitively, though, it seems reasonable to me, in the light of sources of uncertainty about value and the strength of the forces tending to cause price to return to value. By this definition, I think almost all markets are efficient almost all of the time. 'Almost all' means at least 90%" Black (1986, pp. 529, 533).
4. Fisher Black joined Goldman Sachs in 1984 and worked there until his death in 1995 (https://en.wikipedia.org/wiki/Fischer_Black).
5. For brevity, we do not report descriptive statistics of variables for samples used in other tests; they will be provided on request.
6. For brevity, Table 2 does not report coefficient estimates for control variables; we report these in Table IA1 of the Internet Appendix. Table IA1 shows that the sign of statistically significant coefficient estimates is consistent with that reported in prior studies when the model specification ignores that *IVOL*'s estimated relation with $R_{t\to t+1}$ varies with the proportion of ex ante (expost) overvalued observations in the sample. For instance, $R_{t\to t+1}$ is negatively related to firm size (*Size*), net stock issues (*NetStkIssue*), positive accruals (*PosTtlAcc*), and asset growth (*AssetGrowth*), and positively related to the book-to-market ratio (*B/M*) and profitability (*PosIB/BE*).
7. We choose not to use analysts' earnings-per-share (EPS) forecasts from I/B/E/S because the model-based approach allows us a broader sample in terms of both time periods and firms covered.

8. $LnP/V(t) = Ln(P/V^E) = Ln(P/V) + Ln(V/V^E) = Ln(P/V) + (-(LnV^E - LnV))$, where $Ln(\cdot)$ is the natural logarithm transformation operator, P is the market value, V is the intrinsic value, and V^E is the estimated intrinsic value.
9. We assign -0.999 to firms whose $R_{t \to t+1}$ is -1.
10. The six factors are the small-minus-big (SMB) factor, the high-minus-low (HML) factor, the momentum (MOM) factor, the robust-minus-weak (RMW) factor, the conservative-minus-aggressive (CMA) factor and the market factor.
11. The percentage of observations encountering the short selling of their common shares is lower for the top LnP/V quintile than for the fourth LnP/V quintile, possibly because firms in the top quintile are more likely to aggressively fight short arbitrageurs and/or because shareholders of these firms are less willing to lend their shares, since they can benefit more from selling highly overvalued stocks than from collecting lending fees (Lamont, 2012).

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Appendix 1

The equity overvaluation likelihood measure

The overvaluation likelihood measure (*LnP/V*) is the difference between the natural logarithm of the market value of equity and the natural logarithm of the estimated intrinsic value of equity. We adopt Rhodes–Kropf *et al.* (2005) method to estimate the intrinsic value of equity. We use *LnP/V* as our primary measure of overvaluation likelihood.

Rhodes–Kropf *et al.* (2005) method takes the intrinsic value of equity as a function of accounting items. The details of their method are as follows. First, we estimate the following equation separately for each of Fama-French's 12 industry groups each year:

$$\begin{aligned} LnME_{i,t} = \alpha_{0jt} + \alpha_{1jt}LnBE_{i,t} + \alpha_{2jt}NegNI_{i,t} + \alpha_{3jt}LnAbsNI_{i,t} \\ + \alpha_{4jt}NegNI_{i,t} * LnAbsNI_{i,t} + \alpha_{5jt}LEV_{i,t} + \varepsilon_{i,t} \end{aligned} \tag{A.1}$$

where i is firm i; t is fiscal year t; j is industry j; $LnME$ is the natural logarithm of market value of equity; $LnBE$ is the natural logarithm of book value of equity; $LnAbsNI$ is the natural logarithm of the absolute value of net income; $NegNI$ is an indicator variable that equals 1 if net income is negative (0 otherwise); and LEV is the book leverage (that is, the ratio of total liabilities to total assets). All variables are measured at the end of fiscal year t. The sample period is 1950–2015. Untabulated results show that the R^2 ranges from 64.96 to 98.33%, indicating that the accounting items in Equation (A.1) explain most within-industry variance in firm-level market value at a given time.

