

# Joint method using Akamatsu and discrete wavelet transform for image restoration

Jihad Maulana Akbar and De Rosal Ignatius Moses Setiadi  
*Department of Informatics Engineering, Faculty of Computer Science,  
Dian Nuswantoro University, Semarang, Indonesia*

## Abstract

Current technology makes it easy for humans to take an image and convert it to digital content, but sometimes there is additional noise in the image so it looks damaged. The damage that often occurs, like blurring and excessive noise in digital images, can certainly affect the meaning and quality of the image. Image restoration is a process used to restore the image to its original state before the image damage occurs. In this research, we proposed an image restoration method by combining Wavelet transformation and Akamatsu transformation. Based on previous research Akamatsu's transformation only works well on blurred images. In order not to focus solely on blurry images, Akamatsu's transformation will be applied based on Wavelet transformations on high-low (HL), low-high (LH), and high-high (HH) subunits. The result of the proposed method will be comparable with the previous methods. PSNR is used as a measure of image quality restoration. Based on the results the proposed method can improve the quality of the restoration on image noise, such as Gaussian, salt and pepper, and also works well on blurred images. The average increase is around 2 dB based on the PSNR calculation.

**Keywords** Akamatsu transform, Blurred image, Discrete wavelet transform, Image Denoising, Image restoration

**Paper type** Original Article

## 1. Introduction

An image is a discrete representation of data that has layout information and the color intensity of an object [1]. Nowadays, an image can be easily converted into digital content, but sometimes there is additional noise in the image during camera capture so that the image appears damaged. This operation of repairing damage is called image restoration. Damage can occur when shooting can be caused by the quality of the digitizer, poor camera focus, and signal noise from sensors, so that noise or image appears blurry [2]. Image restoration is the process to obtain a clean original image of the damaged image or that exposed to noise [3,4].

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The image restoration has been done with various methods, such as Akamatsu transform [5], Wavelet Transform [6–8], Anscombe transforms [9], Stochastic Resonance [7], Singular Value Decomposition (SVD) [6,10], Adaptive Histogram Equalization [6].

Karungaru et al. [5] used Akamatsu transformation to restore the image. Akamatsu transform is a transformation technique developed by Norio Akamatsu. Akamatsu transform produces a differential value showing the characteristics of the image and by scaling between the integral and differential values can perform image restoration. From this integral and differential values, there is a scale adjustment between the integral and the differential values to obtain the image of the restoration result. The result of restoration with this method is very good only on the blurred image because it can increase the energy in the image, but produce side effects as other noises appear sharper.

Megha et al. [6] proposed Adaptive Histogram Equalization and DWT techniques to restore the image. The first step of image restoration is done by decomposing using Discrete Wavelet Transform to get four low-low (LL), low-high (LH), high-low (HL), and high-high (HH) sub-domains. Sub-band LL is selected then copied to LL2 to be applied to Adaptive Histogram Equalization. Sub-band LL and LL2 subsequently decomposed using SVD to produce  $U_1$ ,  $S_1$ ,  $V_1$  and  $U_2$ ,  $S_2$ ,  $V_2$ . Then the calculation of both SVD decomposition to determine the improvement factor. The results of this restoration method are good for removing noise but making the energy in the image is reduced.

SVD is a transformation that has been widely used in digital image processing [10–12]. SVD is a technique for obtaining geometric features of an image. Basically, SVD is used to handle matrices that do not have an inverse. The SVD has several characteristics that the singular value change does not significantly alter the overall image and the singular value contains illumination information, while the singular vector retains the information from the image.

Wavelet Transform is a technique for stable signals and not stationary. Because of its efficiency in analyzing local discontinuities of a signal, the Wavelet Transform has been widely used in various disciplines [13–16]. DWT is one form of Wavelets Transform in which the image signal is passed through a pile of analysis filters followed by a subtraction operation. DWT decomposes images into 4 sub-domains: low-low (LL), low-high (LH), high-low (HL), and high-high (HH). LL is the low-frequency signal which is the approximation of the original image. LH and HL are intermediate frequency signals. And HH is a high-frequency signal. LH, HL, and HH represent changes in information in the image [13,17]. So DWT is very good at representing a signal with singularity. However, DWT is not suitable for representing fine image signals and colored noise so it needs to be combined with other methods to improve image restoration results [18].

