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L^2-convergence of Yosida approximation for semi-linear backward stochastic differential equation with jumps in infinite dimension

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Abstract

Purpose – The main motivation of this paper is to present the Yosida approximation of a semi-linear backward stochastic differential equation in infinite dimension. Under suitable assumption and condition, an L^2-convergence rate is established.
Design/methodology/approach – The authors establish a result concerning the L^2-convergence rate of the solution of backward stochastic differential equation with jumps with respect to the Yosida approximation.
Findings – The authors carry out a convergence rate of Yosida approximation to the semi-linear backward stochastic differential equation in infinite dimension.
Originality/value – In this paper, the authors present the Yosida approximation of a semi-linear backward stochastic differential equation in infinite dimension. Under suitable assumption and condition, an L^2-convergence rate is established.

Keywords Backward stochastic differential equation with jumps, Gelfand triple, Yosida approximation
Paper type Research paper

1. Introduction

Backward stochastic differential equation (BSDE) was performed first by Pardoux and Peng [1] who proved the existence and uniqueness of adapted solutions, under suitable square-integrability assumptions, on the coefficients and on the terminal condition. Later, several

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To the memory of Professor Habib Ouerdiane (1953-2023).

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authors have been attracted to this area and have provided many applications such as in stochastic games and optimal control [2–4], partial differential equations [5] and numerical approximation [6].

The main motivation of this paper is to carry out a convergence rate of the Yosida approximation to the semi-linear backward stochastic differential equation with jumps in infinite dimension. More precisely, let H be a separable Hilbert space with inner product $\langle , \rangle_H$ and H^* its dual space. Let V be a uniformly convex Banach space, such that $V \subset H$ continuously and densely. For its dual space V^*, it follows that $H^* \subset V^*$ continuously and densely. Then by the identification of H and H^* via the Riesz isomorphism, we get

$$V \subset H \subset V^*. \tag{1}$$

(V, H, V^*) is called a Gelfand triple.

Following [7], we introduce A which is a linear bounded operator such that $A: D(A) = V \to V^*$, where $D(A) = \{v \in V, Av \in H\}$. Using [8], we introduce the Yosida approximation A_λ, $\lambda > 0$ of A defined as

$$A_\lambda x := \frac{1}{\lambda} J(x - J_\lambda x), \tag{2}$$

where J: $V \to V^*$ is the duality mapping defined by Definition 2.1, and J_λ: $V \to V$ is the resolvent of the operator A is defined by

$$J_\lambda x := (J + \lambda A)^{-1} Jx. \tag{3}$$

This Yosida approximation is used to approximate the following semi-linear backward stochastic differential equation in infinite dimension:

$$\begin{cases} dY_t & = AY_t dt + f(t, Y_t, Z_t, Q_t)dt + Z_t dW_t + \int_E Q_t(x)\tilde{N}(dt, dx) \\ Y_T & = X \in H. \end{cases} \tag{4}$$

where W is a cylindrical Wiener process, and $\tilde{N}$ is the compensated Poisson random measure. Using the following family of approximating equations:

$$\begin{cases} dY_t^\lambda & = A_\lambda Y_t^\lambda dt + f\left(t, Y_t^\lambda, Z_t^\lambda, Q_t^\lambda\right)dt + Z_t^\lambda dW_t + \int_E Q_t^\lambda(x)\tilde{N}(dt, dx) \\ Y_T & = X \in H, \end{cases}$$

where $\lambda > 0$, and and A_λ is the Yosida approximation, we establish the existence and uniqueness of the solution of (4).

Many authors have been devoted to the case of BSDE in infinite dimensional spaces such as [9–11].

Hu and Peng [10] proved the existence and the uniqueness of the solution (Y, Z) of this semi-linear backward stochastic evolution equations. This kind of equation appears in many topics as those by Bensoussan [12, 13] and Hu and Peng [14] for the case with no jumps who have studied the maximum principles for stochastic control systems in infinite dimensional spaces and the theory of optimal control and controllability for stochastic partial differential equations.

Existence and uniqueness of a strong solution of (4) was obtained in Ref. [7] by considering a special case of a backward stochastic evolution equation for Hilbert space valued processes. This, in turn, is studied by taking finite dimensional projections and then taking the limit.

This is the Galerkin approximation method which has been used by several authors (See, e.g. Ref. [15]).

Yosida approximations of stochastic differential equations in infinite dimension have been studied in Refs. [16–20]. The authors consider Yosida approximations of various classes of stochastic differential equations with Poisson jumps.

The authors in Ref. [21] prove the existence and uniqueness of a solution for a class of backward stochastic differential equations driven by a geometric Brownian motion with a sub-differential operator by means of the Moreau-Yosida approximation method (see Ref. [22] for this used method). Using approximation tools, the authors provide a probabilistic interpretation for the viscosity solutions of a kind of non-linear variational inequalities.

In the same area, the authors in Ref. [23] deal with a class of mean-field backward stochastic differential equations, with sub-differential operator corresponding to a lower semi-continuous convex function. Using Yosida approximation tools, the authors establish the existence and uniqueness of the solution. As an application, they give a probability interpretation for the viscosity solutions of a class of non-local parabolic variational inequalities.

The authors in Ref. [24] propose and analyze multivalued stochastic differential equations (MSDEs) with maximal monotonous operators driven by semimartingales with jumps. They introduce some methods of approximation of solutions of MSDEs based on discretization of processes and Yosida approximation of the monotonous operator. Their paper studies the general problem of stability of solutions of MSDEs with respect to the convergence of driving semimartingales.

Bahlali *et al.* [25] deal with reflected backward stochastic differential equation (RBSDE) with both monotone and locally monotone coefficient and squared integrable terminal data. Existence and uniqueness of the solution are established with a polynomial growth condition on the coefficient and using Yosida approximation tools. An application to the homogenization of multivalued partial differential equations is given by the authors.The aim of our paper differs from the one proposed in Ref. [26], as it concentrates on BSDEs instead of SDEs. Additionally, it differs from the approach described in Ref. [7] by integrating the idea of L^2-convergence of Yosida approximation. This integration offers a possible technique for solving multivalued differential equations.