Second, we compute the rolling average of α_{kjt} to obtain $\overline{\alpha_{kjT}} = (1/(T-1949)) \sum_{t=1950}^{t=T} \alpha_{kjt}$, $k = 0$ to 5, $j = 1$ to 12, $T = 1966$ to 2015. Our use of the rolling average avoids look-forward bias. We start the computation in 1966 to ensure the reliability of $\overline{\alpha_{kjT}}$. Following Rhodes–Kropf *et al.* (2005), we compute the estimated intrinsic value of equity (log) (LnV) as

$$LnV_{i,T} = \overline{\alpha_{0jT}} + \overline{\alpha_{1jT}}LnBE_{i,T} + \overline{\alpha_{2jT}}NegNI_{i,T} + \overline{\alpha_{3jT}}LnAbsNI_{i,T} + \overline{\alpha_{4jT}}NegNI_{i,T} * LnAbsNI_{i,T} + \overline{\alpha_{5jT}}LEV_{i,T} \quad \text{(A.2)}$$

and use the difference between the natural logarithm of the market value of equity ($LnME_{i,T}$) and the natural logarithm of the estimated intrinsic value of equity (log) ($LnV_{i,T}$) to gauge the relative overvaluation likelihood.

Prior studies provide initial evidence about the validity of LnP/V for gauging the relative overvaluation likelihood. Rhodes–Kropf *et al.* (2005) show that the pattern of merger and acquisition activities varies with LnP/V as theoretically predicted; Chi and Gupta (2009) find that LnP/V is positively related to subsequent income-increasing earnings management, which is consistent with Jensen's (2005) prediction that equity overvaluation induces managers to inflate reported earnings to sustain the overvaluation. If LnP/V, as a measure of the relative overvaluation likelihood at the fiscal year-end of t, has adequate validity, we expect that firms with larger LnP/V, than firms with smaller LnP/V, will (1) deliver higher returns in fiscal year t and lower returns in fiscal year $t+1$ (Frankel & Lee, 1998); (2) encounter higher percentages of outstanding common shares shorted at the fiscal year-end of t, since short arbitrageurs are expected to target overvalued stocks (Karpoff & Lou, 2010); (3) make more stock issues in fiscal year $t+1$ because firms tend to issue stocks when their stocks are overvalued (Baker & Wurgler, 2002); and (4) generate higher percentages of sales from acquisitions in fiscal year $t+1$ because overvalued firms are more likely than undervalued firms to engage in acquisitions (Rhodes–Kropf *et al.*, 2005).

To compare observations with larger and smaller LnP/V, we sort observations into five equal groups according to LnP/V. Table A1 presents the comparison results. R_t is stock return over the 12-month period of fiscal year t and R_{t+1} is stock return over the 12-month period of fiscal year $t+1$. Table A1 shows that when the computation is moved from the bottom to the top LnP/V quintile, R_t increases monotonically and R_{t+1} decreases monotonically. $Short_t$ (short interests) is the percentage of outstanding common shares shorted at the end of fiscal year t; $NetStkIssue_{t+1}$ (net stock issues) is change in the natural logarithm of split-adjusted shares outstanding from the end of fiscal year t to the end of fiscal year $t+1$; $SalesByACQ_{t+1}$ (sales contributed by acquisitions) is the percentage of sales arising from acquisitions in fiscal year $t+1$. Table A1 shows that (1) $Short_t$ and $SalesByACQ_{t+1}$ increase monotonically from the bottom to the top LnP/V quintile; (2) $NetStkIssue_{t+1}$ first decreases and then increases monotonically; (3) observations in the top LnP/V quintile make significantly more net stock issues than those in the bottom quintile in fiscal year $t+1$; (4) the percentage of observations with positive net stock issues or with nonzero sales from acquisitions or with more than 20% sales from acquisitions in fiscal year $t+1$ increases monotonically from the bottom to the top LnP/V quintile; and (5) the percentage of observations encountering the short selling of their common shares at the end of fiscal year t is much higher for the top LnP/V quintile than for the bottom LnP/V quintile [11]. Together with the findings of Rhodes–Kropf *et al.* (2005) and Chi and Gupta (2009), these comparison results suggest that LnP/V well captures the relative overvaluation likelihood of observations. That is, observations with larger LnP/V are more likely to be overvalued than those with smaller LnP/V.