Akamatsu transform proved excellent for restoring the image blur due to produce a different value which is characteristic of the image. This value will be strengthened and can reduce the blur in the image. While the use of low sub-band on wavelet transforms can reduce noise but make significant energy changes in the image. Based on this background, this research proposes to combine DWT and Akamatsu to get more optimal results of image restoration. Sub-bands LH, HL, and HH were chosen to apply Akamatsu transformation. To test the quality of restoration method will be given various image noise such as Gaussian noise, salt & pepper, and blur. The result of the restoration will be calculated by the PSNR and compared with the method contained in the research Karungaru et al. [5] and Megha P et al. [6].

## 2. Akamatsu and wavelet transform

### 2.1 Akamatsu transform

Akamatsu transformation is a transformation that produces the integral and differential values of an input signal [5,19]. Akamatsu transformation requires a target signal that is

defined as  $P(x,y)$  Where the target signal is in the transformation. With the presence of the target signal, the result of the transformation will be divided into two parts, namely the right of the target signal and the target signal left. The signals to the right of  $P(x,y)$  are defined as  $VRS(x)$  and left as  $VLS(x)$ . The target signal is used to determine the result of Akamatsu transformation. To see more clearly the description of the target signal on Akamatsu transform can see [Figure 1](#).

To achieve the target signal required the calculation of the integral value and the differential value. To get an Integral value can use Eq. (1).

$$A(x) = \frac{[AvrSR(x) + AvrSL(x)]}{2} \tag{1}$$

with  $AvrSR(x)$  and  $AvrSL(x)$  obtained from Eqs. (2) and (3).

$$AvrSR(x) = \frac{S_R(x)}{N} \tag{2}$$

$$AvrSL(x) = \frac{S_L(x)}{N} \tag{3}$$

While the first order of the integral value of the Akamatsu transform is defined by Eq. (4).

$$[A(x)]^{1st} = A(x) \tag{4}$$

Thus, the member values of  $VRS(x)$  and  $VLS(x)$  can be written by Eqs. (5) and (6).

$$VRS(x) = P(x + 1, y), P(x + 2, y), \dots, P(x + N, y) \tag{5}$$

$$VLS(x) = P(x - 1, y), P(x - 2, y), \dots, P(x - N, y) \tag{6}$$

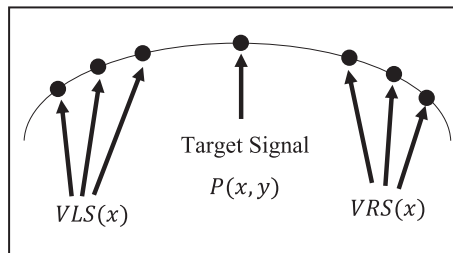
$N$  is an optional constant value that can be changed in every order. While the sum of  $VRS(x)$ , defined as  $S_R$  can be calculated by Eq. (7).

$$S_R = \sum_{k=1}^N P(x + k) \tag{7}$$

and the sum of  $VLS(x)$ , defined as  $S_L$  which can be calculated by Eq. (8).

$$S_L = \sum_{k=1}^N P(x - k) \tag{8}$$

Having obtained the value of Integral it can be obtained deferential value. The differential value ( $D$ ) is obtained by calculating the difference of the original pixel value with the integral value found in Eq. (9).



**Figure 1.**  
Target signal in  
Akamatsu Transform.

$$[D(x)]^{1st} = P(x) - [A(x)]^{1st} \quad (9)$$

In previous research, the Akamatsu transformation was calculated in the x-direction. But in this research Akamatsu is also applied in the y-direction. Akamatsu transforms applied in y-direction also divides the signal into two parts corresponding to the target signal  $P(x, y)$ . The section located above the  $P(x, y)$  is called  $VTS(y)$ , which can be calculated by Eq. (10) and the part that is under the  $P(x, y)$  is called  $VBS(y)$ , which can be calculated by Eq. (11).