This paper is composed of four sections. Section 2 introduces some notations, the Yosida approximation approach and preliminaries results. Section 3 establishes a result concerning the L^2-convergence rate of the solution of backward stochastic differential equation with jumps with respect to the Yosida approximation. In Section 4, we carry out a convergence rate of the Yosida approximation to the semi-linear backward stochastic differential equation in infinite dimension.

2. Preliminaries and notations

Let $(\Omega, \mathcal{F}, P)$ be a probability space with filtration $(\mathcal{F}_t)_{t\in[0,T]} \in \mathcal{F}$. Let Ξ, H be two separable Hilbert spaces, and H^* be the dual space of H. Let V be a Banach space dense in H. Let us assume that V is uniformly convex with uniformly convex dual V^*. It follows that $H^* \subset V^*$ continuously and densely. Then, by the identification of H and H^* via the Riesz isomorphism, we get

$$V \subset H \subset V^*.$$

The Milman-Pettis theorem (see, e.g. Yosida [[27], p. 127]) states that every uniformly convex Banach space is reflexive. So, V is a reflexive Banach space.

Following [28], we introduce a cylindrical Wiener process in Ξ as a family $(W(t), t \geq 0)$, of linear applications $\Xi \to L^2(\Omega)$ such that:

(1) For every $h \in \Xi$, $\{W(t)h, t \geq 0\}$ is a real (continuous) Wiener process,

(2) For every $h, k \in \Xi$ and $t, s \geq 0$, $E(W(t)h, W(t)k) = (t \wedge s)(h,k)_\Xi$.

Let $(E, \mathcal{B}(E))$ be a measurable space, where E is a topological vector space. Furthermore, let $\xi(t)$ be a Lévy process on E and be denoted by $\nu(dx)$, the Lévy measure of ξ. Denote by $L^2(\nu)$ the L^2-space of square integrable $H-$ valued measurable functions associated with ν.

Set $p(t) = \Delta\xi(t) = \xi(t) - \xi(t-)$. Then $p = \{p(t), t \in D_p\}$ is a stationary Poisson point process on E with characteristic measure ν. Denote by $N(dt, dx)$ the Poisson counting measure associated with the Lévy process, $N(t,A) = \sum_{s \in D_p,\ s \leq t} I_A(p(s))$. Denote by $\tilde{N}(dt,dx) = N(dt,dx) - dt\nu(dx)$ the compensated Poisson random measure. The filtration is defined as follows

$$(\mathcal{F}_t = \sigma(W_s, N(s,A), \quad A \in \mathcal{B}(E), \quad s \leq t), t \geq 0).$$

We denote by $\mathcal{P}$ the predictable $\sigma-$ field on $\Omega \times [0, T]$. Introduce now the following spaces:

(1) $L^2(0, T, H)$: the set of all $\mathcal{F}_t-$progressively measurable processes takes its values in H, such that

$$\|x\| = \left(\mathbb{E}\int_0^T \left|x(t)\right|^2 dt\right)^{\frac{1}{2}} < \infty.$$

(2) $L_2(\Xi, H)$: the set of the Hilbert-Schmidt operators from Ξ to H, that is,

$$L_2(\Xi,H) = \left\{\psi \in L(\Xi,H) \middle| \sum_{i=1}^{\infty} \middle| \psi e_n|_H^2 < \infty\right\}$$

where $\{e_n\}_{n=1}^{\infty}$ is an orthonormal basis on Ξ. The set $L_2(\Xi, H)$ is a Hilbert space.

(3) $L^2(\nu)$: L^2- space of square integrable $H-$ valued measurable functions $Q: H \to H$ associated with ν, *that is,*

$$|Q|_{L^2(\nu)}^2 = \int_0^t |Q_s|_H^2 d\nu(s) < \infty.$$

Moreover, beside the same hypotheses on the cylindrical Wiener process, we have:

(1) A positive number $T > 0$;

(2) A map f: $[0, T] \times \Omega \times V \times L_2(\Xi, H) \times L_2(\nu) \to H$.

(3) A final data $X \in L^2(\Omega, \mathcal{F}_T, H)$.

(4) A bounded linear operator A: $D(A) = V \to V^*$, where $D(A) = \{v \in V, Av \in H\}$. We assume that the operator A is monotone, meaning:

$${}_V\langle v, Av\rangle_{V^*} \geq 0, \forall v \in D(A). \tag{5}$$

Now, we assume the following useful hypothesis denoted by **Hyp.1**:

(1) f is measurable from $\mathcal{P} \times \mathcal{B}(H) \times \mathcal{B}(L_2(\Xi, H) \times L_2(\nu))$ to $\mathcal{B}(H)$ and $\mathbb{E} \int_0^T |f(s, 0, 0, 0)|_H^2 ds < +\infty$.

(2) There exists a constant $C > 0$, such that P almost surely for almost every $t \in [0, T]$, the following holds for all Y^1, $Y^2 \in H$, Z^1, $Z^2 \in L_2(\Xi, H)$ and Q^1, $Q^2 \in L_2(\nu)$:

$$|f\left(t, Y_t^1, Z_t^1, Q_t^1\right) - f\left(t, Y_t^2, Z_t^2, Q_t^2\right)|_H \leq C\left(\left|Y_t^1 - Y_t^2\right|_H + \left\|Z_t^1 - Z_t^2\right\|_{L_2(\Xi, H)} + \left\|Q_t^1 - Q_t^2\right\|_{L_\nu^2}\right).$$

In most cases, the duality mapping defined here is multivalued.

Definition 2.1. The duality mapping J: V → V* is defined by:

$$J(x) = \left\{x^* \in V^* \middle| x^*(y) := \langle x, y \rangle_H\right\}, \quad \forall y \in V. \tag{6}$$

Under hypotheses of V and V^*, we get the following result:

Theorem 2.2. [20] Let V be a Banach space. If V* is strictly convex, then the duality mapping J: V → V* is single-valued.

For the detailed proof, see Theorem 1.2 in Ref. [8].