Appendix 2

Table A1. Validity of *LnP/V* as the overvaluation likelihood measure

Variable	*LnP/V:Q1* *N*	*LnP/V:Q1* Mean	*LnP/V:Q2* *N*	*LnP/V:Q2* Mean	*LnP/V:Q3* *N*	*LnP/V:Q3* Mean	*LnP/V:Q4* *N*	*LnP/V:Q4* Mean	*LnP/V:Q5* *N*	*LnP/V:Q5* Mean	*Q5 - Q1* Diff	*Q5 - Q1* *t*
R_t	49,244	−10.52%	49,244	4.51%	49,244	12.16%	49,244	21.35%	49,244	39.67%	50.19%	120.00
R_{t+1}	46,259	31.15%	46,259	18.91%	46,259	13.90%	46,259	9.58%	46,259	4.58%	−26.57%	−42.24
$Short_t$	46,391	0.43%	46,392	0.75%	46,392	1.12%	46,392	1.49%	46,392	1.82%	1.38%	78.37
% ($Short_t > 0$)	46,391	24.63%	46,392	38.25%	46,392	48.36%	46,392	55.07%	46,392	52.01%	27.38%	89.39
$NetStkIssue_{t+1}$	46,384	4.32%	46,385	3.06%	46,385	3.62%	46,385	4.55%	46,385	7.18%	2.86%	25.09
% ($NetStkIssue_{t+1} > 0$)	46,384	52.38%	46,385	59.08%	46,385	64.67%	46,385	69.12%	46,385	76.75%	24.37%	80.23
$SalesByACQ_{t+1}$	43,605	0.91%	43,606	1.33%	43,606	1.58%	43,606	1.98%	43,606	2.60%	1.69%	33.87
% ($SalesByACQ_{t+1} > 0$)	46,384	4.26%	46,385	6.67%	46,385	8.11%	46,385	9.99%	46,385	12.65%	8.39%	45.05
%($SalesByACQ_{t+1} > 0.2$)	43,605	1.86%	43,606	2.69%	43,606	3.18%	43,606	4.05%	43,606	5.38%	3.52%	27.91

Note(s): This table presents comparisons between different LnP/V groups along dimensions that prior studies find vary with equity overvaluation. LnP/V is the difference between the natural logarithm of the market value of equity at the end of fiscal year t and the natural logarithm of the estimated intrinsic value of equity obtained using accounting information from financial statements of fiscal year t; $LnP/V(t)$: Qi indicates the i-th quintile of $LnP/V(t)$, $i = 1$ to 5; R_t (R_{t+1}) is stock return over the 12-month period of fiscal year t ($t+1$); $Short_t$ is the percentage of outstanding common shares shorted at the fiscal year-end of t; $NetStkIssue_{t+1}$ is change in the natural logarithm of split-adjusted shares outstanding from fiscal year t to fiscal year $t+1$; and $SalesByACQ_{t+1}$ is the percentage of sales arising from acquisitions in $t+1$

Table A2.
Variable definitions[a]