$$VTS(y) = P(x, y + 1), P(x, y + 2), \dots, P(x, y + N) \quad (10)$$

$$VBS(y) = P(x, y - 1), P(x, y - 2), \dots, P(x, y - N) \quad (11)$$

Similar to the x-direction Akamatsu transform, the integral value of  $A(y)$  is obtained by Eq. (12).

$$A(y) = \frac{[AvrS_T(y) + AvrS_B(y)]}{2} \quad (12)$$

With  $AvrS_T(y)$  and  $AvrS_B(y)$  derived from Eqs (13) and (14).

$$AvrS_T(y) = \frac{S_R(y)}{N} \quad (13)$$

$$AvrS_B(y) = \frac{S_L(y)}{N} \quad (14)$$

With the first known order from Akamatsu transform then it can be continued to the next order with the above equation as many as n order, so that is obtained Eq. (15) to calculate Akamatsu transform.

$$\begin{aligned} [A(x)]^{1st} &= A(x) \\ [D(x)]^{1st} &= P(x) - [A(x)]^{1st} \\ [A(x)]^{nth} &= A(x) \\ [D(x)]^{nth} &= P(x) - [A(x)]^{nth} \end{aligned} \quad (15)$$

After obtained the integral and differential values, the new pixel value for x-direction is obtained by Eq. (16).

$$P_{new}(x, y) = up\_enh \times D(x) + down\_enh \times A(x) \quad (16)$$

As for y-direction, a new pixel value is obtained by Eq. (17).

$$P_{new}(x, y) = up\_enh \times D(y) + down\_enh \times A(y) \quad (17)$$

where  $up\_enh$  is an increasing value to increase the value of the differential waveform to obtain a clearer image. While  $down\_enh$  is increasing value to decrease the integral waveform value so that the new pixel value does not exceed the 255 limit value.

## 2.2 Discrete wavelet transform

The use of wavelet transforms has been widely used in digital image processing and pattern recognition [20]. This technology also takes an important role in digital image processing. Decomposition using wavelet transform produces two-dimensional functions of time and space. Wavelet decomposition is also helpful in recognizing singularity details by analyzing the time frequencies of non-stationary signals [21–23]. Furthermore, outside there have been

many forms of developed wavelet transforms and each has different characteristics. The discrete wavelet transform is one form of the wavelet transform, such as complex wavelet transform (CWT), Slantlet transforms (SLT) [23,24]. DWT has many good features such as multi-resolution capabilities and space-frequency localization property that makes DWT can be applied to the entire image and can be reconstructed in several image size [25]. In DWT, images are divided into 4 sub-bands: Low-Low (LL), Low-High (LH), High-Low (HL), High-High (HH). The subband distribution process is carried out with two kinds of filters namely low pass and high pass filter. There are various kinds of filters that can be used such as Daubechies and Haar which are the most popular filters [26–28]. Haar filters have the advantage of being simple in structure so that they can reduce memory usage and are preferred for analyzing compact discrete signals [27]. Each sub-band holds important information about the image. LL is representative of the image and is the sub-band most similar to the original image before it is decomposed. LH, HL, and HH are wavelets of variations vertically, horizontally, and diagonally [6,13,21,25].

Decomposition on DWT can be done using Eqs. (18) and (19) [29].

$$\Phi_{j,m,n}(x,y) = 2^{\frac{j}{2}}\varphi(2^j - m, 2^j - m, 2^j y - n) \tag{18}$$

$$\Psi_{j,m,n}^i(x,y) = 2^{\frac{j}{2}}(2^j - m, 2^j - m, 2^j y - n), i = \{H, V, D\} \tag{19}$$

where:  $i$  = wavelet {HL,LH,HH},  $m, n$  = size of image

Figure 2 is a description of sub-bands decomposition process using in DWT using filter Haar. where 2↓1: downsample columns and 1↓2: downsample rows.

Figure 2 shows the decomposition process at the first level. In its development, decomposition is also done on several levels to get the most appropriate results. But in this research decomposition is only done at one level. Where later each subband will be carried out further different processes, according to the characteristics of each subband.