Definition 2.3. The inverse mapping J^{-1}: V* → V is defined by:

$$J^{-1}\left(x^*\right) = \{y \in V \text{ such that} \langle z, y \rangle_H = \langle z, x \rangle_H\}, \quad \forall z \in V. \tag{7}$$

The inverse mapping J^{-1}: $V^* \to V$ is single-valued. For the proof, see [[29], Proposition 32.22] and [[20], Proposition 3.13.].

We will now provide an approximation of the operator A, as mentioned in Ref. [8].

Definition 2.4. For every x ∈ V and λ > 0, the Yosida approximation of A is defined by the operator A_λ: V → V* as

$$A_\lambda x := \frac{1}{\lambda} J(x - J_\lambda x), \tag{8}$$

where the resolvent J_λ: V → V of the operator A is defined by $J_\lambda x = x_\lambda$, with x_λ as a unique solution to the equation:

$$0 = J(x_\lambda - x) + \lambda A x_\lambda. \tag{9}$$

The uniqueness of x_λ was proved by [20] [Proposition 3.17. p. 36]. According to [8] [Proposition 1.3], A_λ is single-valued, monotone, bounded on bounded subsets and semi-continuous from V to V^*. The resolvent can be written as

$$J_\lambda x := (J + \lambda A)^{-1} J x. \tag{10}$$

Lemma 2.5. Equation (8) can be reformulated as:

$$A_\lambda(x) = \left(A^{-1} + \lambda J^{-1}\right)^{-1} x, x \in V. \tag{11}$$

Proof. Let $x \in V$ and $J_\lambda(x)$ be the resolvents of the operator A defined by equation (10). By the definition of the Yosida approximation and the homogeneity of J^{-1} (see Ref. [20]), Equation (8) can be written as

$$J_\lambda(x) = x - \lambda J^{-1}(A_\lambda(x)).$$

Using the fact that $A_\lambda(x) = A(J_\lambda(x))$ for all $x \in V$ ([[20], Proposition 3.19]) and inserting this into the resolvent equation (9), we obtain $A_\lambda(x) = A(x - \lambda J^{-1}(A_\lambda(x)))$ or equivalently, $x = (A^{-1} + \lambda J^{-1})(A_\lambda(x))$. Since A_λ is single-valued, we conclude (11).

3. Yosida approximation

Let H be a separable Hilbert space and V a Banach space such that the space $V \subset H$ is reflexive and dense in H. We identify H with its dual space H^*, and V with its dual space V^*. Then, we get

$$V \subset H \subset V^*$$

We denote by $|\cdot|_V, |\cdot|_{V^*}, |\cdot|_H$, the norms in V, V^* and H, respectively, and by $\langle , \rangle$ the duality product between V and V^*. We introduce the following application:

$$A : \Omega \to \mathcal{L}\left(V, V^*\right),$$

which verifies the following coercivity condition **(L1)**:

There exist $c_1 \geq 0$, $c_2 \in \mathbb{R}$ such that for all $v \in V$, $t \in [0, T]$, we have

$$(\mathbf{L1})\, 2\,{}_{V^*}\langle Av, v\rangle_V + c_1|v|_H^2 \geq c_2|v|_V^2.$$

In this section, we are interested in the Yosida approximation of the following semi-linear backward stochastic differential equation in infinite dimension:

$$\begin{cases} dY_t &= AY_t dt + f(t, Y_t, Z_t, Q_t)dt + Z_t dW_t + \int_E Q_t(x)\tilde{N}(dt, dx) \\ Y_T &= X \in H. \end{cases} \tag{12}$$

Let us consider the family of approximating equations of (12)

$$\begin{cases} dY_t^\lambda &= A_\lambda Y_t^\lambda dt + f\left(t, Y_t^\lambda, Z_t^\lambda, Q_t^\lambda\right)dt + Z_t^\lambda dW_t + \int_E Q_t^\lambda(x)\tilde{N}(dt, dx), \ \lambda > 0 \\ Y_T &= X \in H. \end{cases} \tag{13}$$

Remark 3.1. Note that, for all $\lambda > 0$, the operator A_λ being linear and bounded [[8], Proposition 2.2], it is checked by the standard Picard–Lindelof iteration methods [7] that the triplet $(Y^\lambda, Z^\lambda, Q^\lambda)$ is a classical solution of (13), and it verifies for all $t \in [0, T]$, that

$$Y_t^\lambda = X - \int_t^T \left(A_\lambda Y_u^\lambda + f\left(u, Y_u^\lambda, Z_u^\lambda, Q_u^\lambda\right)\right)du - \int_t^T Z_u^\lambda dW_u - \int_t^T \int_E Q_u^\lambda(x)\tilde{N}(du, dx). \tag{14}$$

The following result establishes the existence and the uniqueness of the solution of (12).

Theorem 3.2. [[7], Theorem 4.1] Assume that $X \in L^2(\Omega, \mathcal{F}_T, H)$. Under Hypothesis **Hyp.1** and Condition **(L1)**, equation (12) has a unique progressively measurable process solution $(Y, Z, Q) \in H \times L_2(\Xi, H) \times L^2(\nu)$ such that:

(1) $\mathbb{E}[\int_0^T \left|Y_t\right|_H^2 dt] < \infty$, $\mathbb{E}[\int_0^T \left|Z_t\right|_{L_2(\Xi,H)}^2 dt] < \infty$, $\mathbb{E}[\int_0^T \left|Q_t\right|_{L^2(\nu)}^2 dt] < \infty$.

(2) $Y_t = X - \int_t^T (AY_s + f(s, Y_s, Z_s, Q_s))ds - \int_t^T Z_s dW_s - \int_t^T \int_E Q_s(x)\tilde{N}(ds, dx)$.

The following results will be used to prove our main result about the L^2 convergence rate.