Variables used in the main test	
AssetGrowth	Change in the natural logarithm of assets per split-adjusted share
Beta-CMA	Factor loading[b] on the conservative-minus-aggressive factor
Beta-HML	Factor loading on the high-minus-low factor
Beta-MktRf	Factor loading on the market factor
Beta-MOM	Factor loading on the momentum factor
Beta-RMW	Factor loading on the robust-minus-weak factor
Beta-SMB	Factor loading on the small-minus-big factor
IB/BE (profitability)	Income before extraordinary items divided by book equity at the beginning of the year
IVOL (idiosyncratic volatility)	Standard deviation of residuals from a regression that takes daily excess returns as a function of daily excess market returns and daily returns to the small-minus-big, high-minus-low, momentum, robust-minus-weak, and conservative-minus-aggressive factors
B/M	Natural logarithm of the ratio of the book value of equity to the market value of equity
LnP/V(t) (overvaluation likelihood)	Difference between the natural logarithm of the market value of equity on 06/30 of *t* and the natural logarithm of the estimated intrinsic value of equity obtained using the latest accounting information available by 06/30 of *t*. Estimation details are in Appendix 1
LnP/V(t):Qi	An indicator variable that equals 1 if *LnP/V(t)* is in the *i*-th annual quintile (0 otherwise), $i = 1$ to 5
Momentum	Stock return over the 11-month period ending on 05/31 of *t*
NegIB (loss)	An indicator variable that equals 1 if income before extraordinary items is negative (0 otherwise)
NetStkIssue (net stock issues)	Change in the natural logarithm of split-adjusted shares outstanding from 06/30 of $t-1$ to 06/30 of *t*
NegTtlAcc (negative total accruals)	*TtlAcc* for firms with negative accruals (0 otherwise)
PosIB/BE (positive profitability)	*IB/BE* for firms with positive *IB/BE* (0 otherwise)
PosTtlAcc (positive total accruals)	*TtlAcc* for firms with positive accruals (0 otherwise)
$R_{t\rightarrow t+1}$	Stock return over 07/01 of *t* to 06/30 of $t+1$, $t = 1966$ to 2015
Size	Natural logarithm of the market value of equity on 06/30 of *t*
TtlAcc (total accruals)	Change in operating working capital per split-adjusted share divided by total assets per split-adjusted share
ZeroNetStkIssue	An indicator variable that equals 1 if *NetStkIssue* equals 0 (0 otherwise)
Other variables	
AbsAutoCorr	Absolute value of the first-order autocorrelation of daily returns, computed using data from 07/01 of $t-1$ to 06/30 of *t*
AvgPrice	Average of *Price(t)* and *Price(t + 1)*

(*continued*)

Table A2.

Variables used in the main test	
AvgSizePct	Annual rank of the average of firm size on 06/30 of t and firm size on 06/30 of $t+1$, scaled to have a minimum of 0 and a maximum of 1
$CMA_{t \to t+1}$	Annualized return to the conservative-minus-aggressive (CMA) factor over 07/01 of t to 06/30 of $t+1$
$HML_{t \to t+1}$	Annualized return to the high-minus-low (HML) factor over 07/01 of t to 06/30 of $t+1$
IVOL3MON	Standard deviation of residuals from a regression that takes daily excess returns as a function of daily excess market returns and daily returns to the small-minus-big, high-minus-low, momentum, robust-minus-weak, and conservative-minus-aggressive factors, computed using daily data from 04/01 of t through 06/30 of t
IVOL5Year	Standard deviation of residuals from a regression that takes monthly excess returns as a function of monthly excess market returns and monthly returns to the small-minus-big, high-minus-low, momentum, robust-minus-weak, and conservative-minus-aggressive factors, computed using monthly data from 07/01 of $t-5$ through 06/30 of t with at least 12 observations
$IVOL : Qi$	The i-th quintile of $IVOL$, $i = 1$ to 5
$LnR_{t \to t+1}$ (continuously compounded return)	Natural logarithm of 1 plus $R_{t \to t+1}$, where $R_{t \to t+1}$ is stock return over 07/01 of t to 06/30 of $t+1$
$MOM_{t \to t+1}$	Annualized return to the momentum (MOM) factor over 07/01 of t to 06/30 of $t+1$
MTB(t) (market-to-book)	Ratio of the market value of equity on 06/30 of t to the latest book value of equity available by 06/30 of t
MTB(t):Qi	An indicator variable that equals 1 if *MTB(t)* is in the ith annual quintile (0 otherwise), $i = 1$ to 5
P/V-F&L(t) (the price-to-value ratio)	Ratio of the market value of equity on 06/30 of t to the estimated intrinsic value of equity (*V-F&L*) that is obtained by incorporating model-based earnings predictions and the industry-specific cost of equity into the empirically tractable version of the residual income valuation model introduced in Frankel and Lee (1998). We adopt Hou, van Dijk, and Zhang's (2012) model-based approach to forecasting earnings and apply Fama and French's (1993) three-factor model to estimate the industry-specific cost of equity
P/V-F&L(t):Qi	An indicator variable that equals 1 if *P/V-F&L(t)* is in the ith annual quintile (0 otherwise), $i = 1$ to 5
PctChgV	Percentage change in the estimated intrinsic value of equity from 06/30 of t to 06/30 of $t+1$
Price(t) (ex ante stock price)	Stock price on 06/30 of t
Price(t + 1) (ex post stock price)	Stock price on 06/30 of $t+1$
PriceDelay	1 - (R^2 of the restricted model/R^2 of the non-restricted model), where the non-restricted model is specified as $r_{i,l} = \alpha_i + \beta_i R_{m,l} + \sum_{n=1}^{4} \delta_i^{(-n)} R_{m,l-n} + \varepsilon_{i,l}$, $r_{i,l}$ is the return on stock i in week l, $R_{m,l}$ is the return for the CRSP value-weighted market index in week l, and the restricted model constrains $\delta_i^{(-n)} = 0$