While reconstruction is done by inverse DWT using Eq. (20).

$$f(x,y) = \frac{1}{\sqrt{MN}} \sum_m \sum_n w_\varphi(j_0, m, n) \varphi_{j_0,m,n}(x,y) + \frac{1}{\sqrt{MN}} \sum_{i=H,V,D} \sum_{j=j_0}^m \sum_n w_\psi^i(j_0, m, n) \Psi_{j_0,m,n}^i(x,y) \tag{20}$$

### 3. Proposed restoration scheme

The proposed restoration scheme in this study is to combine DWT and Akamatsu on the input image. The input image is an image that has been given manipulations such as

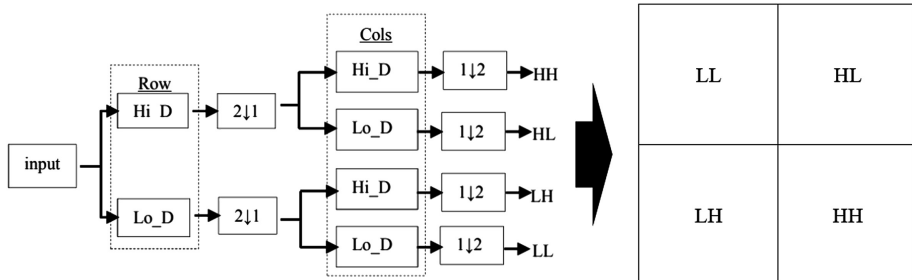


Figure 2. Sub-bands decomposition in DWT using filter Haar.

Gaussian noise, salt and pepper, and blurring. To be able to see more clearly the stages of the proposed scheme could see [Figure 3](#).

Here is a detailed stage of the proposed restoration process in [Figure 3](#):

1. Decompose the input image using DWT to get 4 subband LL, HL, LH, and HH.
2. Select subband HL, LH, and HH to apply Akamatsu transformation.
3. Apply y-direction Akamatsu transform on subband LH to get a new LH subband.
4. Apply x-direction Akamatsu transform on subband HL to get a new HL subband.
5. Apply two Akamatsu transforms on the HH subband. First, apply Akamatsu transform x-direction and the second Akamatsu transform y-direction. To obtain a new HH subband get the average value of both Akamatsu transforms with [Eq. \(21\)](#).

$$HH_{new} = \frac{(HH_{x-direction} + HH_{y-direction})}{2} \quad (21)$$

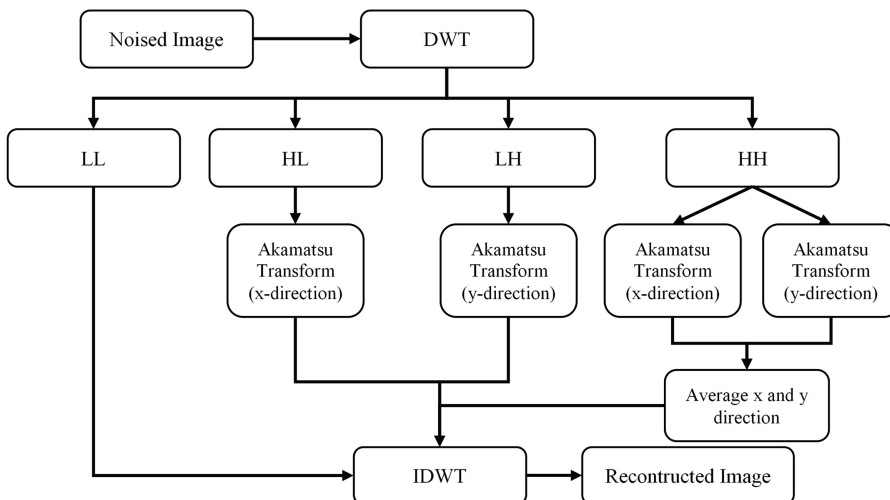
6. Perform a DWT inverse to get the image of the restoration.