Remark 3.3. The coercivity condition **(L1)** of the operator A is transferred to its Yosida approximation A_λ, which follows directly from [[17], Lemma 3.10] and [[17], Proof of Proposition 5.1]. There exist $\tilde{c}_1 \geq 0, \tilde{c}_2 > 0$ such that for all $v \in V$, $t \in [0, T]$

$$2_{V^*}\langle Av, v\rangle_V + \tilde{c}_1|v|_H^2 \geq \tilde{c}_2|v|_V^2.$$

Lemma 3.4. Under Conditions (**L1**), and **Hyp.1,** there exists $C > 0$, such that for all $\lambda > 0$, we have

$$\sup_{t\in[0,T]}\mathbb{E}\left[\left|Y_t^\lambda\right|_H^2\right] + \int_t^T \mathbb{E}\|Z_s^\lambda\|_{L_2(\Xi,H)}^2 ds + \int_t^T \mathbb{E}\left[\left|Y_t^\lambda\right|_V^2\right]ds + \int_t^T \mathbb{E}|Q_s^\lambda|_{L^2(\nu)}^2 ds \leq C. \quad (15)$$

Proof. For fixed $\lambda > 0$, we can apply the Itô formula to $|Y_t^\lambda|_H^2$ and we obtain:

$$\begin{aligned}\mathbb{E}\left|Y_t^\lambda\right|_H^2 + \int_t^T \mathbb{E}\left\|Z_s^\lambda\right\|_{L_2(\Xi,H)}^2 ds + \int_t^T \mathbb{E}|Q_s^\lambda|_{L^2(\nu)}^2 ds \quad &\leq \quad \mathbb{E}Y_T|_H^2 - 2\int_t^T \mathbb{E}_{V*}\left\langle A_\lambda Y_s^\lambda, Y_s^\lambda\right\rangle_V ds \\ &- \quad 2\int_t^T \mathbb{E}\left\langle f\left(s, Y_s^\lambda, Z_s^\lambda, Q_s^\lambda\right), Y_s^\lambda\right\rangle_H ds.\end{aligned}$$

Then, by using the coercivity condition (**L1**) of A_λ and Cauchy-Schwartz inequality for $\alpha_1 > 0$, we get

$$\begin{aligned}\mathbb{E}|Y_t^\lambda|_H^2 + \mathbb{E}\int_t^T \|Z_s^\lambda\|_{L_2(\Xi,H)}^2 ds + \int_t^T E|Q_s^\lambda|_{L^2(\nu)}^2 ds \leq \mathbb{E}|Y_T^2|_H + \frac{1}{\alpha_1}\int_t^T \mathbb{E}|f\left(s, Y_s^\lambda, Z_s^\lambda, Q_s^\lambda\right)|_H^2 ds \\ + \alpha_1\int_t^T \mathbb{E}\left(\left|Y_s^\lambda\right|_H^2\right)ds \\ + \int_t^T\left[-\tilde{c}_2\mathbb{E}\left|Y_s^\lambda\right|_V^2 + \tilde{c}_1\mathbb{E}\left|Y_s^\lambda\right|_H^2\right]ds.\end{aligned}$$

Then, by using **Hyp.1**, we obtain:

$$\begin{aligned}&\mathbb{E}|Y_t^\lambda|_H^2 + \int_t^T \mathbb{E}\|Z_s^\lambda\|_{L_2(\Xi,H)}^2 ds + \int_t^T \mathbb{E}|Q_s^\lambda|_{L^2(\nu)}^2 ds \\ &\leq \mathbb{E}|Y_T|_H^2 + \frac{C}{\alpha_1}\int_t^T \mathbb{E}\left(\left|Y_s^\lambda\right|_H^2 + \left|Z_s^\lambda\right|_{L_2(\Xi,H)}^2 + \left|Q_s^\lambda\right|_{L_2(\nu)}^2\right)ds \\ &+ \int_t^T \mathbb{E}\left[-\tilde{c}_2\left\|Y_s^\lambda\right\|_V^2 + \tilde{c}_1\left|Y_s^\lambda\right|_H^2\right]ds \\ &+ \frac{C}{\alpha_1}\int_t^T \mathbb{E}|f(s,0,0,0)|_H^2 ds + \alpha_1\int_t^T \mathbb{E}|Y_s^\lambda|_H^2 ds.\end{aligned}$$

Therefore, for α_1 large enough, we obtain:

$$\begin{aligned}&\mathbb{E}|Y_t^\lambda|_H^2 + \left(1 - \frac{C}{\alpha_1}\right)\int_t^T \mathbb{E}\|Z_s^\lambda\|_{L_2(\Xi,H)}^2 ds + \tilde{c}_2\int_t^T \mathbb{E}|Y_s^\lambda|_V^2 ds + \int_t^T \mathbb{E}|Q_s^\lambda|_{L^2(\nu)}^2 ds \leq \mathbb{E}|Y_T|_H^2 \\ &\quad + C_3\int_t^T \mathbb{E}|Y_s^\lambda|_H^2 ds + C_3\int_t^T\left[f(s,0,0,0)^2\right]ds\end{aligned}$$

where $C_3 = \alpha_1 + \frac{C}{\alpha_1}$ is independent of λ. By the Gronwall lemma, we finally obtain the expression (15).

The following remark plays a fundamental role in the convergence rate of Yosida approximation.

Remark 3.5. According to [[8], proposition 2.2], A_λ verifies the boundedness condition

$$\|A_\lambda x\|_V{}^* \leq \|Ax\|_V{}^*,$$

for all $x \in D(A)$ on [0, T] and by using the fact that $D(A) = V$ and we get

$$\|A_\lambda Y_s^\lambda\|_{V^*}^2 \leq C\|Y_s^\lambda\|^2.$$

under Condition (**L1**) and **Hyp.1,** we then obtain by applying lemma 3.4:

$$\limsup_{\lambda\to 0} \int_t^T \mathbb{E}\left[\left|A_\lambda Y_s^\lambda\right|_{V^*}^2\right] ds < \infty. \tag{16}$$

4. Convergence of Yosida approximation

In this section, we prove a convergence rate of Yosida approximation to the following semi-linear backward stochastic differential equation in infinite dimension:

$$\begin{cases} dY_t &= AY_t dt + f(t, Y_t, Z_t, Q_t)dt + Z_t dW_t + \displaystyle\int_E Q_t(x)\tilde{N}(dt, dx) \\ Y_T &= X \in H. \end{cases} \tag{17}$$