(*continued*)

Table A2.

Variables used in the main test

ProfitabilityShock	Difference between profitability of $t+1$ and the expected profitability of $t+1$ obtained using the method introduced in Hou and van Dijk (2019)
$R_{f,\,t\rightarrow t+1}$	Annualized one-month T-bill rate over 07/01 of t to 06/30 of $t+1$
$R_{M,\,t\rightarrow t+1}$	Annualized return on the market portfolio over 07/01 of t to 06/30 of $t+1$
RetSkewness	Return skewness computed using daily return data from 04/01 of t to 06/30 of t
$RiskAdjR_{t\rightarrow t+1}$ (risk-adjusted return)	Computed as $R_{i,\,t\rightarrow t+1} - R_{f,t\rightarrow t+1} - (b_i \times (R_{m,t\rightarrow t+1} - R_{f,t\rightarrow t+1}) + s_i \times SMB_{t\rightarrow t+1} + h_i \times HML_{t\rightarrow t+1} + m_i \times MOM_{t\rightarrow t+1} + r_i \times RMW_{t\rightarrow t+1} + c_i \times CMA_{t\rightarrow t+1})$ where i is firm i; $R_{i,\,t\rightarrow t+1}$ is the stock return over 07/01 of t through 06/30 of $t+1$ for firm i; $R_{f,\,t\rightarrow t+1}$ is the annualized one-month T-bill rate over the same period; $R_{M,\,t\rightarrow t+1}$ is the annualized return on the market portfolio over the same period; $SMB_{t\rightarrow t+1}$ / $HML_{t\rightarrow t+1}$ / $MOM_{t\rightarrow t+1}$/ $RMW_{t\rightarrow t+1}$ / $CMA_{t\rightarrow t+1}$ is the annualized return to the small-minus-big/high-minus-low/momentum/robust-minus-weak/conservative-minus-aggressive factor over the same period; b_i, s_i, h_i, m_i, r_i, and c_i are factor loadings obtained using daily return data from 07/01 of t through 06/30 of $t+1$
$RMW_{t\rightarrow t+1}$	Annualized return to the robust-minus-weak (RMW) factor over 07/01 of t to 06/30 of $t+1$
SizePct(t)	Annual rank of firm size on 06/30 of t, scaled to have a minimum of 0 and a maximum of 1
SizePct$(t+1)$	Annual rank of firm size on 06/30 of $t+1$, scaled to have a minimum of 0 and a maximum of 1
$SMB_{t\rightarrow t+1}$	Annualized return to the small-minus-big (SMB) factor over 07/01 of t to 06/30 of $t+1$
$StdLnR_{t\rightarrow t+1}$	Standardized $LnR_{t\rightarrow t+1}$ with a mean of 0 and a standard deviation of 1
$StdR_{t\rightarrow t+1}$	Standardized $R_{t\rightarrow t+1}$ with a mean of 0 and a standard deviation of 1
StkLiq (stock liquidity)	-1 x the natural logarithm of Abdi and Ranaldo's (2017) effective bid-ask spread estimate, computed using daily close, high, and low prices from 07/01 of $t-1$ to 06/30 of t

Note(s): [a]Unless stated otherwise, all variables are computed using the latest accounting and market information available by 06/30 of t
[b]These factor loadings and *IVOL* are computed using daily data from 07/01 of $t-1$ to 06/30 of t, $t = 1966$ to 2015

Appendix 3

Internet appendix are available in online for this article.

Corresponding author

Zhuo (June) Cheng can be contacted at: afzcheng@polyu.edu.hk