#### 4. Implementation and results

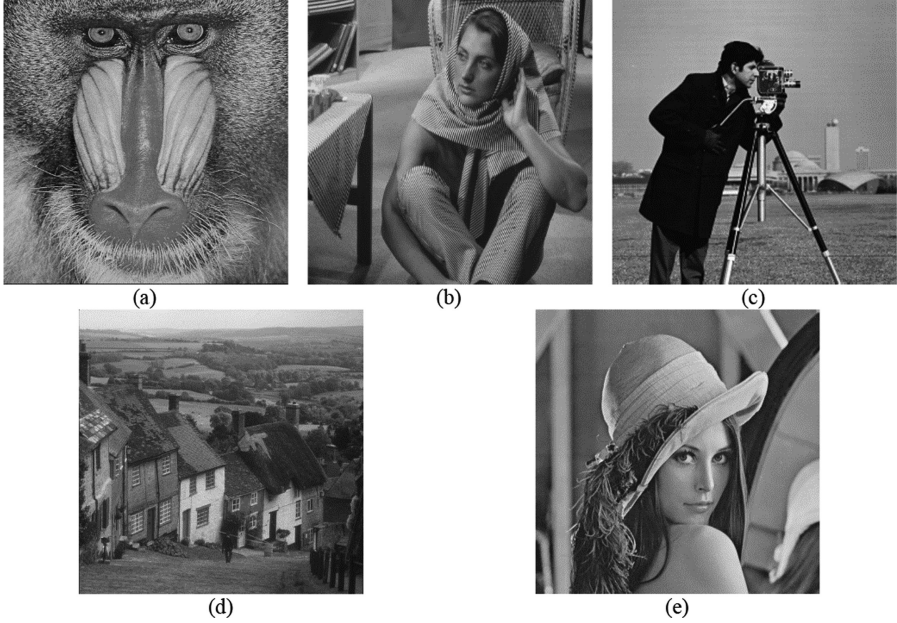
In this study, the proposed method will be simulated using five grayscale images with 512x512 size. This image is a standard image that can be downloaded on the internet. After the image is downloaded, the image is immediately used without preprocessing. In this study, the whole trial process uses Matlab R2015a software. The image used is shown in [Figure 4](#).

To test the proposed method, three kinds of manipulations are a blur, Gaussian noise, and salt and pepper using Matlab function. [Figure 5](#) shows a sample that has been given manipulation.

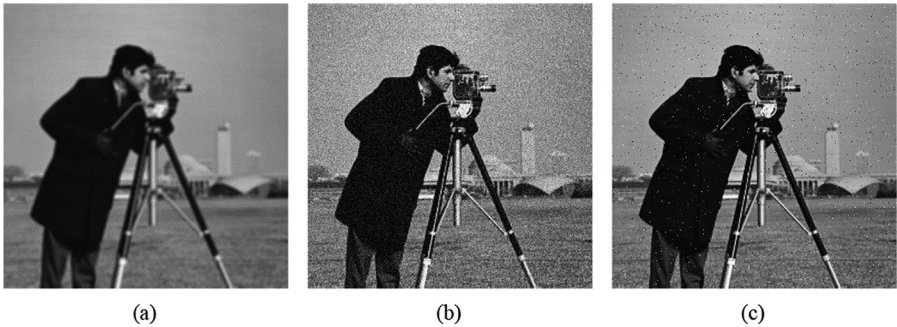
Peak to Signal Noise Ratio (PSNR) is used to determine the degree of damage to the noised image. The value of PSNR is obtained by comparing the original image and the noised image. [Equation \(22\)](#) is used to calculate PSNR.



**Figure 3.**  
The flow of Proposed  
Restoration Scheme.



**Figure 4.**  
Original Image Used  
{(a) Babbon; (b)  
Barbara; (c)  
Cameraman; (d)  
Goldhill; (e) Lena}.



**Figure 5.**  
Noise Cameraman  
Image {(a) Blur; (b)  
Gaussian Noise 0.01; (c)  
Salt and Pepper 0.01}.

$$PSNR = 10 \log_{10} \frac{255^2}{\sum_{h=1}^{H-1} \sum_{g=1}^{G-1} O_i(h, g) - N_i(h, g)} \quad (22)$$

where:  $h, g$  is the size of the image,  $O_i$  is an original image, and  $N_i$  is a noised image.

Table 1 shows the PSNR value of the original image after given three kinds of noise models.