Proposition 4.1. Let Y^λ be the solution to the backward stochastic differential equation (12), and assume that **Hyp.1** holds. Let $\lambda, \mu > 0$, then there exists $D > 0$, such that:

$$\sup_{t\in[0,T]} \mathbb{E}|Y^\lambda - Y^\mu|_H^2 + \int_t^T \mathbb{E}\|Z_s^\lambda - Z_s^\mu\|_{L_2(\Xi,H)}^2 ds + \int_t^T \mathbb{E}|Q_s^\lambda - Q_s^\mu|_{L^2(\nu)}^2 ds \leq D(\lambda + \mu).$$

Proof. Let us denote by Y_t^λ and Y_t^μ two Yosida approximation to

$$\begin{cases} dY_t &= AY_t dt + f(t, Y_t, Z_t, Q_t)dt + Z_t dW_t + \displaystyle\int_E Q_t(x)\tilde{N}(dt, dx) \\ Y_T &= X \in H. \end{cases} \tag{18}$$

by Itô formula, then the expectation, we get

$$\begin{aligned} \mathbb{E}|Y_t^\lambda - Y_t^\mu|_H^2 + \int_t^T \mathbb{E}\|Z_s^\lambda - Z_s^\mu\|_{L_2(\Xi,H)}^2 ds + \int_t^T \mathbb{E}|Q_s^\lambda - Q_s^\mu|_{L^2(\nu)}^2 ds &= -2\int_t^T \mathbb{E}_V\left\langle Y_s^\lambda - Y_s^\mu, \left(A_\lambda Y_s^\lambda - A_\mu Y_s^\mu\right)\right\rangle_V{}^* ds \\ &\quad - 2\int_t^T \mathbb{E}\left\langle Y_s^\lambda - Y_s^\mu, f\left(t, Y_s^\lambda, Z_s^\lambda, Q_s^\lambda\right) - f\left(t, Y_s^\mu, Z_s^\mu, Q_s^\mu\right)\right\rangle_H ds. \end{aligned}$$

By definition of A_λ and the bijectivity of J_λ, we have $I = J_\lambda + J^{-1}(\lambda A_\lambda)$. Hence:

$$\begin{aligned} {}_V\left\langle Y_s^\lambda - Y_s^\mu, A_\lambda Y_s^\lambda - A_\mu Y_s^\mu\right\rangle_{V_*} &= {}_V\left\langle\left(J_\lambda Y_s^\lambda + \lambda J^{-1} A_\lambda Y_s^\lambda\right) - \left(J_\mu Y_s^\mu + \mu J^{-1} A_\mu Y_s^\mu\right), \left(A_\lambda Y_s^\lambda - A_\mu Y_s^\mu\right)\right\rangle_{V_*} \\ &= {}_V\left\langle\left(J_\lambda Y_s^\lambda - J_\mu Y_s^\mu, \left(A_\lambda Y_s^\lambda - A_\mu Y_s^\mu\right)\right\rangle_{V^*} + {}_V\left\langle J^{-1}\left(\lambda A_\lambda Y_s^\lambda\right) - J^{-1}\left(\mu A_\mu Y_s^\mu\right), \left(A_\lambda Y_s^\lambda - A_\mu Y_s^\mu\right)\right\rangle_{V^*}. \end{aligned}$$

So by using Lemma 2.5, we obtain $A_\lambda = AJ_\lambda$ and $A_\mu = AJ_\mu$. Then the monotonicity of A (5) and the fact that J^{-1} is the duality map from V^* to $V^{**} = V$, the first aforementioned term is positive, so we get

$$
\begin{aligned}
-_V\left\langle Y^\lambda_s - Y^\mu_s, A_\lambda Y^\lambda_s - A_\mu Y^\mu_s\right\rangle_{V^*} &\le -{}_V\left\langle J^{-1}\left(\lambda A_\lambda Y^\lambda_s\right) - J^{-1}\left(\mu A_\mu Y^\mu_s\right), \left(A_\lambda Y^\lambda_s - A_\mu Y^\mu_s\right)\right\rangle_{V^*} \\
&= -\frac{1}{\lambda}{}_V\left\langle J^{-1}\left(\lambda A_\lambda Y^\lambda_s\right), \lambda A_\lambda Y^\lambda_s\right\rangle_{V^*} - \frac{1}{\mu}{}_V\left\langle J^{-1}\left(\mu A_\mu Y^\mu_s\right), \mu A_\mu Y^\mu_s\right\rangle_{V^*} \\
&\quad + {}_V\left\langle J^{-1}\left(\lambda A_\lambda Y^\lambda_s\right), A_\mu Y^\mu_s\right\rangle_{V^*} + {}_V\left\langle J^{-1}\left(\mu A_\mu Y^\mu_s\right), A_\lambda Y^\lambda_s\right\rangle_{V^*} \\
&\le -\lambda\left|A_\lambda Y^\lambda_s\right|^2_{V^*} - \mu\left|A_\mu Y^\mu_s\right|^2_{V^*} + \mu\left|A_\mu Y^\mu_s\right|_{V^*}\left|A_\lambda Y^\lambda_s\right|_{V^*} \\
&\quad + \lambda\left|A_\mu Y^\mu_s\right|_{V^*}\left|A_\lambda Y^\lambda_s\right|_{V^*} \le \frac{\lambda+\mu}{2}\left(\left|A_\lambda Y^\lambda_s\right|^2_{V^*} + \left|A_\mu Y^\mu_s\right|^2_{V^*}\right),
\end{aligned}
$$

where we have used the elementary inequality $2ab \le a^2 + b^2$. Here, by applying the expectation and Lipschitz condition **Hyp.1** of f, we get

