MATHEMATICAL AND ECONOMIC THEORY OF ROAD PRICING

DRESNER (ed.)
Research in Transportation Economics

HENSHER & BUTTON (eds.)
Handbooks in Transport

Transport Policy
Editors: Moshe Ben-Akiva, Yoshitsugu Hayashi and John Preston

Transportation Research Part A: Policy and Practice
Editor: Phil Goodwin

Transportation Research Part B: Methodological
Editor: Fred Mannering

MATHEMATICAL AND ECONOMIC THEORY OF ROAD PRICING

HAI YANG
Department of Civil Engineering
The Hong Kong University of Science and Technology
Hong Kong, P. R. China

HAI-JUN HUANG
School of Management
Beijing University of Aeronautics and Astronautics
Beijing 100083, P. R. China

Emerald

United Kingdom – North America – Japan
India – Malaysia – China

Emerald Group Publishing Limited
Howard House, Wagon Lane, Bingley BD16 1WA, UK

First edition 2005

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

ISBN: 978-0-08-045671-3

ISOQAR QUALITY ASSURED ISO 9001
UKAS QUALITY MANAGEMENT 026
Certificate number 1985

Awarded in recognition of Emerald's production department's adherence to quality systems and processes when preparing scholarly journals for print

INVESTOR IN PEOPLE

ACRONYMS

AC	Average Cost
BLPP	Bi-Level Programming Problem
BOT	Build-Operate-Transfer
BPR	Bureau of Public Roads
BLVI	Bi-Level Variational Inequality
CN	Cournot-Nash
DUE	Deterministic User Equilibrium
EB	Economic Benefit
EPEC	Equilibrium Problem with Equilibrium Constraints
KKT	Karush-Kuhn-Tucker
LP	Linear Program
MC	Marginal Cost
MPEC	Mathematical Program with Equilibrium Constraints
OD	Origin -Destination
SC	Social Cost
SO	System Optimum
SW	Social Welfare
STEN	Space-Time Expanded Network
SUE	Stochastic User Equilibrium
UB	User Benefit
UE	User Equilibrium
VI	Variational Inequality
VOT	Value Of Time

Glossary of Notation

a	a link $a \in A$
A	the set of all links
$\bar{A}$	the subset of links subject to a toll charge
$\tilde{A}$	the subset of links with observed flows
$B_w(d_w)$	the benefit function between OD pair $w \in W$
$B_w^m(d_w^m)$	the benefit function of user class $m \in M$ between OD pair $w \in W$
c_{rw}	the travel cost on path r connecting OD pair $w \in W$
$\mathbf{c}$	the vector of path travel costs
c_{rw}^m	the travel cost on path r between OD pair w by users of class $m \in M$
C_a	the capacity of link $a \in A$
d_w	the travel demand between OD pair $w \in W$
$\mathbf{d}$	the vector of OD demands
$\hat{D}_w$	the potential demand or the upper-bound of demand between OD pair $w \in W$
d_w^m	the travel demand of user class m between OD pair $w \in W$
$\hat{D}_w^m$	the potential demand of user class m between OD pair $w \in W$
$D_w(\mu_w)$	the demand function between OD pair $w \in W$
$D_w^{-1}(d_w)$	the inversed demand function between OD pair $w \in W$
E_v^u	Elasticity of v to u, $E_v^u = (u/v)\mathrm{d}v(u)/\mathrm{d}u$
f_{rw}	the flow on path $r \in R_w$
$\mathbf{f}$	the vector of path flows
f_{rw}^m	the flow of user class $m \in M$ on path $r \in R_w$ between OD pair $w \in W$
G	(N, A) be a network with node set N and link set A
K	the set of Cournot-Nash (CN) players or the set of vehicle types
$I_a(y_a)$	the construction cost of link a as a function of link capacity y_a
m	a user class $m \in M$
M	the set of all user classes with different values of time
N	the set of all nodes
r	a path or a route $r \in R$
R_w	the set of all paths connecting OD pair $w \in W$