Furthermore, the restoration of the noised image with the proposed method. The proposed method combines two methods, namely wavelet transform with Haar filter and then transformed again with Akamatsu transform. Wavelet transform is performed to get four subbands, where three of the four subbands are further processed using Akamatsu transform. The processing of the Akamatsu transformation in each DWT subband is adjusted to the characteristics of each subband to optimized restoration results. Perform the

Image	Noise Type	PSNR	Joint method for image restoration
Lena	Gaussian	20.0750	
	Salt & Pepper	25.3945	
	Blur	27.4355	
Barbara	Gaussian	20.0847	
	Salt & Pepper	25.4396	
	Blur	23.5422	
Baboon	Gaussian	20.0327	
	Salt & Pepper	25.6330	
	Blur	20.6008	
Cameraman	Gaussian	20.3898	
	Salt & Pepper	25.0744	
	Blur	26.1126	
Goldhill	Gaussian	20.1134	
	Salt & Pepper	25.2362	
	Blur	26.7294	

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**Table 1.**  
PSNR of the  
noise image.

Akamatsu y-direction transform on the LH subband, the x-direction on the HL subband and the xy-direction on the HH subband. This is done because the LH subband has a middle frequency horizontally, HL has a vertically middle frequency and HH has a diagonal high frequency [6,13,21,25]. Table 2 shows the results and PSNR values of the reconstructed images from the proposed restoration method.

## 5. Comparative and analysis

To evaluate the performance of the proposed method, the restoration of the proposed method were compared with the results of method restoration on the research of Karungaru et al. [5] and Megha et al. [6]. In this research, both methods have been replicated to compare the results of image restoration. This is done because the data set of images used are different, so it needs to be replicated so that it can be compared with the same image dataset. Comparative measurements were made with PSNR shown in Table 3.

As can be seen in Table 3, the PSNR value of the proposed method of restoration appears better than the two previous methods. Visually the difference of image result of restoration can be seen in Figures 6–8.

Based on the PSNR values contained in Table 3, proves that the results of image restoration by the proposed method appear superior to the previous method. Visually, Figures 6–8 also show that the proposed method appears to be superior. Only, in the blur, the results of restoration by the method [5] appear more clear and sharp when compared with the proposed method. However, the result of restoration on the method [6] creates new noise in the form of lines on the edge of the image, as shown in Figure 6(a).

## 6. Conclusions

Based on the results of the analysis and comparison, it can be concluded that the proposed method has better restoration compared to the method [5] and method [6] in all types of manipulations. The restoration of Gaussian noise and salt and pepper manipulations also reduce and smooth the noise generated. While the restoration using the method [5] even causes a blur effect. This happens because the Akamatsu transformation has characteristics that can change the image information by changing the differential value with the *down\_enh* value. While the proposed method applies changes to this information on sub-band HL, LH,

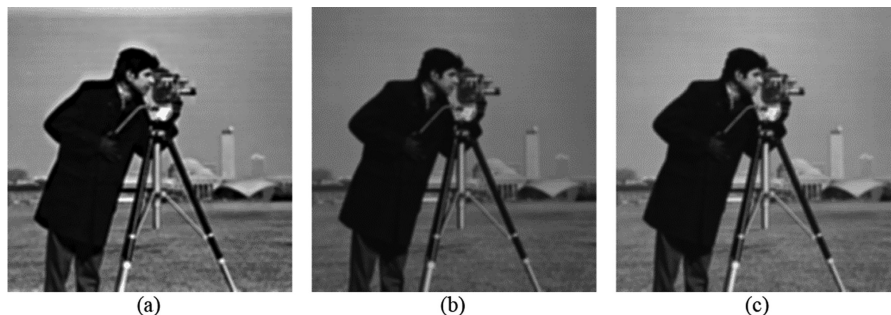


Image	Noise type		
	Blur	Gaussian	Salt and pepper
Baboon	 20.5861	 22.2379	 24.0013
Barbara	 23.5623	 23.2848	 25.5292
Cameraman	 26.1307	 25.3617	 29.2486
Goldhill	 26.7336	 24.7563	 28.4402
Lena	 27.4395	 24.9357	 28.9584