$$
\begin{aligned}
&\mathbb{E}|Y^\lambda_t - Y^\mu_t|^2_H + \int_t^T \|Z^\lambda_s - Z^\mu_s\|^2_{L_2(\Xi,H)}ds + \int_t^T \mathbb{E}|Q^\lambda_s - Q^\mu_s|^2_{L^2(\nu)}ds \le \\
&\frac{\lambda+\mu}{2}\left[\int_t^T \mathbb{E}\left|A_\lambda Y^\lambda_s\right|^2_{V^*}ds + \int_t^T \mathbb{E}\left|A_\mu Y^\mu_s\right|^2_{V^*}ds\right] \\
&-2\int_t^T \left\langle Y^\lambda_s - Y^\mu_s, f\left(t, Y^\lambda_s, Z^\lambda_s, Q^\lambda_s\right) - f\left(t, Y^\mu_s, Z^\mu_s, Q^\mu_s\right)\right\rangle_H ds \\
&\le \alpha\int_t^T \mathbb{E}|Y^\lambda_s - Y^\mu_s|^2_H ds + \frac{1}{\alpha}\int_t^T \mathbb{E}\left[f\left(t, Y^\lambda_s, Z^\lambda_s, Q^\mu_s\right) - f\left(t, Y^\mu_s, Z^\mu_s, Q^\mu_s\right)\right]^2_H ds \\
&+\frac{\lambda+\mu}{2}\left[\int_t^T \mathbb{E}\left|A_\lambda Y^\lambda_s\right|^2_{V^*}ds + \int_t^T \mathbb{E}\left|A_\mu Y^\mu_s\right|^2_{V^*}ds\right] \le \alpha\int_t^T \mathbb{E}|Y^\lambda_s - Y^\mu_s|^2_H ds \\
&+\frac{C}{\alpha}\int_t^T \mathbb{E}\left[\left|Y^\lambda_s - Y^\mu_s\right|^2_H + \left|Z^\lambda_s - Z^\mu_s\right|^2_{L_2(\Xi,H)} + \left|Q^\lambda_s - Q^\mu_s\right|^2_{L_2(\nu)}\right]ds \\
&+\frac{\lambda+\mu}{2}\left[\int_t^T \mathbb{E}\left|A_\lambda Y^\lambda_s\right|^2_{V^*}ds + \int_t^T \mathbb{E}\left|A_\mu Y^\mu_s\right|^2_{V^*}ds\right] \le \left(\alpha + \frac{C}{\alpha}\right)\int_t^T \mathbb{E}|Y^\lambda_s - Y^\mu_s|^2_H ds \\
&+\frac{C}{\alpha}\int_t^T \mathbb{E}\left[\left|Z^\lambda_s - Z^\mu_s\right|^2_{L_2(\Xi,H)} + \left|Q^\lambda_s - Q^\mu_s\right|^2_{L_2(\nu)}\right]ds \\
&+\frac{\lambda+\mu}{2}\left[\int_t^T \mathbb{E}\left|A_\lambda Y^\lambda_s\right|^2_{V^*}ds + \int_t^T \mathbb{E}\left|A_\mu Y^\mu_s\right|^2_{V^*}ds\right].
\end{aligned}
$$

Then, we obtain

$$
\begin{aligned}
\mathbb{E}|Y^\lambda_t - Y^\mu_t|^2_H &\le \mathbb{E}|Y^\lambda_t - Y^\mu_t|^2_H + \int_t^T \mathbb{E}\|Z^\lambda_s - Z^\mu_s\|^2_{L_2(\Xi,H)}ds \\
&+ \int_t^T \mathbb{E}\left|Q^\lambda_s - Q^\mu_s\right|^2_{L^2(\nu)}ds \le \left(\alpha + \frac{C}{\alpha}\right)\int_t^T \mathbb{E}|Y^\lambda_s - Y^\mu_s|^2_H ds + B_{\lambda,\mu},
\end{aligned}
\tag{19}
$$

where

$$B_{\lambda,\mu} = \frac{C}{\alpha}\int_t^T \mathbb{E}\Big[\big|Z_s^\lambda - Z_s^\mu|^2_{L_2(\Xi,H)} + \big|Q_s^\lambda - Q_s^\mu|^2_{L_2(\nu)}\Big]ds$$
$$+\frac{\lambda+\mu}{2}\left[\int_t^T \mathbb{E}\Big|A_\lambda Y_s^\lambda|^2_{V^*}ds + \int_t^T \mathbb{E}\Big|A_\mu Y_s^\mu|^2_{V^*}ds\right].$$

Using Gronwall lemma, this shows that $\mathbb{E}|Y_t^\lambda - Y_t^\mu|_H^2 \le B_{\lambda,\mu}e^{\frac{C}{\alpha}(T-t)}$, which plugged in the inequality (19) provides

$$\begin{aligned}\mathbb{E}|Y_t^\lambda - Y_t^\mu|_H^2 + \int_t^T \mathbb{E}\|Z_s^\lambda - Z_s^\mu\|^2_{L_2(\Xi,H)}ds + \int_t^T \mathbb{E}|Q_s^\lambda - Q_s^\mu|^2_{L^2(\nu)}ds &\le B_{\lambda,\mu}(1+C_1(T-t)e^{(C_1(T-t))})\\ &\le B_{\lambda,\mu}(1+C_2(T-t)),\end{aligned}$$

where $C_1 = \left(\alpha + \frac{C}{\alpha}\right)$ and $C_2 = C_1e^{(C_1(T-t))}$. Then, we have

$$\int_t^T \mathbb{E}\|Z_s^\lambda - Z_s^\mu\|^2_{L_2(\Xi,H)}ds + \int_t^T \mathbb{E}|Q_s^\lambda - Q_s^\mu|^2_{L^2(\nu)}ds \le \mathbb{E}|Y_t^\lambda - Y_t^\mu|_H^2 + \int_t^T \mathbb{E}\|Z_s^\lambda$$
$$-Z_s^\mu\|^2_{L_2(\Xi,H)}ds + \int_t^T \mathbb{E}|Q_s^\lambda - Q_s^\mu|^2_{L^2(\nu)}ds \le (1+C_2(T-t))$$
$$\times\left[\frac{C}{\alpha}\int_t^T \mathbb{E}\Big[\big\|Z_s^\lambda - Z_s^\mu\|^2_{L_2(\Xi,H)} + \big|Q_s^\lambda - Q_s^\mu|^2_{L_2(\nu)}\Big]ds\right.$$
$$\left.+\frac{\lambda+\mu}{2}\left[\int_t^T \mathbb{E}\Big|A_\lambda Y_s^\lambda|^2_{V^*}ds + \int_t^T \mathbb{E}\Big|A_\mu Y_s^\mu|^2_{V^*}ds\right]\right].$$