R	$=\bigcup_{w\in W} R_w$
t_a^0	the free-flow travel time (moving time on link $a\in A$
$t_a(v_a)$	the travel time on link $a\in A$ as a function of link flow v_a
$\tilde{t}_a(v_a)$	the marginal social travel time on link $a\in A$, including the congestion externality, $\tilde{t}_a(v_a)=t_a(v_a)+v_a\,\mathrm{d}t_a(v_a)/\mathrm{d}v_a$
$t_a(v_a, y_a)$	the link travel time as a function of both flow and capacity variables
$\mathbf{t}(\mathbf{v})$	the vector of link travel times
u_a	toll charge on link $a\in A$
$\mathbf{u}$	the vector of link tolls
v_a	the flow on link $a\in A$
$\mathbf{v}$	the vector of link flows
v_a^m	the flow of user class m on link $a\in A$
v_a^k	the link flow of vehicle type $k\in K$ or CN player $k\in K$
w	an OD pair $w\in W$
W	the set of all OD pairs
W^k	the set of OD pairs of which users are controlled by CN player $k\in K$
W^K	$=\bigcup_{k\in K} W^k$
W^m	the set of OD pairs of which users are controlled by UE class $m\in M$
W^M	$=\bigcup_{m\in M} W^m$
y_a	the capacity of a new link a or capacity increase of an existing link a
$\mathbf{y}$	the vector of link capacity variables
β	the users' value of time
β_m	the average value of time of user class $m\in M$
δ_{ar}	1, if link $a\in A$ is on path $r\in R$ and 0, otherwise
Δ	the link/path incidence matrix, $\Delta=[\delta_{ar}]$
ε	a predetermined tolerance for stopping iterative process
Λ_{rw}	1 if path $r\in R_w$ and 0, otherwise
Λ	the OD/path incidence matrix, $\Lambda=[\Lambda_{rw}]$
μ_w	the Lagrange multiplier associated with the demand conservation constraint of OD pair $w\in W$, or the generalized travel cost between OD pair $w\in W$ at equilibrium
μ_w^m	the generalized travel cost of user class $m\in M$ between OD pair $w\in W$
μ	the vector of generalized OD travel costs

Ω	the feasible region of link flows and OD flows
Ω_f	the feasible region of path flows
Ω_v	the feasible region of link flows
Ω^k	the feasible region of links flows of CN player $k \in K$
Ω^m	the feasible region of link flows of a UE class $m \in M$
Ω^U	the feasible region of link flows of UE player
$\nabla_{\mathbf{v}} \mathbf{t}(\mathbf{v})$	the Jacobian matrix of link travel time function **t** with respect to **v**
$\in$	element membership
$\subset$	proper set inclusion
$\cup$	union of sets
∞	infinity
$\lvert \cdot \rvert$	the cardinality of a finite set
$\lVert \cdot \rVert$	the Euclidean norm
$\arg\min_x F(x)$	the set of x attaining the minimum of the function $F(x)$

PREFACE

Road pricing as an effective means of both managing road traffic demand and raising additional revenue for road construction has been studied extensively by both transportation researchers and economists. Practical implementation has been progressing rapidly and electronic road pricing schemes have been proposed and tested worldwide. It is likely that over the next few years, with increasing public acceptability, there will be greater use of road pricing. The incontestable fact is that there is a great need for the development of efficient road-use pricing models.

Despite a few monographs and journal special issues in the literature that have been devoted to the topic in recent years, most studies have focused on empirical studies, policy experience, environmental issue of road congestion and road pricing. There is still scope for methodological development of advanced road pricing systems, such as dynamic pricing, integrated road and transit pricing, as well as practical toll charging schemes in general road networks. This book is intended to deal with a number of timely topics including: fundamentals of user-equilibrium problems; principle of marginal-cost pricing applied to road pricing; existence of optimal link tolls for system optimum under multi-class, multi-criteria, multiple equilibrium behaviors; social and spatial equity issues in road pricing; optimal design of private toll roads; simultaneous determination of optimal toll levels and locations; trial-and-error implementation of marginal-cost pricing on networks with unknown demand functions; dynamic road pricing.

This book would appear to be the first book devoted exclusively to the mathematical and economic investigation of road-use pricing in general congested networks, which aims at alleviating traffic congestion, improving transport conditions and enhancing social welfare. It constitutes an update of the state of the art of the latest research, mainly by the authors and their colleagues. The book is targeted at students, professionals and scientists who are studying and working in relevant transportation fields.

We are most thankful to a number of individuals for their help during our work in this field. First and foremost we would like to acknowledge the contribution and continuing encouragement of Prof. M.G.H. Bell of Imperial College London (ICL). In fact, the manuscript was finalized during the first author's sabbatical leave at ICL in early 2005, and he is particularly grateful to Prof. Bell for his hospitality during a very fruitful and enjoyable stay in ICL. Thanks are also due to Prof. R. Lindsey of the University of Alberta, Prof. E.

Verhoef of the Free University of Amsterdam, Prof. W.H.K. Lam of the Hong Kong Polytechnic University and Prof. S.C. Wong of the University of Hong Kong throughout our study of the various road pricing problems and in the preparation of this book. We highly appreciate our research collaboration that produced some of the material included in the book. We also wish to greatly thank three former Ph.D students at the Hong Kong University of Science and Technology, Dr. Qiang Meng (currently at the National University of Singapore), Dr. Xiaoning Zhang (currently at the Tongji University of Shanghai) and Dr. Judith Wang (currently at the University of Auckland) for their contributions to various topics in the book. Thanks also go to Miss Wei Xu and Mr Xiaolei Guo (currently PhD students at the Hong Kong University of Science and Technology) who helped in discussions in earlier drafts of the book. Funding for the research was provided by the Research Grants Council of the Hong Kong Special Administrative Region (HKSAR) and the National Natural Science Foundation, P.R. China.

Hai Yang
Hai-Jun Huang

31 May 2005