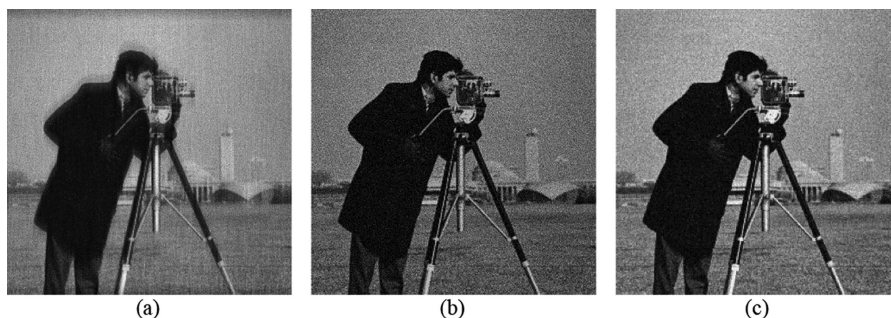
**Table 2.**  
Reconstructed results  
with PSNR values  
using the proposed  
method.

**Table 3.**  
Comparison of PSNR  
value proposed  
method, the method  
in [5] and the method  
in [6].

Image	Noise type	Proposed method	The method in [5]	The method in [6]
Lena	Gaussian	24.9357	23.4659	17.0212
	Salt & Pepper	28.9584	25.9581	19.0746
	Blur	27.4395	26.3382	19.2786
Barbara	Gaussian	23.2848	23.0617	18.0083
	Salt & Pepper	25.5292	25.1769	20.3796
	Blur	23.5623	23.0914	19.6397
Baboon	Gaussian	22.2379	22.1477	17.1517
	Salt & Pepper	24.0013	23.8401	19.2273
	Blur	20.5861	20.4113	17.5664
Cameraman	Gaussian	25.3617	22.8571	17.2854
	Salt & Pepper	29.2486	24.7714	18.6267
	Blur	26.1307	25.3826	18.8358
Goldhill	Gaussian	24.7563	23.2829	17.1602
	Salt & Pepper	28.4402	25.4686	18.7861
	Blur	26.7336	25.6285	18.6463



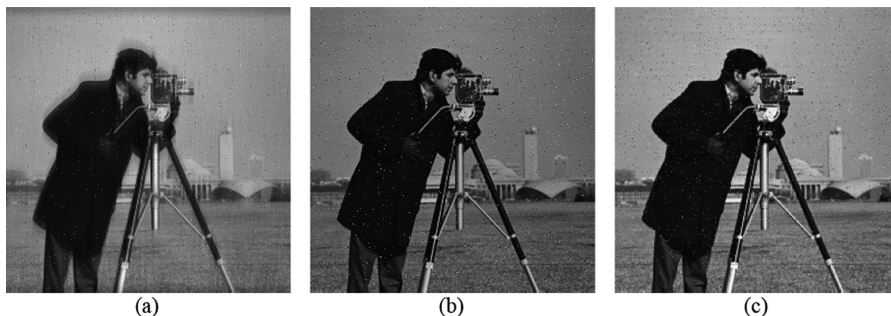
**Figure 6.**  
Reconstructed  
Cameraman Image  
from Blur  
manipulations  
{(a) Method in [5];  
(b) Method in [6];  
(c) Proposed Method}.



**Figure 7.**  
Reconstructed  
Cameraman Image  
from Gaussian Noise  
manipulations  
{(a) Method in [5];  
(b) Method in [6];  
(c) Proposed Method}.

and HH. So after the inverse DWT, the original image characteristics did not change significantly but can eliminate the noise that occurs in the image. Method [6] has a different character, although noise is reduced, the energy in the image of the restoration in the image is also reduced so that the image appears darker and the resulting PSNR value is less good. So it can be concluded that the proposed method can work better on various manipulations and can maintain the value of image approximation. In future studies, several wavelet filters can

**Figure 8.**  
Reconstructed  
Cameraman Image  
from Salt and Peppers  
manipulations {(a)  
Method in [5]; (b)  
Method in [6]; (c)  
Proposed Method}.



be used, as well as improved wavelet transforms and wavelet transform levels to analyze the results so that more optimal restoration is obtained. The use of other transformations can also be combined to get better image restoration.

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**Corresponding author**

De Rosal Ignatius Moses Setiadi can be contacted at: [moses@dsn.dinus.ac.id](mailto:moses@dsn.dinus.ac.id)