By subtraction, we have:

$$\left[1-(1+C_2(T-t))\frac{C}{\alpha}\right)\Bigg]\int_t^T \mathbb{E}\Big\|Z_s^\lambda - Z_s^\mu\|^2_{L_2(\Xi,H)} + \Big\|Q_s^\lambda - Q_s^\mu\|^2_{L_2(\nu)}ds$$
$$\le\frac{\lambda+\mu}{2}\left[\int_t^T \mathbb{E}\Big|A_\lambda Y_s^\lambda|^2_{V^*}ds + \int_t^T \mathbb{E}\Big|A_\mu Y_s^\mu|^2_{V^*}ds\right].$$

For α larger than $(1 + C_2(T - t))C$, this provides that there exists $D > 0$, such that

$$\int_t^T \mathbb{E}\|Z_s^\lambda - Z_s^\mu\|^2_{L_2(\Xi,H)} + |Q_s^\lambda - Q_s^\mu|^2_{L_2(\nu)}ds \le D\frac{\lambda+\mu}{2}\left[\int_t^T \mathbb{E}\Big|A_\lambda Y_s^\lambda|^2_{V^*}ds + \mathbb{E}\Big|A_\mu Y_s^\mu|^2_{V^*}ds\right].$$

By using the same idea for the jump part and plugging in (19), we deduce that

$$\mathbb{E}|Y_t^\lambda - Y_t^\mu|_H^2 \le D\left(\frac{\lambda+\mu}{2}\right)\left[\int_t^T \mathbb{E}\Big|A_\lambda Y_s^\lambda|^2_{V^*}ds + \int_t^T \mathbb{E}\Big|A_\mu Y_s^\mu|^2_{V^*}ds\right].$$

Using that A_λ and A_μ, we verify the boundedness condition introduced in Remark 3.5, and the result holds.

Remark 4.2. By using Lemma 3.4, for λ goes to 0, we prove that $(Y^{\lambda}, Z^{\lambda}, Q^{\lambda})$ goes to the triplet (Y, Z, Q) in the space $L^2(\Omega, H) \times L_2(\Xi, H) \times L_2(\nu)$.

The following theorem shows that the limit (Y, Z, Q) is a solution of equation (12).

Theorem 4.3. Under **Hyp.1**, we have

$$\sup_{t\in[0,T]}\mathbb{E}|Y_t - Y_t^{\lambda}|_H^2 + \int_t^T \mathbb{E}\|Z_s - Z_s^{\lambda}\|_{L_2(\Xi,H)}^2 ds + \int_t^T \mathbb{E}|Q_s^{\lambda} - Q_s|_{L^2(\nu)}^2 ds \to 0. \tag{20}$$

where $(Y, Z, Q) \in L^2(\Omega, H) \times L_2(\Xi, H) \times L^2(\nu)$ is the unique solution of (12).

Proof. Proposition 4.1 yields $(Y^{\lambda})_{\lambda\geq 0}$ and $(Z^{\lambda})_{\lambda\geq 0}$ which are predictable Cauchy family approximating equations in complete spaces $L^2(\Omega, H)$ and $L^2(\Xi, H)$ and $(Q^{\lambda})_{\lambda\geq 0}$, a progressively measurable Cauchy family approximating equations in $L^2(\nu)$; then there exists a predictable processes Y, Z and Q, respectively $\mathcal{F}$-progressively measurable such that the sequences $(Y^{\lambda})_{\lambda\geq 0}$ and $(Z^{\lambda})_{\lambda\geq 0}$ and $(Q^{\lambda})_{\lambda\geq 0}$ converge, respectively, toward Y in $L^2(\Omega, H)$ Z in $L_2(\Xi, H)$ and Q in $L^2(\nu)$.

Now, it is sufficient to prove that this triplet (Y, Z, Q) coincides with the solution of (12). Therefore,

$$\begin{aligned}
&\mathbb{E}\left|Y_t - X + \int_t^T AY_u + f(u, Y_u, Z_u)du + \int_t^T Z_u dW_u + \int_t^T \int_E Q_u \tilde{N}(du, dx)\right|_H^2 \\
&\leq 2\mathbb{E}|Y_t - Y_t^{\lambda}|_H^2 + 2E\left|Y_t^{\lambda} - X + \int_t^T AY_u\right. \\
&\left. + f(u, Y_u, Z_u)du + \int_t^T Z_u dW_u + \int_t^T \int_E Q_s(x)\tilde{N}(ds, dx)\right|_H^2 \\
&\leq 8\left[\mathbb{E}\left|\int_t^T AY_u - A_{\lambda}Y_u^{\lambda}du\right|_H^2 + \mathbb{E}\left|\int_t^T \left(f(u, Y_u, Z_u, Q_u) - f\left(u, Y_u^{\lambda}, Z_u^{\lambda}, Q_u^{\lambda}\right)\right)du\right|_H^2\right. \\
&\left. + \mathbb{E}\left|\int_t^T (Z_u - Z_u^{\lambda})dW_u\right|_H^2 + \mathbb{E}\left|\int_t^T \int_E (Q_u - Q_u^{\lambda})\tilde{N}(du, dx)\right|_H^2\right] + 2\mathbb{E}|Y_t - Y_t^{\lambda}|_H^2 \\
&= 8[I_1 + I_2 + I_3 + I_4] + 2I_5.
\end{aligned}$$

We estimate each term separately. First note that, thanks to Hille Yosida approximation and [[17], Lemma 3.9], we have

$$\lim_{\lambda\to 0} A_{\lambda}x = Ax, \quad \textit{for all } x \in D(A). \tag{21}$$

then

$$\begin{aligned}
I_1 &\leq 2\left[\mathbb{E}\int_t^T \left|A_{\lambda}\left(Y_u - Y_u^{\lambda}\right)\right|_H^2 du + \left[\mathbb{E}\right|\int_t^T |(A - A_{\lambda})Y_u|_H^2 du \\
&\leq \int_t^T \left[-\tilde{c}_2\mathbb{E}\left|Y_u - Y_u^{\lambda}\right|_V^2 + c_1\mathbb{E}\left|Y_u - Y_u^{\lambda}\right|_H^2\right] ds \\
&\leq C\int_t^T \mathbb{E}\left|Y_u - Y_u^{\lambda}\right|_H^2\Big] ds \to 0
\end{aligned}$$

when $\lambda \to 0$ using Proposition 4.1. The term I_2 is estimated by applying the Lipschitz condition with respect to Cauchy Schwartz inequality, and this yields

$$\begin{aligned} I_2 &\leq 2\mathbb{E}\left|\int_t^T \left(f(u, Y_u, Z_u, Q_u) - f\left(u, Y_u^\lambda, Z_u^\lambda, Q_u^\lambda\right)\right) du\right|_H^2 \\ &\leq (T-t)\mathbb{E}\int_t^T \left|f(u, Y_u, Z_u, Q_u) - f\left(u, Y_u^\lambda, Z_u^\lambda, Q_u^\lambda\right)\right|_H^2 du \\ &\leq C(T-t)\mathbb{E}\int_t^T \left(\left|Y_u - Y_u^\lambda\right|_H^2 + \left|Z_u - Z_u^\lambda\right|_{L_2(\Xi,H)}^2 + \left|Q_u - Q_u^\lambda\right|_{L_2(\nu)}^2\right) du \\ &\to 0 \end{aligned}$$

where $\lambda \to 0$.

Finally, the terms I_3, I_4 and I_5 are covered by Proposition 4.1. Then the results holds.

Corollary 4.4. Assume that **Hyp.1** holds, then there exists a unique triplet $(Y, Z, Q) \in L^2(\Omega, H) \times L_2(\Xi, H) \times L^2(\nu)$ which satisfies (12), such that:

$$\sup_{t\in[0,T]}\mathbb{E}\left|Y_t - Y_t^\lambda\right|_H^2 + \int_t^T \mathbb{E}\|Z_s - Z_s^\lambda\|_{L_2(\Xi,H)}^2 ds + \int_t^T \int_E \mathbb{E}|Q_s^\lambda - Q_s|_{L^2(\nu)}^2 ds \leq C\lambda. \qquad (22)$$

Proof. Thanks to Proposition 4.1, we compute:

$$\begin{aligned} &\sup_{t\in[0,T]}\mathbb{E}|Y_t^\lambda - Y_t|_H^2 + \int_t^T \mathbb{E}\|Z_s - Z_s^\lambda\|_{L_2(\Xi,H)}^2 ds + \int_t^T \mathbb{E}|Q_s^\lambda - Q_s|_{L^2(\nu)}^2 ds \leq 2\sup_{t\in[0,T]}\mathbb{E}|Y_t^\lambda \\ &\quad - Y_t^\mu|_H^2 + 2\sup_{t\in[0,T]}\mathbb{E}|Y_t^\mu - Y_t|_H^2 + 2\int_t^T \mathbb{E}\|Z_s^\mu - Z_s\|_{L_2(\Xi,H)}^2 ds + 2\int_t^T \mathbb{E}\|Z_s^\lambda \\ &\quad - Z_s^\mu\|_{L_2(\Xi,H)}^2 ds + 2\int_t^T \mathbb{E}|Q_s^\mu - Q_s|_{L_2(\nu)}^2 ds + 2\int_t^T \mathbb{E}|Q_s^\lambda - Q_s^\mu|_{L^2(\nu)}^2 ds \leq 2D(\lambda+\mu) \\ &\quad + 2\sup_{t\in[0,T]}\mathbb{E}|Y_t^\mu - Y_t|_H^2 + 2\int_t^T \mathbb{E}\|Z_s^\mu - Z_s\|_{L_2(\Xi,H)}^2 ds + 2\int_t^T \mathbb{E}|Q_s^\mu - Q_s|_{L^2(\nu)}^2 ds. \end{aligned}$$

Then, when μ goes to zero, applying Lebesgue dominated convergence theorem yields:

$$\sup_{t\in[0,T]}\mathbb{E}|Y_t - Y_t^\lambda|_H^2 + \int_t^T \mathbb{E}\|Z_s - Z_s^\lambda\|_{L_2(\Xi,H)}^2 ds + \int_t^T \mathbb{E}|Q_s^\lambda - Q_s|_{L^2(\nu)}^2 ds \leq 2D\lambda.$$

Example 4.5. Let an open set $\Lambda \subset \mathbb{R}^d$, and denote by $C_0^\infty(\Lambda)$ the set of all infinitely differentiable real valued functions defined on Λ with compact support. For $u \in C_0^\infty(\Lambda)$ let us define

$$\|u\|_{1,2} := \left(\int \left(\left|u(\xi)\right|^2 + \left|\Delta u(\xi)\right|^2\right) d\xi\right) \frac{1}{2}.$$

Let us define $H_0^{1,2}(\Lambda)$ by the completion of $C_0^\infty(\Lambda)$ with respect to $\|\cdot\|_{1,2}$. Then, for $A = -\Delta$ and $H_0^{1,2} \subset L^2 \subset (H_0^{1,2})^*$, A satisfies **(L1)**.

Proof. For the detailed proof, we refer to [28] [p. 62].

Example 4.6. [27] For the case V = H = V*. If A is a Lipschitz function, the Yosida approximation [8] [Proposition 2.3] is given by

$$A_\lambda x = \frac{1}{\lambda}(x - J_\lambda x), \;\; x \in H. \tag{23}$$

where the resolvent J_λ of A is defined on H by

$$J_\lambda = (I + \lambda A)^{-1}. \tag{24}$$

A satisfies **(L1)**.

Proof. For more details, we refer to [[28], p. 59].

Example 4.7. [28] Let $p > 2, \Gamma \in R^n$, let $V := L^p(\Gamma), H := L^2(\Gamma)$ and $V^* := \left(L^{\frac{p}{p-1}}(\Gamma)\right)$, we define A: D(A) = V → V*, by $Au := -u|u|^{p-2}$, $u \in V$. Then, A satisfies (**L1**).

Proof. For a detailed proof, we refer to [28] [p. 61] [].

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